Journal of Interdisciplinary Mathematics

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjim20

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Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
DOI : 10.1080/09720502.2022.2040855

On fuzzy intense separation axioms in fuzzy ideal topological space

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Abstract

The research is based on three stages in the construction process. The first stage is to give new definitions for three different types of Fuzzy sets that we called FI-set, FC-set and SFI-set. The second stage is to use what was defined in the previous stage in order to construct also three new definitions of internal, external and boundary fuzzy sets. Finally, the new concepts and separation axioms were defined using the definitions from the previous two stages.

Subject Classification: *94D05, 39B82, 54D10.*

Keywords: *Fuzzy sets, Ideal space, Fuzzy ideal space, Fuzzy point.*

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1. Introduction

A Fuzzy sets first introduced by [1] in 1956, this concept has attracted many researchers to study and deepen it and to provide new definitions based on it. In 1999 D. Molodtsov tackle soft set theory which depends on ambiguous and unclear mathematical objects. Also, in this paper, the connection between these two concepts was made. A fuzzy sets and soft set and new ideas and theories and examples were presented that illustrate the relationship between the two concepts. These new definitions and ideas can also be generalized and used in the two researches [3,4].

A fuzzy set $\tilde{A} = \{(X, f_A(x); \forall x \in X, f(x) \in [0,1]\},$ where is a fuzzy from X to [0,1] and a fuzzy point

$$P_x^{\alpha}(y) = \begin{cases} \alpha & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

And $P_x^{\alpha}(y) \in \tilde{A}$ iff $\alpha \leq f_A(x), \forall x \in X,$ $\tilde{A} \vee \tilde{B} = \{(x, \max(f_A(x), g_B(x))); \forall x \in X\},$ $\tilde{A} \wedge \tilde{B} = \{(x, \min(f_A(x), g_B(x))); \forall x \in X\}.$ And $1_x = \{(x,1); \forall x \in X\};$ $0_x = \{(x,0); \forall x \in X\};$ finally $1_x - \tilde{A} = \{(x, 1 - f_A(x)); \forall x \in X\}$ [5] and we denoted Γ^x be the family of all fuzzy sets defined on X in other word any fuzzy set $\tilde{A}$ is subset of $X * [0,1],$ so we see that Γ^x = power of $X * [0,1] = P(X * [0,1]).$ The fuzzy ideal topology $(1_x, \tilde{\tau}_f, \tilde{J}_f),$ where $\tilde{\tau}_f$ is fuzzy ideal topology defined by change [5] and $\tilde{\tau}_f$ is the fuzzy ideal defined by Sarkar [5], the collection $N(P_x^{\alpha})$ is the system fuzzy neighborhood of P_x^{α} and $\tau_f(P_x^{\alpha}) = \{\tilde{U} \in \tau;\ P_x^{\alpha} \in \tilde{U}\}$ the symbolizes $\tilde{A}q\tilde{H}$ iff $\exists y \in X\, s.t.\ f_A(y) + g_B(y) > 1$ which call $\tilde{A}$ quasi coincident with $\tilde{B}$ and $\tilde{A}q\tilde{H}$ iff; $\forall x \in X\, s.t.\ f_A(x) + g_B(x) \leq 1.$

2. Building Foundations

Definition 2.1 :

1. $\tilde{A}^* = \vee\{P_x^{\alpha}; \forall U \in q - N(P_x^{\alpha}),\ \exists y \in X\, s.t.\, f_u(y) +\ g_u(y) - 1 > h_u(y),\ \forall \tilde{I} \in \tilde{J}\}.$ Now generate a new types of fuzzy $\psi -$ operator.
2. $\psi(\tilde{A}) = \tilde{1} - (\tilde{1} - \tilde{A})^*$ iff $= \vee\{P_x^{\alpha}; \forall \tilde{U} \in \tilde{\tau}_f(P_x^{\alpha}),\ \forall y \in X,\ \{f_u(y) -\ g_u(y), 0\}\, s.t.\, h_J(y)$ for some $F \in \tilde{\tau}_f\}.$ And more information and properties of fuzzy $\psi -$ operators and fuzzy local function it is possible to review the research papers [6,7].

