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DISCHARGE CHARACTERISTICS FOR TRAPEZOIDAL COMPOUND LABYRINTH

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ABSTRACT

Replacement weirs are commonly used to pass changed or higher than design storm flows. Such a situation could arise due to climate change or urbanization impacts. Labyrinth weirs generally have higher discharge capacities and therefore are used for this purpose. Trapezoidal compound labyrinth weir is proposed in this study. The proposed compound weir acts as a staged labyrinth weir, where the trapezoidal section is utilized to satisfy design discharge requirements and functions as a compound weir during high flow events. However, there is a lack of sufficient information for the hydraulic design of compound labyrinth weirs. The main goal of this study was to investigate the efficiency of discharge of trapezoidal compound labyrinth weirs. Laboratory experiments were conducted in a rectangular laboratory flume. The flow characteristics for eight physical models with sidewall angles (α) ranging from 6^o to 90^o with half round crest were studied, and head-discharge relationships were assessed. The measured discharges were compared with predicted values using a modified form of the weir equation derived from energy conservation. The error percentage of discharge was calculated between the calculated discharge and the observed discharge when discharge particularly was separated through the notches on the weir. The results of this study showed that the compound discharge coefficient (Cdc) is increased initially with low $H't/P'$ values due to the nappe interference of falling jets being not so severe, then decreases with increasing $H't/P'$ for all sidewall angles because of the increasing collision of falling jets. The empirical equations were generated by curve-fitting for rang sidewall angles 6^o to 90^o and half round crest. These equations were validated for $0.05 \leq H't/P' < \sim 0.75–0.9$. The error percentage was as large as 15% for $\alpha = 20^o$ when discharge particularly was separated to the notches of weir (low stage).

Keywords: labyrinth weirs, head-discharge relationships.

1. INTRODUCTION

The design of hydraulic structures usually depends on flow estimates. However, in the last few years, flow behaviour has changed due to climate change (Karaeren, 2014). Labyrinth weirs provide an alternative rehabilitation measure through the upgrading of older weir. This is because labyrinth weirs consist of a series of linear weirs providing a longer crest length compared with a normal weir installed within the same width of channel, and are thus able to increase the discharge capacity for a given head. For sites where the width is restricted and a high capacity of discharge is required, a labyrinth weir is an effective and economical solution (Khode et al., 2012). Labyrinth weirs consist of different geometric shapes such as rectangular (e.g., Taylor, 1968), semi-circular (e.g., Ghodsian et al., 2002), triangular (e.g., Ghodsian, 2009), and trapezoidal (e.g., Crookston and Tullis, 2012). The discharge coefficient depends on the geometry of the channel and the labyrinth weir and flow characteristics (Bagheri and Heidarpour,2010).

The hydraulic behaviour of labyrinth weirs has been of interest to researchers and practitioners for many years. Early studies were conducted by Taylor (1968) on the behaviour of labyrinth weirs, providing information on the hydraulic performance and general design of this type of weir. Hay and Taylor (1970) developed design criteria based on Taylor's work. Darvas (1971) developed a family of curves to evaluate labyrinth spillway performance. Houston (1982) extensively studied a physical model to assess different labyrinth geometries and approach conditions. Tullis et al. (1995) conducted a large experimental study to investigate the performance of the labyrinth weir. They found a family of design curves to indicate the relationship between the coefficient of discharge and headwater and the weir height (Ht/P) for various side wall angles (α) of 6° to 18° and they obtained additional curves for weir sidewall angles of 25˚ and 35˚ by extrapolation. Discharge equations were also proposed. Savage et al. (2004) identified discharge errors of up to ±25% in the method proposed by Tullis et al. (1995). Tullis et al. (2007) carried out work on Dog River Dam in Georgia by using a physical model of the labyrinth weir. They provided a dimensionless head-discharge relationship for submerged labyrinth weirs. Khode et al. (2011) conducted experimental studies on trapezoidal labyrinth weirs for side wall angles 6˚, 8˚, 10˚, 16˚, 21˚, 26˚ and 30˚. Khode et al. (2012) extended these experimental works for a wider range of flow conditions. Savage et al. (2016) showed that the coefficient of discharge design curves found by Crookston and Tullis (2012) is suitable for headwater ratios (Ht/P) of up to 2.0 or more. The previous studies stated that there is a lack of adequate information related to compound labyrinth weirs (labyrinth weir with notches). Therefore, proposed models in the present study can provide sufficient information for understanding the flow characteristics through a notch or entire of labyrinth weirs. Moreover, this information will help the engineers to improve the performance of weir especially during sudden increasing in discharges due to climate change by using compound labyrinth weirs. The purpose of this study was to systematically investigate the hydraulic performance of trapezoidal compound labyrinth weirs with a half-round crest, using a range of side wall angle (α) configurations.

