On Solving Pursuit Problems with Fuzzy Parameters

Bushra Hussien Aliwi
University of Babylon

On Solving Pursuit Problems with Fuzzy Parameters

Bushra Hussien Aliwi

Department of Mathematics-Education College for Pure Sciences -Babylon University-Iraq

Tel:+9647811197373 and email: bushrahussien.aliwi@yahoo.com

Abstract : This work is on suppose a solution for the pursuit problems through finding the true trajectory for some object from function(s) defined on interval(s) and change the parameters that defined on it by suppose the parameters as fuzzy numbers .

Key words: Pursuit Curves , Fuzzy Numbers ,Fuzzy Parameters .

Introduction :

A pursuit problem or what called "pursuit curves" is to find or drawing a bending path of an object(s) traces out when it moves to other fixed or moving point(s). There are numerous pursuit problems in different applications, many researches were introduced to solve these problems ,most try to solve the equation of pursuit curve depending on the object and the point reach if it constant or moving with many parameters control these problems .The age of these problems is old .Here we will see some pursuit problem and many explanations and processing that were presented by researchers .

A Pursuit curves were considered in general by the French scientist Pierre Bouguer in 1732.and the term "pursuit curve" was first defined by George Boole in his "Treatise on differential equations" in 1859. In 1638 ,in the story" The man in the moone" by Francis Godwin gave an idea for a pursuit curves through migration of 25 swans in trajectory toward the moon in curve [1] . Another problem is to model aircraft or submarines following targets, or predators following prey. For example, a hawk follows a sparrow, a large fish chases a small fish, or in trying a fox to catches a rabbit [1][2],also at the dog chases to the cat at one speed .

From the military pursuit problems is to control on path of self-guided missile intercept an enemy plane in the sky that couldn't be changes (path) at any time into the plane . From additional pursuit problems is the cooperative MAS study that consider four hunters against sole prey all are in random places the aim is to control on a prey that moving randomly[3], at each instant we can singularly study the motion of each one of actions without others or (pursuit curves) ,in condition, don't move out of the environment .Another example for pursuit problems for path into a moving body is what is the path of a hawk at point (x,y) sees a sparrow flying in one direction at speed $\boldsymbol{v}$,the hawk speed *w* is constant in direction to the sparrow if the starts point is (a,0), then t=0, and the sparrow at (0,0) [4] .

In this paper we want to solve similar pursuit problem for finding path of moving toward fixed point in different parameters influence on the speed of object (swimmer)and the conditions of environment (water) through take fuzzy numbers instead real numbers as parameters values ,so we must know about Fuzzy numbers (Triangular Fuzzy number(TFN),and Trapezoidal Fuzzy number(TrFN))that will suppose in the texted numerical example .

1. Fuzzy Numbers :

Fuzzy number is expressed as a fuzzy set with support of membership function is a part of real number set $\boldsymbol{R}$, and the membership function is $\boldsymbol{\mu(x)}: \boldsymbol{R} \to [\boldsymbol{0}, \boldsymbol{1}]$ with features to be convex and normal, that is[5][6];

$$\boldsymbol{max_x\, \mu(x) = 1} \quad \ldots (1)$$

and

$$\boldsymbol{\mu[\lambda x_1 + (1-\lambda)x_2] \geq min[\,\mu(x_1), \mu(x_2)]} \quad \ldots (2)$$

Where $\boldsymbol{x},\ \boldsymbol{x_1},\ \boldsymbol{x_2} \in \boldsymbol{R}, \forall \boldsymbol{\lambda} \in [\boldsymbol{0}, \boldsymbol{1}]$.

Since the boundary of this interval is infinite, the interval is also a fuzzy set. Generally a fuzzy interval is represented by two end points $\boldsymbol{a_1}$ and $\boldsymbol{a_3}$ and a peak point $\boldsymbol{a_2}$ as $(\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3})$

2.1 Triangular Fuzzy number(TFN)

Is the most popular one. It is a fuzzy number represented with three points as follows : given by three ordinary numbers $\widetilde{\boldsymbol{A}} = (\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3})$, [5][7]:

$$\boldsymbol{\mu(a)} = \begin{cases} \boldsymbol{0} & , \quad \boldsymbol{a < a_1} \\ \dfrac{\boldsymbol{a - a_1}}{\boldsymbol{a_2 - a_1}} & , \quad \boldsymbol{a_1 \leq a \leq a_2} \\ \dfrac{\boldsymbol{a_3 - a}}{\boldsymbol{a_3 - a_2}} & , \boldsymbol{a_2 \leq a \leq a_3} \\ \boldsymbol{0} & , \quad \boldsymbol{a > a_3} \end{cases} \quad \ldots .(3)$$