Definition 2.2 : A fuzzy set $\tilde{K}$ in the fuzzy ideal topological space $(\tilde{1}_x, \tilde{\tau}_f, \tilde{J}_f)$ is said to be:

1. Fuzzy intense set (*FI* –set), if $\exists$ fuzzy point P_x^α and $\tilde{U} \in \tilde{\tau}_f(P_x^\alpha)$, $\forall y \in X$, $\max\{f_u(y) \quad - g_{\tilde{k}^*}(y), \quad 0\} \le h_J(y)$, for some $J \in \tilde{J}_f$. The collection of all *FF*-sets denoted by *FIC*.
2. Fuzzy complement intense set (*FC*-set), if $\exists$ fuzzy point P_x^α s.t. $\tilde{U} \in \tilde{\tau}_f(P_x^\alpha)$, $\forall y \in X$, $\max\{f_u(y) - g_{(1-k)^*}(y), \quad 0\} \le h_J(y)$, for some $J \in \tilde{J}_f$. The collection of all *FC*-sets denoted by *FCC*.
3. Specifically fuzzy intense set (*SFI*-set), if $\forall$ fuzzy point P_x^α, $\tilde{U} \in q - N(P_x^\alpha)$, $\exists y \in X$ s.t. $f_u(y) - g_{k^*}(y) > h_J(y)$, for any $J \in \tilde{J}_f$. The family of all *SFI*-set denoted by *SFIC*.
4. Fuzzy inner (finer) of a fuzzy point P_x^α, if $\exists$ a fuzzy open $\tilde{U}$ s.t. $\alpha \le f_u(x) \le g_k(x)$.
5. The fuzzy ideal space $(\tilde{1}_x, \tilde{\tau}_f, \tilde{J}_f)$ is said to be fuzzy highly connected (*Fh*-connected) if satisfy the feature $\forall \tilde{U} \in \tilde{\tau}$ iff $\widetilde{U^*} = \tilde{1}$.

Example 2.3 : Let $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ be fuzzy ideal space and $X = \{1,2,3\}$,

$$A = \{1,2\},\ f_A(x) = \begin{cases} 0.5 & \text{if } x=1 \\ 0.3 & \text{if } x=2 \\ 0 & \text{if } x=3 \end{cases}$$

$$D = \{2,3\},\ g_D(x) = \begin{cases} 0.5 & \text{if } x=3 \\ 0.7 & \text{if } x=2 \\ 0 & \text{if } x=1 \end{cases},\ \tilde{D} = \{(1,0),(2,0.7),(3,0.5)\}.$$

$$C = \{1,3\},\ k_C(x) = \begin{cases} \dfrac{x^2}{10} & \text{if } x \in C \\ 0 & \text{if } x \notin C \end{cases},\ \tilde{C} = \{(1,0.1),(2,0),(3,0.9)\}.$$

$$J = \{2,3\},\ h_i(x) = \begin{cases} \dfrac{2x+1}{10} & \text{if } x \in J \\ 10 & \text{if } x \notin J \end{cases},\ \tilde{J} = \{(1,0),(2,0.5),(3,0.7)\}.$$

$$\tilde{D} \wedge \tilde{C} = \{(1,0),(2,0),(3,0.5)\},$$

$$\tilde{D} \vee \tilde{C} = \{(1,0),(2,0.7),(3,0.9)\},$$

$$\tilde{\tau}_f = \{\tilde{1}, \tilde{0}, \tilde{D}, \widetilde{C}, \tilde{D} \wedge \tilde{C}, \tilde{D} \vee \tilde{C}\},$$

$$\tilde{J}_f = \{\tilde{0}, \tilde{J}\} \cup \{\tilde{J}, \tilde{J} \le \tilde{J}\},$$

$$\psi(A^*) = \{(1,0),(2,0.7),(3,0.5)\},$$

so $\tilde{A}$ is *FI*- set. The important properties of the *FI*-sets, *FC*-sets and *SFI*-sets are given in the following proposition.

Proposition 2.4 : Let $(\tilde{1},\tilde{\tau}_f,\tilde{J}_f)$ be a fuzzy ideal topological space. The following features are hold.

1. Every fuzzy set is, either *FI*- set or *FC*- set.
2. If $\{\widetilde{K_\lambda},\lambda\in\Gamma\}$ are the family of *FI*- sets, then $\vee_{\lambda\in\Gamma}\widetilde{K_\lambda}$ is a *FI*- set.
3. If $\vee_{\lambda\in\Gamma}\widetilde{K_\lambda}$ is a *FC*- set, then $\forall,\ \lambda\in\Gamma,\ \widetilde{K_\lambda}$ is *FC*- set.
4. If $\{\widetilde{K_\lambda},\lambda\in\Gamma\}$ are the family of *FC*- sets, then $\wedge_{\lambda\in\Gamma}\widetilde{K_\lambda}$ is a *FC*- set.
5. If $\tilde{J}_f\wedge\tilde{\tau}_f=\tilde{0}$ and $\tilde{H}$ is *FC*- set, then $\exists$ a fuzzy point P_x^α and $\tilde{U}\in\tilde{\tau}_f(P_x^\alpha)$, $\forall y\in X$, $\max\{f_u(y)-g_H(y),0\}\ \leq K_J(y)$, for some $J\in\widetilde{J_f}$.
6. If $\tilde{J}_f\wedge\tilde{\tau}_f=\tilde{0}$, then $\forall\tilde{0}\neq\tilde{U}\in\tilde{\tau}_f$, $\tilde{U}$ is *FI*- set.
7. If $\widetilde{K^*}=\tilde{0}$, then $\tilde{K}$ is *FI*- set.

