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To cite this article: Mustafa Hasan Hadi, May Alaa Abdul-Khaleq AL-Yaseen & Luay A. Al-Swidi (2021) Forms weakly continuity using weak  $\omega$  – open sets, Journal of Interdisciplinary Mathematics, 24:5, 1141-1144, DOI: [10.1080/09720502.2020.1790747](https://doi.org/10.1080/09720502.2020.1790747)

To link to this article: <https://doi.org/10.1080/09720502.2020.1790747>



Published online: 07 Oct 2020.



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## Forms weakly continuity using weak $\omega$ – open sets

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### Abstract

In this paper, we introduced new types of weak continuity by weakly open sets we have named perfectly  $\omega$ -continuous, contra- $\omega$ -continuous and approximately  $\omega$ -continuous. As well as we created the relationship between them and also provided some theorems about these types of weak continuous.

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*Subject Classification:* 54C05, 54C08, 54C10.

*Keywords:* ap- $\omega$ -continuous, ap- $\omega$ -open, ap- $\omega$ -closed, ultra- $\omega$ -open map.

### 1. Introduction

In 1970, Levine [5] presented his notion generalization closed sets ( $g$  – closed) in a topological space Dontcher. J.[2] introduced notion contra continuous map by open set in 1996. Noiri.T, Al-Omari.A. and Noorani. M. S. M. [7] introduced type of the sets are called weak  $\omega$  – open sets by  $\omega$  – closedset.

To know more properties about weak  $\omega$  – open see [7]. In this research, we have circulated  $\omega$  – closed and  $\omega$  – open we have name  $g\omega$  – closed and  $g\omega$  – open respectively, through which we have known a new kind of continuity we have named contra –  $\omega$  – continuous, perfectly

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$\omega$  – continuous and approximately  $\omega$  – continuous we have shown the relationship between them and also presented some theorems about them. We also introduced new types of closed maps and called them approximately  $\omega$  – closed map and ultra –  $\omega$  – closed map, perfectly –  $\omega$  – closed map, we have proved some of the characteristics that concern them. In addition [1] introduced new types of continuous functions and names of generalizations of continuity. In a year 2013 C.Janaki and V. Jeyanthi introduce and investigate the notion of contra  $\pi$ gr-continuous,almost contra  $\pi$ gr-continuous functions and discussed the relationship with other contra continuous functions and obtained their characteristics [4]. Also Md. Hanif Page [3] Study Almost Contra gs-continuous functions. Luay. A. Al Swidi and Mustafaknew the types of continuity using weak  $\omega$  –  $\omega$  – Open Sets

## 2. Auxiliary Results

**Definition 2.1.** [7] : Let  $x$  be a point  $x$  in  $(X, T)$ . We call  $x$  is condensation point of  $A$  if for every  $U$  in  $T$  and  $x$  in  $U$ , the set  $U \cap A$  is uncountable.

**Definition 2.2.** [7] : Let  $A \subseteq X$ . If  $A$  contains all its condensation points then it is called  $\omega$  – closed. The complement of the  $\omega$  – closed set is called  $\omega$  – open”.

**Remark 2.3.** [7] : It is clear that any open set is  $\omega$  – open

**Definition 1.4.** [7] : Let  $A$  be a subset of a space  $X$ .  $A$  is called”

1.  $b$  –  $\omega$  – open if  $A \subseteq \text{int}_{\omega}(cl(A)) \cup cl(\text{int}_{\omega}(A))$  and we called  $b$  –  $\omega$  – closed set to the complement of the  $b$  –  $\omega$  – open set .
2.  $\beta$  –  $\omega$  – open if  $A \subseteq cl(\text{int}_{\omega}(cl(A)))$  and we called  $\beta$  –  $\omega$  – closed set to the complement of the  $\beta$  –  $\omega$  – open set .
3.  $\alpha$  –  $\omega$  – open if  $A \subseteq \text{int}_{\omega}(cl(\text{int}_{\omega}(A)))$  and we called  $\alpha$  –  $\omega$  – closed set to the complement of the  $\alpha$  –  $\omega$  – open set .
4. pre –  $\omega$  – open if  $A \subseteq \text{int}_{\omega}(cl(A))$  and we called pre –  $\omega$  – closed set to the complement of the pre –  $\omega$  – open set.

