Gray Image Quantization Method Based on New Threshold Optimizing Technique

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***Abstract*— Images are one of the most important media information in our world it used as inputs to many functions like classification, content-based image retrieval, image feature extraction, and many image processing purposes. In these fields, the optimal image quantization and reduction of gray levels methods are considered a significant preprocess that increases the efficiency and performance of the applications and enhances the precision of its results. Image quantization has a big effect on the use of huge image datasets used in these applications whereas it reduces the size of images by retaining features and the general structure of the image. This paper proposes a new image quantization method depend on optimal selection for threshold values depending on the PSNR metric. This method uses one of the Greedy programming algorithms that is the Divide and Conquer strategy. In this strategy, the selection of the threshold value that represents an optimum division position in the image gray levels range depends on the heuristic function. The heuristic function decides whichever threshold value (new gray level) from the image gray levels range is an optimal PSNR if it is selected and added to the quantized image compared with the original image. The experimental results show that the proposed method enhances the quantized image quality compared with standard quantization methods as standard uniform quantization and k-mean clustering method and other methods. The proposed method gets efficient results in PSNR and SSIM quality measures with increasing average ranged in [1.2%-5.5%] and [2%-6.5%], respectively, compared with other methods considering different gray levels and different image types. The method time complexity analysis is explained. The experimental results show the time consuming of applying the proposed method on many images and two big datasets.**

Keywords—quantization, gray levels reduction, Divide and Conquer, PSNR, SSIM

I. Introduction

Usually, quantization is used as preprocessing for many applications and image processing functions to perform more fast performance and efficient results with huge datasets. Also, there are applications such as image retrieval that have a good performance if it contents a quantization preprocessing [1][2][3]. Quantization is defined as a process of reducing the range of the original image gray levels into a desired limited number of gray levels and produces a quantized image [4]. The reduction of gray levels means reducing the number of bits that need to represent each pixel in the image. If the number of gray levels is L, then there are log2L=N (L=2N) bits are required for each pixel [5]. Many types of quantization methods developed across successive decades. Uniform quantization considers as one of the simplest and fast quantization methods to reduce the gray levels in equal spaces [6]. If the range of the origin image gray levels is between [Kmin, Kmax], and the desired number of gray levels is L=2m, then the formula of uniform quantization will be as:

$$Q(i,j) = P\left(\left(\frac{I(i,j)-K_{min}}{K_{max}-K_{min}}\right) * L - 1\right) \quad (1)$$

Where, $Q(i,j)$ is a quantized image, $I(i,j)$ is the original image, $P()$ represents the cut or around the number. Uniform quantization's resulted image does not take into consideration the spatial information of the original image, which means, this method does not depend on the real gray levels and the distribution of the image histogram. As well, this method produces many false contours lines as a deformation to the resulted image. These entire reasons make this method has a low quality compared with many other quantization methods [7]. The quantization process can be considered as a multi-thresholding problem if the number of desired gray levels is few and can consider as a clustering problem if we want to segment the image to several meaningful regions [8]. One of the methods that used as a quantization process in many applications is k-mean clustering. This method depends on arbitrary selection to K centers and finds the distance between each center and all pixels of the image, then assigned the pixels to the nearest centers [9]. This process is repeated until reaching the accepted error rate. This method has many drawbacks like the random initialize for the centers' values, as well as, the computational complexity of the method is depending on the centers' number, data size, and iteration number [10]. There are many non-uniform quantization methods like methods that depend on the histogram of the image and the intensity distribution as well as many other methods like Dithering and Halftoning [5]. Also, there are many gray level reduction methods that are introduced as preprocessing for many applications as the min-max linear compression that compress the gray level values of the image in a definite min-max range. Moreover, gray-level binning holds a specific number of bits of every gray level to have the desired number of gray levels bits. The histogram equalization, also, considered as a quantization method that equalized the histogram to get an equal number of gray levels and then reduces the gray-level number overall to achieve the desire gray level number [11]. The method proposed in this paper can be considered as a non-uniform quantization method. This method aims to find the optimum desired set of gray levels using a quality metric as a heuristic function and divide and conquer strategy [12].