Proposition 2.5 : Let $(\tilde{1},\tilde{\tau}_f,\tilde{J}_f)$ be *Fh*- connected fuzzy ideal space, then following features are true.

1. If $\tilde{K}$ and $\tilde{H}$ are *FC*- sets, then $\tilde{K}\vee\tilde{H}$ is *FC*- sets.
2. If $\tilde{K}$ and $\tilde{H}$ are *FC*- sets, then $\tilde{K}\wedge\tilde{H}$ is *FC*- sets.

Definition 2.6 : A fuzzy set $\tilde{A}$ in fuzzy ideal space $(\tilde{1},\tilde{\tau}_f,\tilde{J}_f)$ it's called:

1. Intense inner and denoted by $\tilde{A}_i$ s.t. $\tilde{A}_i=\{(x,\sup_{\tilde{M}\in FIC}\{f_M(x), f_M(x)\leq g_A(x)\});\ \forall x\in X\}$.
2. Intense external and denoted by $\tilde{A}_e$ s.t. $\tilde{A}_e=\{(x,\inf_{\tilde{F}\in FCC}\{h_F(x), g_A(x)\leq h_F(x)\});\ \forall x\in X\}$.
3. Intense confines and denoted by $\tilde{A}_c$ s.t. $\tilde{A}_c=\{(x,\min\{\sup_{\tilde{M}\in FIC}\{f_M(x), f_M(x)\leq g_A(x)\}\}),\ \inf_{\tilde{F}\in FCC}\{h_F(x),\ h_F(x)+g_A(x)\geq 1\}\};\ \forall x\in X\}$.

Proposition 2.7 : For any fuzzy sets $\tilde{K}$ and $\tilde{H}$ in fuzzy ideal space $(\tilde{1},\tilde{\tau}_f,\tilde{J}_f)$ the following feature are true.

1. $\tilde{A}\subseteq\tilde{A}_e$.
2. $\tilde{A}=\tilde{1}$ is *FC*- set iff $\tilde{A}=\tilde{A}_e$.
3. $\tilde{A}_i\subseteq\tilde{A}$.

4. $\tilde{A} \neq \tilde{0}$ is *FI*- set iff $\tilde{A} = \tilde{A}_i$.
5. $\tilde{1} - \tilde{A}_i = (\tilde{1} - \tilde{A})_e$.
6. $\tilde{1} - \tilde{A}_e = (\tilde{1} - \tilde{A})_i$.
7. $\tilde{A}_c = 1 - (\tilde{A}_i \vee (\tilde{1} - \tilde{A})_i)$.
8. $\tilde{A}_i \vee \tilde{B}_i \subseteq (\tilde{A} \vee \tilde{B})_i$.
9. $\tilde{A}_e \vee \tilde{B}_e \subseteq (\tilde{A} \vee \tilde{B})_e$.
10. $(\tilde{A} \wedge \tilde{B})_i \subseteq \tilde{A}_i \vee \tilde{B}_i$.
11. $\tilde{A}_c = (\tilde{1} - \tilde{A})_c$.
12. If $\tilde{A}_e = \tilde{1}$, then $(\tilde{1} - \tilde{A})^* = \tilde{1}$.
13. $\tilde{A}_c$ is *FC*- set.
14. $\tilde{A} \neq \tilde{0}$ is *FI*- set iff $\tilde{A}_c \wedge \tilde{A} = \tilde{0}$.
15. $\tilde{A} \neq \tilde{1}$ is *FC*- set iff $\tilde{A}_c \subseteq \tilde{A}$.
16. $\tilde{1} \neq \tilde{A} \neq \tilde{0}$ is both *FI*- set and *FC*- set iff $\tilde{A}_c = \tilde{0}$.

Proposition 2.8 : Let $\tilde{A}$ and $\tilde{B}$ are fuzzy sets in the fuzzy ideal space $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ the following are hold:

1. $\tilde{A}_c \subseteq \tilde{A}$ or $\tilde{A}_c \subseteq \tilde{1} - \tilde{A}$.
2. If $\tilde{A} \subseteq \tilde{B}$, then $\tilde{A}_e \subseteq \tilde{B}_e$ and $\tilde{A}_i \subseteq \tilde{B}_i$.
3. For any two different fuzzy points P_x^α and P_y^ℓ, then $(P_x^\alpha)_e \neq (P_y^\ell)_e$ and $(P_x^\alpha)_i \neq (P_y^\ell)_i$.