2.2 Trapezoidal Fuzzy Number(TrFN)

To know about the Trapezoidal Fuzzy Number(TrFN) ,if $\widetilde{\boldsymbol{B}} = (\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}, \boldsymbol{b_4})$,where $-\infty < b_1 \le \boldsymbol{b_2} \le \boldsymbol{b_3} \le \boldsymbol{b_4} < +\infty$,called TrFN. When $\boldsymbol{b_1} > 0$ it be positive trapezoidal fuzzy number, but when $\boldsymbol{b_2} = \boldsymbol{b_3}$ the TrFN change into TFN. the form for membership function is[8];

$$\mu(b) = \begin{cases} \dfrac{b - b_1}{b_2 - b_1} & , \quad b_1 \le b < b_2 \\ 1 & , \quad b_2 \le b \le b_3 \\ \dfrac{b - b_4}{b_3 - b_4} & , \quad b_3 < b \le b_4 \\ 0 & , \quad other \end{cases} \qquad \dots (4)$$

Its interval is represented by two end points $\boldsymbol{b_1}$ and $\boldsymbol{b_4}$ and a two peak points $\boldsymbol{b_2}, \boldsymbol{b_3}$ as $(\boldsymbol{b_1}, \boldsymbol{b_2}, \boldsymbol{b_3}, \boldsymbol{b_4})$.

2. **Pursuit Problem:** The trivial problem that known in these subjects is the reaching to target (fixed, or moving) point from fixed point through passing in changeable conditions(such water, winds, …,est.) .The parameters that control on the conditions are the speed values for body and water that be in opposite or same direction of moving ,and the distance between the two points (positions) .

3. **Applied Example :**. As a new trail and view to solve the pursuit problem in reference [9]. Suppose a person (can or cannot swimming) jumps into a river and swims toward the fixed point in water directly opposite start point. The person's swimming speed is effected by ability of swimming and emergency to continued (if he/she tired). The water motion speed is somewhat constant since it is effected by depth and width of river, volume of water in that place of rive ,… ,etc. .(the other parameters will be discussed in future researches)While the direction of swimming is toward the target point ,the question is ,How could be determine the swimmer's trajectory equation.

Here we will work on the effect of only two parameters (swimming speed and water speed) .

Put the width of river is $\boldsymbol{w}$ and the starting point with coordinate $(\boldsymbol{w}, \boldsymbol{0})$,and the target point is at origin point$(\boldsymbol{0}, \boldsymbol{0})$ .the time is the variable $\boldsymbol{t}$,the position of the swimmer through the path is point $(\boldsymbol{x}, \boldsymbol{y})$ while it change with the time it symbols as $(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{y}(\boldsymbol{t}))$.

Now let us imagine the path is part from unit circle that centered at origin the highest value for speed of water and the speed of swimmer is in point on surround of this region so these parameters will chose from rang(positive half from circle) as fuzzy numbers with highest value **1** (normal fuzzy set, positive values in **[0,1]**). That solved as ordinary differential equation ODF .