Many quality measures that used to evaluate quantization, thresholding, and reduction gray levels processes. There are two quality measures are used in this paper peak signal to noise ratio (PSNR) and structural similarity index (SSIM). Although, PSNR is based on overall image pixels statistics and average calculations, it stills a good and efficient quality measure to evaluating the image processing operations [7]. PSNR can be considered as in" (2)" below:

$$PSNR(A,B) = 10\ log_{10}\left(\frac{MaxPixel^2}{\frac{1}{X\times Y}\sum i \sum j(A(i,j)-B(i,j))^2}\right) \qquad (2)$$

Where *MaxPixel*=2k -1(*k* is the number of bits to represent each pixel), *X* and *Y* are the sizes of images, *A* and *B* are the images that we are compared. SSIM is used to evaluate the similarity between images based on luminance, contrast, and structure. " (3)" below described this measure:

$$SSIM(A,B) = \frac{1}{M}\sum_{i=1}^{M}\left(\frac{2\mu_{ai}\mu_{bi}+d_1}{\mu_{ai}^2+\mu_{bi}^2+d_1}\right)\times\left(\frac{2\sigma_{aibi}+d_2}{\sigma_{ai}^2+\sigma_{bi}^2+d_2}\right) \qquad (3)$$

$$d_1 = (f1 * MaxPixel)^2 \qquad (4)$$

$$d_2 = (f2 * MaxPixel)^2 \qquad (5)$$

Where, ai, bi are all M windows of A, B images. μ, σ are mean and standard deviation for the windows, respectively. f1=0.01, and f2=0.03 are default numbers [13].

II. RELATED WORKS

The quantization and thresholding process is very important processes that are applied to an image to reduce it with keeping its structure and the main features in the resulted image. There are many researchers introduced quantization, gray levels reduction, and thresholding methods that are applied in different applications. Nameirakpam Dhanachandra and Yambem Jina Chanu developed the K-mean method using kernel subtractive as a function to find the cluster centroids. The subtractive method converted the space of the pixel into an upper dimension to make the values of the pixels are separable [9]. Kotte Sowjanya and et al. introduced a multi-thresholding gray levels method based on adaptive wind driven optimization (AWDO) algorithm and applied it on MRI brain images. The method produced the optimum threshold values by maximizing the popular objectives such as between class variance (Otsu method) and Kapur's entropy [14]. R. Srikanth and K. Bikshalu used an Energy Curve, Otsu's and Harmony Search methods as a substitute for the histogram to find the best gray levels using the objective function that maximize the inter-class variance [15]. Wei Liu and et al. introduced a method that adapted a breeding mechanism of Chinese hybrid rice to find the optimal gray levels values and they benefited from Renyi's entropy as a fitness function to optimizing the results [16]. Lorenzo Nichele et al. tested the efficiency of many thresholding algorithms on both empirical and artificial confocal images of Escherichia coli and Staphylococcus aureus. They estimate the misclassification errors based on typical pattern recognition factors and they found that the Bernsen local thresholding gives the greatest results also depending on cell calculating and morphology mechanism [17]. This paper proposed a method to optimizing the quantized gray level values of the images. This new image quantization method uses a Divide and Conquer strategy that depends on optimal selection for threshold values concerning the PSNR metric. The main idea of the research includes a new non-uniform quantization method that depends on optimizing a set of gray levels based on PSNR measure and using divide and conquers strategy. This is the main point of the novelty of the research and this method is not proposed or introduced in any research before.

III. PROPOSED METHOD

The proposed method included quantizing the gray image by reducing the number of gray levels from 255 gray levels to the desired number of gray levels that is determined by the user. The quantized gray levels are generated by finding threshold values by divided the original range of gray levels. This division depends on optimization criteria. The main idea of the proposed method is motivated by the uniform quantization of gray images that reduces the number of gray levels of the image by dividing the gray levels range of the original image into equal periods. The resulted gray levels from the uniform process are distributed uniformly. However, it may be not the real gray levels in the image.