Definition 2.9 : The fuzzy ideal space $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ is said to be:

1. DI_0- space iff for each two different fuzzy points $P_x^\alpha \neq P_y^\ell$, $\exists FI$ - set $\tilde{G}$ s.t. $P_x^\alpha \tilde{\in} \tilde{G}$ and $P_y^\ell \tilde{\notin} \tilde{G}$.
2. DI_1- space iff for each two different fuzzy points $P_x^\alpha \neq P_y^\ell$, $\exists$ two *FI*- set $\tilde{G}$ and $\tilde{H}$ s.t. $P_x^\alpha \tilde{\in} \tilde{G}$ and $P_y^\ell \tilde{\notin} \tilde{G}$ and $P_x^\alpha \tilde{\notin} \tilde{H}$ and $P_y^\ell \tilde{\in} \tilde{H}$.
3. DI_2- space iff for each two different fuzzy points $P_x^\alpha \neq P_y^\ell$, $\exists$ two *FI*- set $\tilde{G}$ and $\tilde{H}$ s.t. $P_x^\alpha \tilde{\in} \tilde{G}$ and $P_y^\ell \tilde{\in} \tilde{H}$.

Theorem 2.10 : *For any fuzzy ideal space $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$, the following statements are equivalent:*

1. *The fuzzy ideal space $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ is DI_1 - space.*
2. *For each fuzzy point P_x^α is FC- set.*
3. *For any fuzzy set $\tilde{U} = \wedge\{\tilde{G} \in FIC;\ \tilde{U} \subseteq \tilde{G}\}$.*

Proof : $(1 \to 2)$ By Proposition 2.7. part (3), $(\tilde{1}-P_x^\alpha)_i \subset P_x^\alpha$ for each $P_y^\ell \subset \tilde{1}-P_x^\alpha$, imply that $P_y^\ell \neq P_x^\alpha$ and by DI_1- space $\exists$ *FI*- set $\tilde{G}$ s.t $P_y^\ell \in \tilde{G}$, $P_x^\alpha \notin \tilde{G}$, so we get $P_y^\ell \in \tilde{G} \subseteq \tilde{1}-P_x^\alpha$ that means $P_y^\gamma \in V\{\tilde{G} \in FIC$, $\tilde{G} \subset \tilde{1}-P_x^\alpha\} = (\tilde{1}-P_x^\alpha)_i$, from this we have $\tilde{1}-P_x^\alpha \subseteq (\tilde{1}-P_x^\alpha)_i$, then $\tilde{1}-P_x^\alpha = (\tilde{1}-P_x^\alpha)_i = (\tilde{1}-P_x^\alpha)_e$ iff $P_x^\alpha = (P_x^\alpha)_e$ and Proposition 2.7. part (2) , P_x^α is *FC*- set.

$(2 \to 3)$ For any fuzzy set $\tilde{U}$ s.t $\tilde{U} \subseteq \wedge\{\tilde{G} \in FIC, \tilde{U} \subseteq \tilde{G}\}$. Let us $P_x^\alpha \subseteq \wedge\{\tilde{G} \in FIC, \ \tilde{U} \subseteq \tilde{G}\}$ and by (2) P_x^α is *FC*- set, so by Definition 2.2. , $\tilde{1}-P_x^\alpha$ is *FI*-set. If possible $P_x^\alpha \notin \tilde{U}$, then $\tilde{U} \subseteq \tilde{1}-P_x^\alpha$ but $P_x^\alpha \notin \tilde{1}-P_x^\alpha$, where contradiction. Therefore $\tilde{U} = \wedge\{\tilde{G} \in FIC, \tilde{U} \subseteq \tilde{G}\}$.

$(3 \to 1)$ Let $P_y^\ell \neq P_x^\alpha$, then $P_x^\alpha \notin P_y^\ell = \wedge\{\tilde{G} \in FIC, P_y^\ell \subseteq \tilde{G}\}$ and $P_y^\ell \notin P_x^\alpha = \wedge\{\tilde{H} \in FIC, P_x^\alpha \subseteq \tilde{H}\}$. Then the fuzzy ideal $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ is DI_1- space.

Theorem 2.11 : *For any fuzzy ideal space* $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$, *the two statements are equivalent:*

1. *The fuzzy ideal space* $(\tilde{1}, \tilde{\tau}_f, \tilde{J}_f)$ *is* DI_2*- space.*
2. *For any fuzzy ideal* $\tilde{U} = \wedge\{\tilde{1}-\tilde{G}; \tilde{G} \in FIC$ *and* $\tilde{U} \subseteq \tilde{1}-\tilde{G}\}$.

3. Discussion and Conclusion

The research is based on finding new concepts of separation axioms by using new concepts of sets that have never been known previously, so we defined a new sets called *FI*- sets, *FC*- sets and *SFI*- sets and we used it to define DI_i- space, i = 0, 1, 2. So this research is the beginning of expanded studies in this area. Also we can use these sets to develop the compactness [8,9].

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Received September, 2021

Revised January, 2022