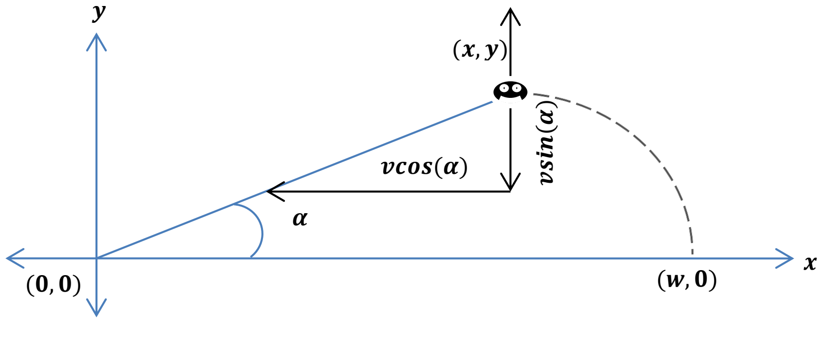


Figure (1) :Trajectory of swimmer through water

4. **Range of Parameters Values :**

The first parameter is speed of water $\boldsymbol{s}$ and second is swimmer's speed $\boldsymbol{v}$ are chosen to find fuzzy number defined by triangle membership function, and $\boldsymbol{s}$ is normal fuzzy set the fuzzy region for the values will depend on linguistic variables (hedges) that give fuzzification for real values ,that is the parameter $\boldsymbol{s}$ is not constant even it supposed constant in this research but it may be high or medium, or low .Also for the parameter $\boldsymbol{v}$ not all persons equaled in swimming speed and the person cannot continued in same speed in swimming(high or medium or low) ,the Table (1) will help us to control on choosing the parameters values through linguistic variables and then giving numerical values for each one ,it is available to take more values linguistically for parameters .

$$\tilde{\boldsymbol{V}} = [a_1, a_3] \quad \text{....(5)}$$

$$\tilde{\boldsymbol{S}} = [b_1, b_4] \quad \text{....(6)}$$

For some what the parameter $\boldsymbol{v}$ depend on $\boldsymbol{s}$ .the simmer must stop if the speed water is very high and his speed is low ,or must has precedes his speed to very high to be able to pass .

These linguistic values correspond to numerical values here we will take the values as in the table below for supposing the linguistic value; **"very $\boldsymbol{x}$" = $\boldsymbol{x^2}$** .

Table(1) Numerical values for Linguistic parameters values .

Linguistic Parameters evaluation	**Numerical values for MFs**
Very high	0.8-1
High	0.6-0.8
Medium(Ordinary)	0.4-0.6
Low	0.2-0.4
Very low(Stop)	0-0.2

In many applications of fuzzy language use to express the evaluation information based on knowledge and experience, that transform or describe the qualitative data into quantitative data as in Table (1) that give the linguistic parameters evaluate for numerical values of MFs .

By comparing the advantages and disadvantages of different quantitative data in terms of represent the experts language values, select trapezoidal fuzzy number to represent the expert preference information and the language evaluation of property values can be represented by trapezoidal fuzzy number, the specific are shown in Table(1) that were approximated to the values in [8].

The Linguistic description for the supreimum of minimum values for division two different fuzzy sets as in Table(2);

Table(2): Linguistic parameters values change .

Speed body / Speed water	**High**	**Medium**	**Low**
Very high	very high	very high	Very high
High	High	High	High
Medium	High	High	High
Low	High	Medium	Low
Very low(stop)	High	Medium	Low

Take $\tilde{\boldsymbol{V}}$ as TFN ,and $\tilde{\boldsymbol{S}}$ as TrFN normal set ,the chosen fuzzy sets features are ; be everywhere positive for positive values support fuzzy sets , symmetric(approximation values) ,and normal(with single maximum =1) .

5. **Fuzzy Solution :** The ordinary solution for this equation as ordinary differential equation is to suppose $\boldsymbol{u} = \frac{y}{x}$

So

$$\boldsymbol{u} + \sqrt{\boldsymbol{1} + \boldsymbol{u^2}} = \boldsymbol{k}\boldsymbol{x}^{-s/v} \quad \text{....(7)}$$

Then:

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{w}{2}[(\frac{x}{w})^{1-(s/v)} - (\frac{x}{w})^{1+(s/v)}] \quad \text{....(8)}$$

Put parameters as fuzzy sets ,then equation(8) will be ;

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{w}{2}[(\frac{x}{w})^{1-(\tilde{S}/\tilde{V})} - (\frac{x}{w})^{1+(\tilde{S}/\tilde{V})}] \quad \text{....(9)}$$

1. when $\tilde{\boldsymbol{S}} = \tilde{\boldsymbol{V}}$,or both has value $\boldsymbol{0}$ or when $\tilde{\boldsymbol{S}}$ has value $\boldsymbol{0}$ or $\tilde{\boldsymbol{V}}$ has value $\boldsymbol{0}$,that mean each has no member;

Then:

$$f(x) = 0$$

or $$f(x) = \infty$$

That has no meaning in practical applications .

2. While when $\tilde{S} = \tilde{V}$, when both $\tilde{S}$ and $\tilde{V}$ has value **1** ,that is mean:
$\mu_{\frac{\tilde{S}}{\tilde{V}}} = 1$,maximum of the division of two fuzzy set(Fuzzy intervals) .