Our proposed method considers the reduction of gray levels in the image will depend on non-uniformed quantization that divided the actual range of gray levels into unequal periods or regions. These unequal periods are split by threshold values selected depending on a heuristic function that optimizes each threshold value before selecting it. To finding these threshold values, a greedy programming method that uses Divide and Conquer strategy has been adapted. First, we start with generating a quantized image with one gray level only (the higher gray level value in the image) then we generate a quantized image with 2 gray levels (one generated threshold value). After that, we generate a quantized image with 4 gray levels (3 threshold values) then a quantized image with 8 gray levels. The chosen threshold values will be the values that give an optimal division to the image gray levels range. The optimal dividing is depending on which threshold value (gray level) in the image gray level range gets a maximum PSNR between the original image and quantized image that resulted from chosen threshold dividing values.

Also, the proposed method can be considered as a multi-thresholding problem for the image histogram. Let I the original image, the range of gray levels is [L, H] (e.g. [4,243]) and the desired number of threshold values is M (determined by the user) ($1..2^{k-1}$). Then, the threshold values will be (th1, th2, th3,…,thm) where thm = H. The quantized image Q will be computed as in "(6)" :

$$Q(i,j)=\begin{cases} th1\ if\ L \le I(i,j) \le th1 \\ th2\ if\ th1 \le I(i,j) \le th2 \\ \quad . \\ \quad . \\ H\ if\ thm-1 \le I(i,j) \le H \end{cases} \qquad (6)$$

The proposed method selects the threshold values using a hieratical division wherein each level the procedure is applied recursively on each division part. The block diagram that introduces the method is illustrated in fig. 1.

The proposed procedure contains the following steps:
Step 1: start with a quantized image with one gray level H only.
Step 2: select the threshold value that represents the division position to the original image gray levels range. We test all gray levels from L to H in the original image and find the gray level Th that has a higher PSNR between the original image and the quantized image after applying Th.
Step 3: the image gray levels will reduce into two gray levels only (Th, H). Where all the pixels that have gray levels that less than or equal to Th will have Th value, otherwise will have H gray level.
Step 4: If the number of threshold values equal to M then stop. Else, repeat step 2 and step 3 with a new quantized image after adding Th and two new gray level ranges; first (L, Th-1) and next with (Th, H) to find two other threshold values.
Figure 2 illustrates the hierarchy of the first three iterations of finding the first eight threshold values that generate the quantized image from 2 to 8 gray levels.
We use the queue as a data structure instead of a stack to arrange the priority of processing the ranges of gray levels that adapts the first in first out strategy.
To illustrate our method we use an X-Ray image as an example to apply our procedure. The X-Ray original image is selected randomly from the RYDLS-20 dataset introduced by Rodolfo M. Pereira et al. [18]. The original image and the histogram of it with the first eight quantized gray levels are illustrated in Fig. 3. Our method is not used histogram but it presented here just to illustrate the non-uniform distribution of image gray levels and the position of selected threshold values for the quantized image.

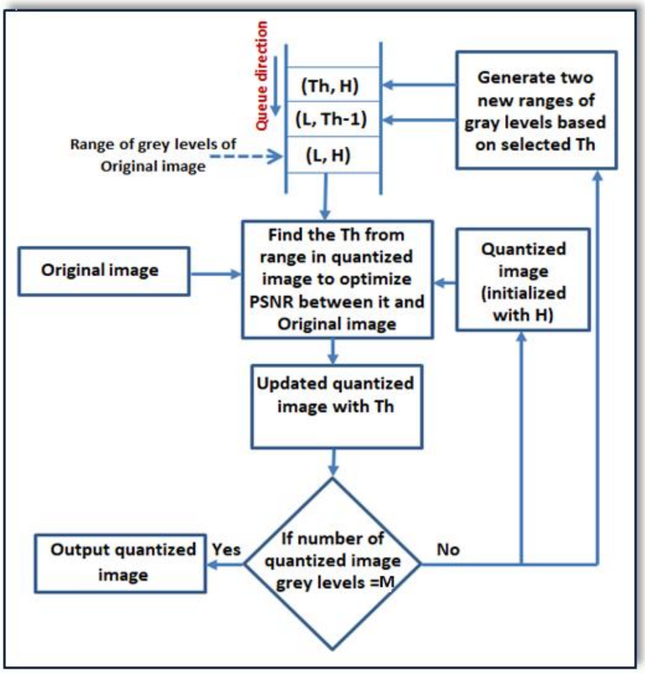

Fig. 1 Block diagram of the proposed method

Iteration	Selected thresholds								No. gray levels
Initial case	H								1
Iteration 1	Th1				H				2
Iteration 2	Th2		Th1		Th3		H		4
Iteration 3	Th4	TH2	Th5	Th1	Th6	Th3	Th7	H	8