2. EXPERIMENTAL SETUP

2.1 Instrumentation

The experimental tests were carried out in a rectangular flume of length 6.5 m; width 0.5 m and depth 0.6 m in a civil laboratory at Deakin University, Geelong, Australia. The flume walls were transparent and made up of acrylic panels supported by steel framework. The longitudinal bed slope of the flume was set to zero. Water was supplied from a tank with dimensions 2.5 m × 1 m×1 m which was location at the end of the flume, and recirculated through the channel with a 200 mm diameter supply pipe using two pumps connected in parallel. The flow rate was controlled by a gate valve. The capacity of each pump was 40 L/sec. Two-point gauges with accuracy ±0.1 mm were used for measuring the water level over both a weir crest and weir notch. The water level at the labyrinth weir varied from 5 to 100 mm, for various values of discharge. A flow meter was utilised to measure the flow rate which ranged from 0.002 m^3/s to 0.08 m^3/s. Approximately 15 to 30 head-discharge of readings were carried out for each tested weir geometry. For each run, the head over the crest of the labyrinth weir was measured at about 2-3 times using a point gauge and calculate average of readings to avoid the water surface curvature effects.

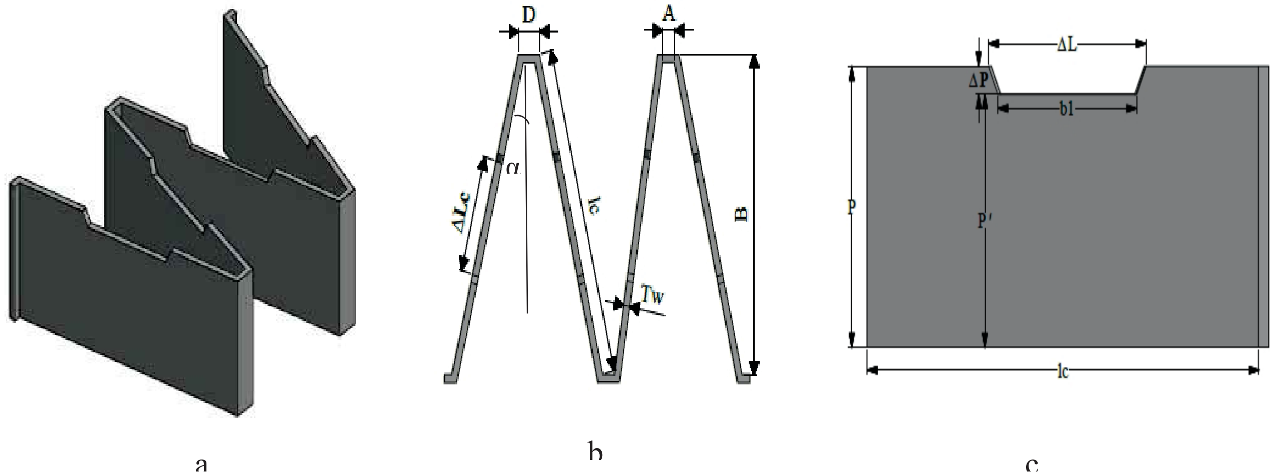

Figure 1. Labyrinth weir geometry. (a) 3D image, (b) top view, (c) side view.