Then $f(x)$ in equation (9) will be :

$$f(x) = \frac{w}{2}[1 - (\frac{x}{w})^2] \qquad \text{.....(10)}$$

We want to solve this equation put $w = 20$ m, that led us to the closed interval$[0, w]$ for change of values for variable x and then y ;

Then $$f(x) = 10\left[1 - \left(\frac{x}{20}\right)^2\right]$$

$$= \left[10 - \frac{x^2}{40}\right] = \left[\frac{400 - x^2}{40}\right] = \left[\frac{(20)^2 - x^2}{2(20)}\right]$$

3. Or when $\tilde{S}$ has value **1** for one or all members .

4. Or $\tilde{V}$,has value **1** for one or all members .

In the cases 3 and 4 each divisions of fuzzy intervals will has the maximum value =1 and the form be as;

$$f(x) = \frac{w}{2}\left[\left(\frac{x}{w}\right)^{1-1} - \left(\frac{x}{w}\right)^{1+1}\right] = \frac{w}{2}\left[1 - \left(\frac{x}{w}\right)^2\right]$$

Which is exactly the equation(10) ,and it is simple in solution .

Now for $\frac{\tilde{S}}{\tilde{V}}$ as division of two fuzzy numbers in the second case ,put $z = \frac{a}{b}$;

Where $(a, \mu_{\tilde{S}}(a)) \in \tilde{S}$ and $(b, \mu_{\tilde{V}}(b)) \in \tilde{V}$, where $\mu_{\tilde{S}}(a)$, $\mu_{\tilde{V}}(b)$ are membership functions for $\tilde{S}, \tilde{V}$ respectively .

And $a \in \tilde{S} = [a_1, a_3] \subset (0, \infty)$ and $b \in \tilde{V} = [b_1, b_4] \subset (0, \infty)$ open real valued intervals

$$[a_1, a_3]/[b_1, b_4] = [a_1/b_1 \cap a_1/b_4 \cap a_3/b_1 \cap a_3/b_4, a_1/b_1 \cup a_1/b_4 \cup a_3/b_1 \cup a_3/b_4 \qquad \text{....(11)}$$

Excluding the case values $b_1 = 0$, $b_4 = 0$

and then ;

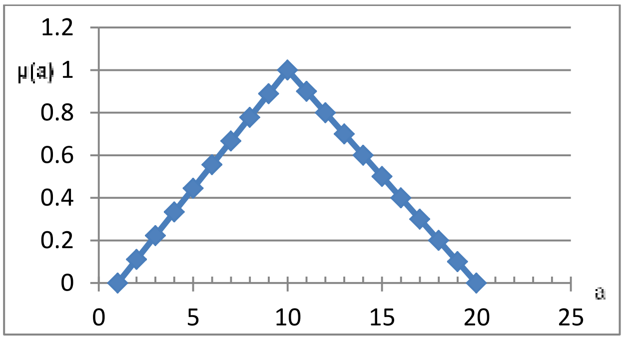
$$\mu_{\frac{\tilde{S}}{\tilde{V}}}(z) = \bigcup_{z=\frac{a}{b}} \left(\mu_{\tilde{S}}(a) \cap \mu_{\tilde{V}}(b)\right) \qquad \text{....(12)}$$

Which corresponds to the form?

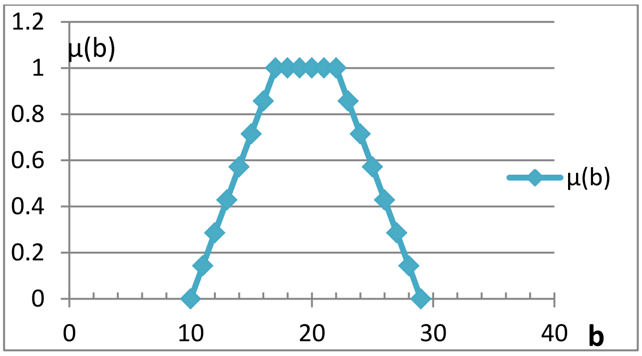
$$\mu_{\frac{\tilde{S}}{\tilde{V}}}(z) = \sup_{z=\frac{a}{b}} min \{\mu_{\tilde{S}}(a), \mu_{\tilde{V}}(b)\} \qquad \text{....(13)}$$

The values chosen as $\tilde{S} = (10, 17, 22, 29)$ **and** $\tilde{V} = (1, 10, 20)$ **.**

From table (4) the values for $\frac{\tilde{S}}{\tilde{V}}$ gave a good results in the intervals$[3, 8]$ for $\tilde{V}$ and $[12, 27]$ for $\tilde{S}$, so we can chose from these values for best implementation .