### 3. The Main Result

**Definition 3.1 :** Let  $(\mathfrak{X}, T)$  be a topological space. Let  $M \subseteq \mathfrak{X}$ . We called  $M$  a generalized  $\omega$  – closed set ( $g\omega$  – closed set) if  $cl_{\omega}(M) \subseteq S$ , whenever  $M \subseteq S$  and  $S$  is an  $\omega$  – open set.

The complement of the generalized  $\omega$  – closed set is called generalized  $\omega$  – open set ( $g\omega$  – open set).

**Definition 3.2 :** A map  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is called to be approximately  $\omega$  – continuous (ap –  $\omega$  – continuous) map if  $cl_{\omega}(G) \subseteq e^{-1}(S)$  whenever  $S$  is an  $\omega$  – open subset  $\mathcal{F}$  and  $G$  is  $g\omega$  – closed subset of  $\mathfrak{X}$ , s.t  $G \subseteq e^{-1}(S)$ .

Let us now recall the definition of the  $\omega$  – continuous map.

**Definition 3.3 :** A function  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is called to be  $\omega$  – continuous if for any  $r \in \mathfrak{X}$  and any  $\omega$  – open set  $D$  of  $\mathcal{F}$  containing  $e(r)$ , there exists  $\omega$  – open set  $S$ , s. t  $e(S) \subseteq D$ .

**Definition 3.5 :** A map  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is called to be contra –  $\omega$  – continuous if  $e^{-1}(D) \subseteq \mathfrak{X}$  is  $\omega$  – closed set for any  $\omega$  – open set  $D$  in  $\mathcal{F}$ .

**Theorem 3.6 :** Let  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  be a map. If  $e$  is contra –  $\omega$  – continuous map, then  $e$  is ap –  $\omega$  – continuous map.

*Proof :* Suppose that  $S \subseteq \mathcal{F}$  is an  $\omega$  – open set and  $G \subseteq \mathfrak{X}$  is  $g\omega$  – closed set s. t.  $G \subseteq e^{-1}(S)$ . Therefore  $cl_{\omega}(G) \subseteq cl_{\omega}(e^{-1}(S))$ . Since  $e$  is contra –  $\omega$  – continuous map, we've got  $e^{-1}(S)$  is an  $\omega$  – closed and hence  $cl_{\omega}(G) \subseteq cl_{\omega}(e^{-1}(S)) = e^{-1}(S)$ . . Thus  $e$  is ap –  $\omega$  – continuous map.

**Definition 3.7 :** A map  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is called to be approximately  $\omega$  – closed set (ap –  $\omega$  – closed set) if  $e(G) \subseteq \text{int}_{\omega}(D)$  whenever  $D \subseteq \mathcal{F}$  is a  $g\omega$  – open set,  $G \subseteq \mathfrak{X}$  is  $\omega$  – closed  $\dot{\cup}$  – closed set and  $e(G) \subseteq D$ .

**Definition 3.8 :** A map  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is said to be approximately  $\omega$  – open (ap –  $\omega$  – open) if  $cl_{\omega}(G) \subseteq e(S)$ , whenever  $S$  is an  $\omega$  – open subset of  $\mathcal{R}$ ,  $G$  is a  $g\omega$  – closed subset of  $\mathcal{F}$  and  $G \subseteq e(S)$ .

**Definition 3.9 :** A function  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is said to be ultra –  $\omega$  – closed if any  $M \subseteq \mathfrak{X}$  is  $\omega$  – closed set,  $e(M)$  is  $\omega$  – open set in  $\mathcal{F}$ .

**Definition 3.10 :** A function  $e : (\mathfrak{X}, T) \rightarrow (\mathcal{F}, T')$  is said to be ultra –  $\omega$  – open if any  $M \subseteq \mathfrak{X}$  is  $\omega$  – open set,  $e(M)$  is  $\omega$  – closed set in  $\mathcal{F}$ .

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*Received May, 2020*