2.2 Model description

In the present work, eight models with sidewall angle (α) configurations of 6, 8, 10, 12, 15, 20, 35 and 90° were investigated. This model were two-cycles of a trapezoidal compound labyrinth weir with a half-round crest. All models were fabricated from acrylic sheets, with thickness of 10 mm. Labyrinth weir geometry is showed in Figure 1. All models were located within 1 metre of the inlet point and had a half-round crest shape with ($R=T/2$) and two complete cycles of labyrinth; a total width of 0.5 m. The details of the physical models are shown in the Table 1. The weir was attached to the flume floor and sidewall by a sealant. This study defines a stage term for any segment of the labyrinth weir crest determined at a different elevation. A notch term defines an opening provided in the walls of a labyrinth weir. The bottom edge of the notch is known as a low stage crest. The upper length of the notch is less than the labyrinth side wall length.

Table 1. Physical model test program

α **degree**	**P** **(cm)**	**B** **(cm)**	**Lc-cycle** **(cm)**	**Tw** **(cm)**	**A** **(cm)**	**D** **(cm)**	**N**	**Trapezoidal hole geometry**			**$\Delta P/P$**	**$\Delta L/lc$**
								$b1$ **(cm)**	**ΔL** **(cm)**	**ΔP** **(cm)**		
6	20	100.8	413.6	1	2	3.80	2	52.8	60.8	4	0.2	0.6
8	20	75.6	313.54	1	2	3.73	2	37.8	45.8	4	0.2	0.6
10	20	60.4	253.5	1	2	3.67	2	28.8	36.8	4	0.2	0.6
12	20	50.2	213.6	1	2	3.61	2	22.8	30.8	4	0.2	0.6
15	20	40.0	173.8	1	2	3.53	2	16.8	24.8	4	0.2	0.6
20	20	29.6	134.3	1	2	3.40	2	10.9	18.9	4	0.2	0.6
35	20	15.6	84.5	1	2	3.04	2	3.4	11.4	4	0.2	0.6
90	20	-	50	1	-	-	-	22	30	4	0.2	0.6

Note: α is the sidewall angle, P is the height of the labyrinth weir, B is the length of the labyrinth weir, Lc-cycle is the total crest length, lc is the crest length for single sidewall of weir, Tw is the weir thickness, A is the inside apex width, D is the outside apex width, N is the number of cycles, $b1$ is the bottom width for the notch, ΔL is the top width for the notch, and ΔP is the notch depth.

2.3 Head discharge relationship

Discharge (Q) over a labyrinth weir under free flow condition can be expressed by the Eq. (1) (Crookston and Tullis, 2012).

$$Q = Cd * \frac{2}{3}\sqrt{2g} * L'c * H_t^{1.5} \tag{1}$$

Eq. (2) represents the discharge Q, through a trapezoidal notch (lower stage) (Henderson, 1966). and Eq. (3) represents the head-discharge relationship over a trapezoidal compound labyrinth weir (Idrees et al.,2016). In addition, it has been utilised to compute the compound discharge coefficients for differing flow conditions:

$$Q = Cd\left(\frac{8}{15} * \sqrt{2g} * \tan\frac{\theta}{2} * H'^{1.5}_t + \frac{2}{3} * \sqrt{2g} * b_1 * H'^{1.5}_t\right) \tag{2}$$

$$Q = Cdc\left[\frac{2}{3} * \sqrt{2g} * L'c * H_t^{1.5} + n\left(\frac{8}{15} * \sqrt{2g} * \tan\frac{\theta}{2} * H'^{1.5}_t + \frac{2}{3}\sqrt{2g} * b_1 * H'^{1.5}_t\right)\right] \tag{3}$$

In Eq. (3), Q = discharge, Cdc = compound coefficient of the discharge, L_c = length of weir crest, b_1= bottom width of the notch, L'_c = length of the labyrinth weir crest after subtracting length of notches (ΔL), ($L'_c = L_c - n*\Delta L$), n = number of notches, g = acceleration due to gravity, H'_t= total head through notch (low stage) and H_t = total head over the weir crest (high stage). The value of H'_t was computed as h' plus the velocity head at the measurement location $\{H'_t = h' + (V^2 / 2g)\}$, The value of H_t was computed as h plus the velocity head at the measurement location $\{H_t = h + (V^2 / 2g)\}$ (see Figure 2). Because there are two crest elevations for a single labyrinth weir, the definition of head (h) should be as for a staged weir.

