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On condensed set in ideal topological spaces

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Abstract

The research relied on two basic and new concepts, starting from the first concept of condensed, which is divided into four different types, where the relationships between them were studied, as well as their relationship to the concept of I-dense, I-irresolvable and other basic concepts in the ideal topological spaces. The second concept is the I-extremal disconnected, and we have shown relationship with the first concept.

Subject Classification: *32C18, 37J05, 46B85.*

Keywords: *Ideal space, Sub condensed, I-dense, I-resolvable.*

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1. Introduction

The concepts defined here, in addition to their characteristics, depend primarily on the concept of ideal defined by K. Kuratowski [1], also the concept of local function defined by R. R.Vaidyanathswamy [2], where there is a constellation of researchers who have enriched this function, whether it is related to fuzzy sets or related with the soft sets [3,4]. An important concept that played a fundamental role in both pure and applied mathematics. The idea of proximity, known by Riesz [5]. So, the local function of any subset H of universal set X is $H^* = \{x \in X, \forall k \in \tau(x)$ s.t $k \cap H$ not number of ideal $I\}$ and $\psi(H) = X - (X - H)^*$, E. Hayashi [6], defined *- dense-in-itself if $k \subset k^*$, also the *-prefect, if $k = k^*$ and the ideal space is called co-dense if $\tau \cap I = \varphi$. J. Dontchev [7], defined I-dense if $k^* = X$, and P. Samuels [8], also defined co-dense if $X^* = X$, but D. Jankovic and T. R. Hamlett [9], prove that the equivalent of $\tau \cap I = \varphi$ if $X^* = X$ and the ideal space satisfy, this is called Hayashi-Samuel spaces.

Proposition 1.2 : [9] Let (X, τ, I) be ideal, the following statement, are hold:

1. $\forall V \in \tau, V \subset \psi(V)$.
2. If (X, τ, I) is Hayashi-Samuel space $\psi(k) \subseteq \mathrm{cl}(k)$, for each $k \subseteq X$.
3. $\psi(H) \subset H^* \subset \mathrm{cl}(H)$, for any $H \subseteq X$.

2. Building Foundations

Definition 2.1 : Let (X, τ, I) be ideal and $\mathcal{A} \subseteq X$,

1. Super condensed if $\psi(\mathcal{A}^*) \subset \psi(\mathcal{A})$.
2. Sub condensed if $[\psi(\mathcal{A})]^* = \mathcal{A}^*$.
3. condensed if super condensed and sub condense.
4. non - condensed if $\psi(\mathcal{A}^*) = \varphi$.
5. ψ - congruent if $\psi(\mathcal{A}^*) = \mathcal{A}$.

Example 2.2 : Let $I = \{\mathcal{A} \subseteq R; \mathcal{A}$ is finite$\}$, be an ideal on the real number R, so $\psi(\mathcal{A}^*) = \psi((IQ^*))$ a not non-condensed sets and for any $r \in R$, the singleton set $\{r\}$ is non-condensed set.

Example 2.3 : Let $X = \{x_a, x_b, x_c\}$, $\tau = \{\varnothing, X, \{x_c\}, \{x_b\}, \{x_c, x_b\}\}$, $I = \{\varnothing, \{x_a\}\}$ Then $\{x_a, x_c\}$, is super condensed but not sub condensed.

In the following proposition, the most important properties and relationships of the concepts of condensed.

Proposition 2.4 : Let (X,τ,I) is ideal. For any subset $\mathcal{A}\subseteq X$, following are hold:

1. If $\mathcal{A}$ is $*-$perfect, then $\mathcal{A}$ is super condensed.
2. If $\mathcal{A}$ is $\psi-$ conguront then $\mathcal{A}$ is sub condensed.
3. If $\mathcal{A}$ is super condensed, then $\mathcal{A}^c$ is sub condensed.
4. If $\mathcal{A}^c$ is subcondensed, then $\mathcal{A}^c$ is super condensed.
5. If $\mathcal{A}$ is condensed, if $\mathcal{A}^c$ is condensed.
6. If $\mathcal{A}$ is super condensed, then $\mathcal{A}^c$ is *I*-dense.
7. If $\mathcal{A}$ is sub condensed and $\mathcal{A}^c$ is non- condensed, then $\mathcal{A}$ is *I*-dense.
8. If $\mathcal{A}$ is $\psi-$ conguront and $\mathcal{A}^c$ is non- condensed, then $\mathcal{A}$ is *I*-dense.
9. If $\mathcal{A}$ is non- condensed and perfect, then $\mathcal{A}$ is *I*-dense.