Figure(2): TFN for swimmer speed for values in table (4) .



Figure(3): TrFN for water speed for values in table (4) .

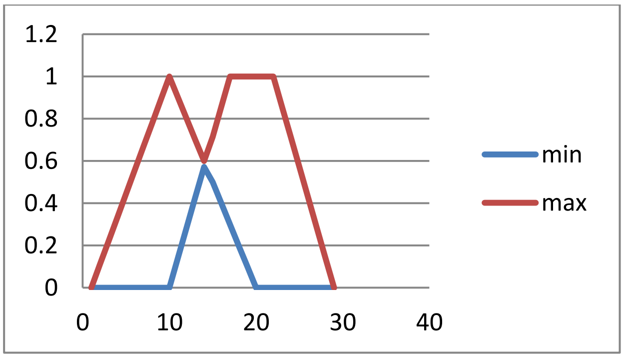
Table(3) Fuzzification through linguistic values for real valued parameters

.

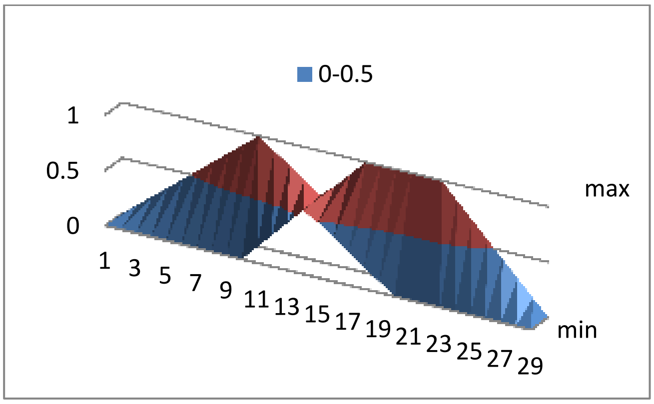
Linguistic Parameters evaluation	**Numerical values for MFs**
Very high	0.8-1
High	0.6-0.8
Medium(Ordinary)	0.4-0.6
Low	0.2-0.4
Very low(Stop)	0-0.2

See the tables(4,5,6,7) in the appendix for the correct values that give the features of values used in this example.

The intersection of MFs that gives the minimum values for speeds or stopping, while when take the maximum values as union of MFs gives highest values for applying, i.e. at MFs of union =1 .When a and b are equaled that is z=1 and the value of MF for z that answered as maximum for minimum between MFs which gives the medium values for application .

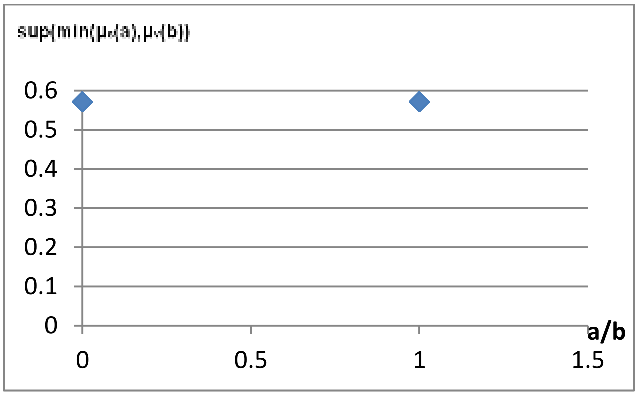


Figure(4) The intersection(minimum) values and union(maximum) values for MFs



Figure(5) Surface figuration for intersection and union values for MFs in figure(4) .

We must know that division of two TFN don't necessarily give a TFN, and so the division of two TrFN .



Figure(6); Supremiun of minimum values for membership functions

6. **Conclusion:** A pursuit curves are used in to modeling the paths for reaching to fixed or moving point(s) .These curves must be under condition(s) such influence of water ,The parameters that control on the conditions are the speed values for body and water ,and the distance between the two positions. Here the idea is to suppose these parameters with fuzzy values defined through MFs with some conditions to get on the general solution for equation of the pursuit problem that occurred, and then suppose arbitrary real values and logical in applied (for example positive for speed) to get on particular solution curve the resulted MF in equation has features of both MF but with larger support values . All curves that will result will help us to predicate the parameters if it unknown such speed of water or speed of body ,and they don't constant in reality .