Head-discharge data were collected for flows over the lower and upper crests. For lower stage flows, the upstream head for the notch (h') was measured from the lower crest elevation defined by P'. The total head Ht' for the lower stage was calculated as the sum of the static head, h' and the velocity head ($V^2/2g$), as shown in Figure 2(a). When the discharge was delivered over the entire labyrinth weir, h was measured according to the high stage of the weir crest [see Figure 2(b)].

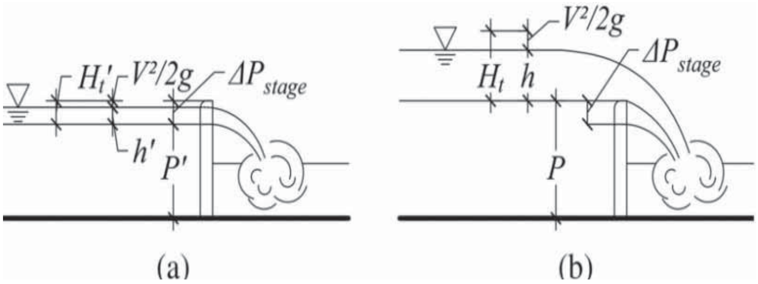

Figure 2. Sketch of hydraulic parameters for flow on: (a) lower stage; (b) upper stage

3. RESULTS AND DISCUSSIONS

3.1 Compound labyrinth weir discharge (multiple crest elevation)

In the present work, the data for compound coefficient of the discharge were collected from Eq. (3). Figure 3 shows the variation of the compound discharge coefficient $Cdc(\alpha)$ with $H't/P'$ (dimensionless total headwater ratio over lower stage to height of notch) for different values of α. The values of Cdc are initially similar and independent of $H't/P'$ when $H't/P'$ is small, increases with $H't/P'$, reaches a maximum and decreases towards a constant value (as a function of α) when $H't/P'$ is large. It is also noted that Cdc decrease to values that are considerably lower for large $H't/P'$ compared to Cdc at the lowest values of $H't/P'$ values tested. This phenomenon is related to the nappe interference of falling jets being not so severe at the lower stage of flow. A Figure 4 showed the behaviour flow through tests. Subsequently, the compound discharge coefficient $Cdc(\alpha^\circ)$ decreases with higher values $H't/P'$ for all curves which find in the Figure 3 because of the increasing collision of falling jets that result from a non-aerated nappe. For all angles α° = 6o to 35^{o} and 90^{o} at $H't/P' = 0.2$, the compound coefficient of discharge $Cdc(\alpha)$ increases with increasing sidewall angle (α) of weir caused by the abrupt removal of the air cavity behind the nappe (Crookston and Tullis, 2011). The curves in Figure 3 show empirical curve-fits, given by Eq. (4) to the data points obtained from the experiments for nonlinear compound weir and Eq. (5) for linear compound weir with $R^2 > 0.99$. The curves fitting procedure were done in IBM SPSS 22 software. The curve fit coefficients for Eqs. 4 and 5 are shown in Table 2 and are valid for $0.05 \leq H't/P' <$ ~0.75–0.9. Because the data display a well-behaved nature and using Eq. 4 for a nonlinear weir and Eq. 5 for a linear weir, the $Cdc(\alpha^\circ)$ curves have been extrapolated to $H't/P'$ 1.0. The empirical Equations 4 and 5 were selected instead of polynomial relationships because of their improved data representation ($R^2 \geq 0.99$) and extrapolation performance (they remain well-behaved up to $H't/P' \leq 2.0$).