Theorem 2.5 : *Let (X,τ,I) be Hayashi-Samuel, the following statement are true for any closed set $\mathcal{A}$:*

1. *$\mathcal{A}$ is sub condensed if $\mathcal{A}\subset[\psi(\mathcal{A})]^*$.*
2. *$\mathcal{A}$ is sub condensed if $[\psi(\mathcal{A})]^*\sqsupseteq\mathcal{A}\sqsupseteq\psi(\mathcal{A}^*)$.*
3. *$\mathcal{A}$ is super condensed if $\mathcal{A}\sqsupseteq\psi(\mathcal{A}^*)$.*

Proposition 2.6 : In any Hayashi-Samuel space (X,τ,I) having the properties:

1. If $\mathcal{A}$ is $\psi-$ conguront then $\mathcal{A}$ isopen super condensed.
2. If $\mathcal{A}$ is non- condensed and perfect, then $(X-\mathcal{A})^*$ is *I*-dense.
3. If $\mathcal{A}\in I$, then $\mathcal{A}$ is non- condensed.
4. For any $\mathcal{A}\subseteq X,\psi(\mathcal{A}^*)\subseteq[\psi(\mathcal{A})]^*$.

Proposition 2.7 : Let (X,τ,I) is ideal, the following are true.

1. If $\mathcal{A}$ is non- condensed, then every subset of $\mathcal{A}$ is also non- condensed.
2. The intersection of any finite collection of non- condensed subset of X is non- condensed.
3. If $\mathcal{A}$ is non- condensed, then for any $J\in I$, then $\mathcal{A}-J$ and $\mathcal{A}\bigcup J$ are non- condensed subset of X.
4. For any ideal $J\subset I$ and $\mathcal{A}$ is non- condensed w.r.t. I.

5. If $\mathcal{A}$ is non- condensed, then $\mathcal{A} \wedge B$ is non- condensed, for any subset B of X.
6. If $\mathcal{A}$ is non- condensed, then $\psi(\mathcal{A}^c)$ is I-dense.
7. If $\mathcal{A}$ is non- condensed, then $\mathcal{A}^*$ is non- condensed.

Proposition 2.8 : Let (X, τ, I) be a ideal, the following are correct.

1. If $\mathcal{A}$ is non- condensed if $X - \mathcal{A}^X$ is I-dense.

2. $X - \mathcal{A}$ is non- condensed if $\psi(\mathcal{A})$ is I-dense.

3. If $\mathcal{A}$ is non- condensed, then $\forall u \in \tau(x)$, $\exists y \neq x \,\text{in}\, \psi(X - \mathcal{A})$ s.t $\psi(X - \mathcal{A}) \bigcap u_x \neq \varphi$.

Proposition 2.9 : Let (X, τ, I) Hayashi-Samuel space

1. For any $u \in \tau, (u, \tau_x)$ is irresoluble.
2. For any $u \in \tau^*$ is I-irresoluble and $\mathcal{A} \in \mathcal{D}^*(x)$ then $\mathcal{A}^c$ is non-condensed.

Definition 2.10 : [10] Let (X, τ, I) be ideal and $\mathcal{A} \subseteq X$, then *-frontier set of $\mathcal{A}$ is denoted by $F^*r(\mathcal{A})$ and $F^*r(\mathcal{A}) = \mathcal{A}^* \bigcap (X - \mathcal{A})^*$.

Through Example 2.2, we see that $F^*r(Q) = R$ and in the Example 2.3, we see that $F^*r(\{x_a, x_c\}) = \varphi$.

Proposition 2.11 : Let $(X, \tau, I))$ is a ideal and $\mathcal{A}$ is I-dense $\subseteq X$. If $\psi(F^*r(\mathcal{A})) = \varphi$ then $\mathcal{A}^c$ is non- condensed set.

Corollary 2.12 : *In the Hayashi-Samuel space* $\forall J \in I$, *then* J^c *is non- condensed set*.

Definition 2.13 : The ideal space is called I-extremely disconnected, if every open, the local function of it, is all open set. Through Example 2.3, we see that (X, τ, I) is not I-extremely disconnected, but If we take $\tau = \{\varnothing, X, \{x_c\}\}$ and $I = \{\varnothing, \{x_a\}\}$, we get that (X, τ, I) is I-extremely disconnected.

Proposition 2.14 : Let (X, τ, I) is a ideal, if X is I-extremely disconnected then every pair of disjoint open sets having disjoint local function.

Corollary 2.15 : *Let* (X, τ, I) *Hayashi-Samuel if X is I-extremely disconnected, then* $\forall u, v \in \tau$ s.t $u \bigcap v = \varphi$, t $u \bigcap v$ *is non-condensed set*.

3. Discussion and Conclusion

We not that these sets are flexible in the process of use, so that they can be used to defining new separation axiom, which is defined topological spaces as well as defined ideal spaces. On the other hand, the weakly open sets can be an defined by using condensed sets [11,12,13,14].

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