7. **References**

1. Andrew J. Simoson," Pursuit Curves for Man in the Moone" ,The Mathematical Association of America, The College Mathematics Journal Vol.38,No.5, November 2007.
2. Russell Herman, "Solving Differential Equations Using Simulink". United States ,published by r. l. herman 2016 http://people.uncw.edu/hermanr/mat361/simulink
3. Zhou Pu-Cheng, Hong Bing-Rong , Huang Qing-Cheng and Javaid Khurshid , " Hybrid Multiagent Reinforcement Learning Approach: The Pursuit Problem", Information Technology Journal 5(6) ,Asian Network for Scientific Information , 2006 .
4. Russell Herman, "A First Course in Differential Equations for Scientists and Engineers". United States ,published by R. L. HERMAN 2016 .http://people.uncw.edu/hermanr/mat361/odebook
5. Reznik L. , "Fuzzy Controllers " ,Great Britain by Biddles Ltd,Guildford and King's Lynn ,1997
6. R. Ezzati, R. Enayati, A. Mottaghi, and R. Saneifard ,"A New Method For Ranking Fuzzy Numbers Without Concerning Of Real Numbers" , TWMS Jour. Pure Appl. Math., V.2, N.2, 2011, pp.256-270
7. Shang Gao, Zaiyue Zhang ,"Multiplication Operation on Fuzzy Numbers" ,Academy Publisher National Natural Science Foundation of China ,Journal of Software ,vol. 4, NO. 4, JUNE 2009
8. Sha Fu, Zhongli Liu, Hangjun Zhou, Dan Song and Yezhi Xiao,"Trapezoidal Fuzzy Number Attitude Indicators Group Decision Making Approaches Based on Fuzzy Language", Asian network for scientific Information ,Journal of Applied Sciences, 14: 2304-2308 ,2014 . http://scialert.net/abstract/?doi=jas.2014.2304.2308
9. Peter V. O'Neil, "Advanced Engineering Mathematics" 7th Edition, United States of America,2010

Appendix (1) :

Table(4):Language Evaluation for Numerical Values of Fuzzy Parameters

a	$\mu_s(a)$	Linguistic value	b	$\mu_v(b)$	Linguistic value	$\mu_s(a) \cap \mu_v(b)$	Linguistic value	$\mu_s(a) \cup \mu_v(b)$	Linguistic value	z	$\cup(\mu_s(a) \cap \mu_v($	Linguistic value
1	0	Stop		0	Stop	0	Stop	0	Stop	∞	0.571428571	Medium
2	0.111111111	Very low		0	Stop	0	Stop	0.111111111	Very low	∞	0.571428571	Medium
3	0.222222222	Low		0	Stop	0	Stop	0.222222222	Low	∞	0.571428571	Medium
4	0.333333333	Low		0	Stop	0	Stop	0.333333333	Low	∞	0.571428571	Medium
5	0.444444444	Medium		0	Stop	0	Stop	0.444444444	Medium	∞	0.571428571	Medium
6	0.555555556	Medium		0	Stop	0	Stop	0.555555556	Medium	∞	0.571428571	Medium
7	0.666666667	High		0	Stop	0	Stop	0.666666667	High	∞	0.571428571	Medium
8	0.777777778	High		0	Stop	0	Stop	0.777777778	High	∞	0.571428571	Medium
9	0.888888889	Very high		0	Stop	0	Stop	0.888888889	Very high	∞	0.571428571	Medium
10	1	Very high	10	0	Stop	0	Stop	1	Very high	1	0.571428571	Medium
11	0.9	Very high	11	0.142857143	Very low	0.142857143	Very low	0.9	Very high	1	0.571428571	Medium
12	0.8	High	12	0.285714286	Low	0.285714286	Low	0.8	High	1	0.571428571	Medium
13	0.7	High	13	0.428571429	Medium	0.428571429	Medium	0.7	High	1	0.571428571	Medium
14	0.6	High	14	0.571428571	Medium	0.571428571	Medium	0.6	High	1	0.571428571	Medium
15	0.5	Medium	15	0.714285714	High	0.5	Medium	0.714285714	High	1	0.571428571	Medium
16	0.4	Medium	16	0.857142857	Very high	0.4	Medium	0.857142857	Very high	1	0.571428571	Medium
17	0.3	Low	17	1	Very high	0.3	Low	1	Very high	1	0.571428571	Medium
18	0.2	Low	18	1	Very high	0.2	Low	1	Very high	1	0.571428571	Medium
19	0.1	Very low	19	1	Very high	0.1	Very low	1	Very high	1	0.571428571	Medium
20	0	Stop	20	1	Very high	0	Stop	1	Very high	1	0.571428571	Medium
	0	Stop	21	1	Very high	0	Stop	1	Very high	0	0.571428571	Medium
	0	Stop	22	1	Very high	0	Stop	1	Very high	0	0.571428571	Medium
	0	Stop	23	0.857142857	Very high	0	Stop	0.857142857	Very high	0	0.571428571	Medium
	0	Stop	24	0.714285714	High	0	Stop	0.714285714	High	0	0.571428571	Medium
	0	Stop	25	0.571428571	Medium	0	Stop	0.571428571	Medium	0	0.571428571	Medium
	0	Stop	26	0.428571429	Medium	0	Stop	0.428571429	Medium	0	0.571428571	Medium
	0	Stop	27	0.285714286	Low	0	Stop	0.285714286	Low	0	0.571428571	Medium
	0	Stop	28	0.142857143	Very low	0	Stop	0.142857143	Very low	0	0.571428571	Medium
	0	Stop	29	0	Stop	0	Stop	0	Stop	0	0.571428571	Medium