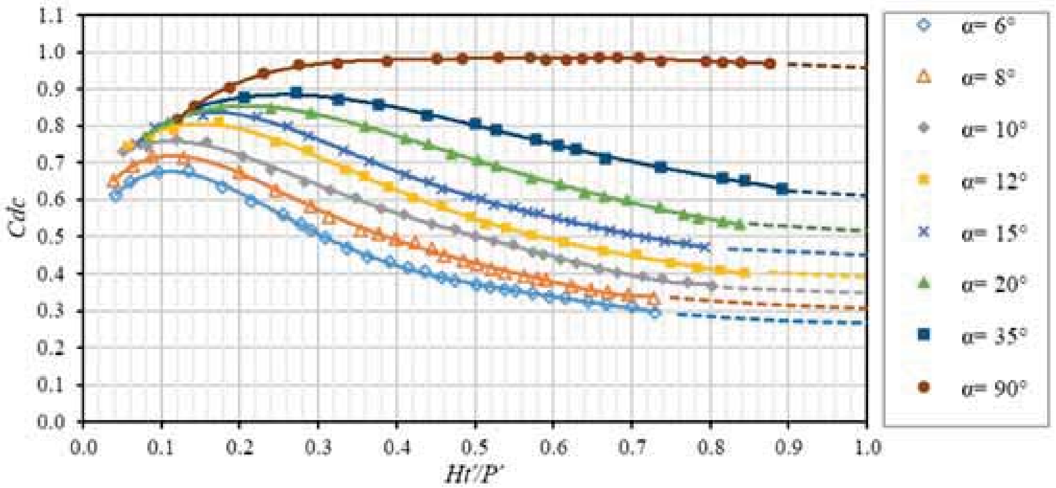

Figure 3. Compound coefficient of discharge (Cdc) versus $H't/P'$ data for the tested compound labyrinth weir geometries

For compound nonlinear weir:

$$Cdc(\alpha^\circ) = A + \frac{B-A}{1+\left(\dfrac{C}{H'_t/P'}\right)^{D}} \tag{4}$$

For compound linear weir:

$$Cdc(90^{\circ}) = A + \frac{1}{A + \frac{B}{\left(H'_t / P'\right)^2} + C\left(H'_t / P'\right)^2} + D \tag{5}$$

Figure 4. Compound Labyrinth weir with α=15° (a) set up the model inside flume, (b) low discharge over notch, (c) high discharge over entire weir

Table 2. Curve-Fit coefficients for a half-round crest for compound Labyrinth weirs and Linear weirs, validated for 0.05 ≤ $H't/P'$ < ~0.75–0.9

α°	A	B	C	D	R^2
6°	0.657	0.292	0.340	3.546	0.989
8°	0.697	0.284	0.404	3.162	0.988
10°	0.756	0.266	0.482	2.621	0.995
12°	0.782	0.370	0.465	3.756	0.99
15°	0.804	0.446	0.476	4.216	0.981
20°	0.494	0.824	0.576	-4.834	0.98
35°	0.578	0.879	0.646	-4.666	0.991
90°	1.904	0.017	0.216	0.491	0.994