Note that the values in yellow color must given attention.

The relation as intersection of values determined as in this table to take minimum value for MFs as ;

Table(5):Language evaluation for intersection of values of Fuzzy Parameters

Speed body **Speed water**	**Very high**	**High**	**Medium**	**Low**	**Stop**
Very high	very high	High	medium	low	very low
High	high	High	medium	low	very low
Medium	medium	Medium	medium	low	very low
Low	ordinary	ordinary	ordinary	low	very low
Very low(stop)	very low	very low	very low	very low	very low

The relation as union of values determined as in this table to take maximum for minimum values for MFs as ;

Table(6):Language evaluation for union of values of Fuzzy Parameters

Speed body **Speed water**	**Very high**	**High**	**Medium**	**Low**	**Stop**
Very high	very high	high	medium	low	very low
High	very high	high	medium	low	very low
Medium	very high	high	medium	low	very low
Low	very high	high	medium	low	very low
Very low(stop)	very high	high	medium	low	very low

Table(7):Numerical evaluation for intersection between a and b values of Fuzzy Parameters

	b																			
a	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0	0	0	0	0	0	0	0	0	0								1	1	1
2	0.111111111	0	0	0	0	0	0	0	0	0	0.111111111	0.111111111	0.11111111	0.11111111	0.11111111	0.11111111	0.11111111	0.11111111	0.111111111	0.111111111

																	1	1		
3	0.222222222	0	0	0	0	0	0	0	0	0	0.142857143	0.222222222	0.222222222	0.222222222	0.222222222	0.222222222	0.222222222	0.222222222	0.222222222	0.222222222
4	0.333333333	0									0.142857143	0.285714286	0.333333333	0.333333333	0.333333333	0.333333333	0.333333333	0.333333333	0.333333333	0.333333333
5	0.444444444	0									0.142857143	0.285714286	0.428571429	0.444444444	0.444444444	0.444444444	0.444444444	0.444444444	0.444444444	0.444444444
6	0.555555556	0									0.142857143	0.285714286	0.428571429	0.555555556						
7	0.666666667	0									0.142857143	0.285714286	0.428571429	0.555555556						

8	0.777777778	0									0.142857143	0.285714286	0.428571429	0.555555556						
9	0.888888889	0									0.142857143	0.285714286	0.428571429	0.555555556						
10	1	0									0.142857143	0.285714286	0.428571429	0.555555556						
11	0.9	0.142857143									0.142857143	0.285714286	0.428571429	0.555555556						
12	0.8	0.285714286									0.142857143	0.285714286	0.428571429	0.555555556						
13	0.7	0.428571429									0.142857143	0.285714286	0.428571429	0.555555556						
14	0.6	0.571428571									0.142857143	0.285714286	0.428571429	0.555555556						
15	0.5	0.714285714									0.142857143	0.285714286	0.428571429	0.555555556						
16	0.4	0.857142857									0.142857143	0.285714286	0.4	0.555555556						
17	0.3	1									0.142857143	0.285714286	0.3	0.555555556						
18	0.2	1									0.142857143	0.2	0.2	0.5555						

														55556						
19	0.1	1									0.1	0.1	0.1	0.5555555556						
20	0	1									0	0	0	0.5555555556						