3.2 Low stage- separated discharge

Head-discharge data for a range of α values were collected when the discharge was separated to the notches. These data were compared with calculated Q values estimated from Eq. 2, using $H't$ values determined from the current experimental test, and using Cd values for a traditional labyrinth weir (without notches) with a half-round crest obtained from (Crookston and Tullis, 2012). The error percentage (i.e., $Q_{calculated}/Q_{actual}$) versus $H't/P'$ is plotted in Figure 5. As shown in Figure 5, the error percentage was increased with increased $H't/P'$ values for all the models of sidewall angles (α) 6o to 90°. The percentage difference for (α=6°) is within (5% to 14.7%), (α=8°) is within (4% to 14.5%), (α=10°) is within (3% to 14.5%), (α=12°) is within (4.5% to 14.5%), (α=15°) is within (4% to 14%), (α=20°) is within (5% to 15%) and (α=35°) is within (6% to 11%). For ($H't/P'$< 0.15), the maximum percentage error was 12% for α=6° and the minimum percentage error was 3% for α=10°. For (0.27 > $H't/P'$ > 0.15), the maximum percentage error was 15% for α=20° and the minimum percentage error was 6% for α=35°. For comparison with linear compound weir (α=90°), the error percentage was within (3.25% to 4.9%). It is likely that some error will be produced when the discharge is calculated through the trapezoidal notch only as well as labyrinth weirs are complex structures and flow behaviour is three dimensional.

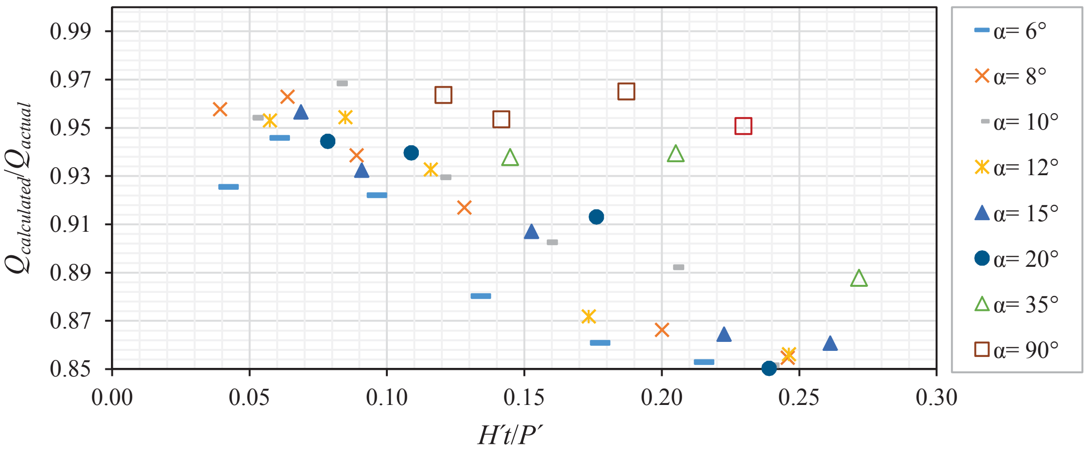

Figure 5. The Relationship between error percentage ($Q_{calculated}/Q_{actual}$) versus $H't/P'$ for discharges isolated through notches.

4. CONCLUSIONS

Labyrinth weirs are a hydraulic structure common in the world because of its favourable aspect of an increasing ability for the weir to pass greater amounts of water in the flood seasons. The labyrinth weir reflects an effective method for improving storage capacity. The objective of this work was to assess and document the hydraulic performance and flow characteristics of a compound labyrinth weir under controlled laboratory conditions. The following conclusions are obtained based on the test results and observations:

1. The compound discharge coefficient increased initially when the water level over the crest of the labyrinth weir reached a higher value, then the curve gradually decreased.
2. The results showed that the compound coefficient of discharge was at its least value for a sidewall angle of 6^o and increased with increasing sidewall angle values under the limited width of the labyrinth weir.
3. The empirical equations were generated by curve-fitting for a range of sidewall angles of 6^o to 90^o and a half-round crest. These equations were validated for $0.05 \leq H't/P' < \sim 0.75–0.9$
4. The empirical equations would be suitable for estimating the compound discharge coefficient over the compound labyrinth weirs. Therefore, this provides a method to predict flow rates for the case of a half-round crest, compound labyrinth weir where there are no definite theoretical head-discharge relationships.
5. The error percentage was as large as 15% for $\alpha=20^o$ when the discharge particularly was separated to the notches on the sidewalls of the weir (low stage).
6. The experimental work provides new insight about flow behaviour that passes over a compound labyrinth weir.

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