Fig. 2 The progress hierarchy of the first three iterations for the proposed method

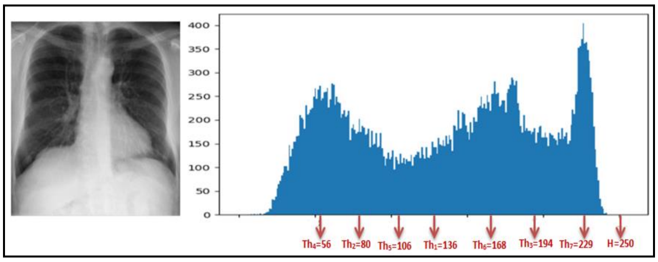

Fig. 3 the origin and histogram of the x-ray image with the first eight gray levels resulted from the proposed method

IV. RESULTS AND DISCOSIONS

A. *Tested Images and Datasets*

To present the results, the proposed method has been implemented with different gray images and quantized to many gray levels and compared with other methods. To illustrate our method we use an X-Ray image with size 200×200 as an example to apply our procedure. The X-Ray original image is selected randomly from the RYDLS-20 dataset that is a very big dataset for COVID 19 X-Ray images introduced by Rodolfo M. Pereira et al. [18]. MRI brain tumor image that used by P. Tamije in [19] is also used to present the results. Also, we use standard gray Lena image with size 256×256 pixels and standard gray Baboon image with size 512×512 pixels to present the results. The proposed method time consuming is computed using all the above images as well as two big datasets. The first dataset used to compute time complexity is the RYDLS-20 dataset (RYDLS) [18]. This dataset contains 1144 (200×200) chest X-ray images for normal, COVID- 19, MERS, Pneumocystis, SARS, Streptococcus, and Varicella diseases classes. The second dataset that used is SARS-COV-2 Ct-Scan Dataset (COV19). It contains (200×200) 2482 CT scans in total 1252 CT scans positive infection (COVID-19) and 1230 CT scans non-infected by SARS-CoV-2 [21].

B. *Expermentals and Discussions*

We use PSNR and SSIM as quality measures to compare the resolution of the resulted quantized image with the original one. PSNR's and SSIM's equations are illustrated in (2) and (3), respectively. Fig. 4 and fig. 5 illustrate the original and the quantized image after applying the proposed method on X-ray and Lina images, respectively, with a different number of gray levels.
To measure the efficiency of our method, we compare the PSNR and SSIM of our method (our) with the standard K-mean clustering (k-mean), uniform quantization (uniform), and Papamarkos and Atsalakis [20] (Papamarkos) methods for different gray levels (4, 8, 16, 32, 64, and 128). Table (I)

illustrated the PSNR accuracy measure results of the process Lina image using the proposed method compared with uniform quantization and Papamarkos and Atsalakis [20] methods where N is the number of gray levels. Table (II) and table (III) illustrate PSNR and SSIM metrics results, respectively. These metrics are applied between the original and quantized X-ray, Brain tumor, and Baboon images when applying the uniform, K-mean, proposed method (our) methods. It's clear, as shown in table I, our proposed method outperforms the uniform and Papamarkos methods in increasing average rate close to 7.19 and 4.63, respectively. Also, from the table (II), the proposed methods outperforms almost the other methods and gets more accurate quantizing results with an increasing average rate ranged in [1.2%-5.5%]. In table II, there is a little difference between the proposed method and the k-mean clustering method when the number of gray levels is small as 4 and 8. Whereas the PSNR of the proposed method is increased compared with the k-mean method as the number of gray levels are increased and the proposed method is outperformed the k-mean method in 32, 64, and 128 gray levels. From table III, it is obvious that all experiments that were applied to evaluate the proposed method compared with other methods show the outperforming of the proposed method in terms of SSIM quality measure.

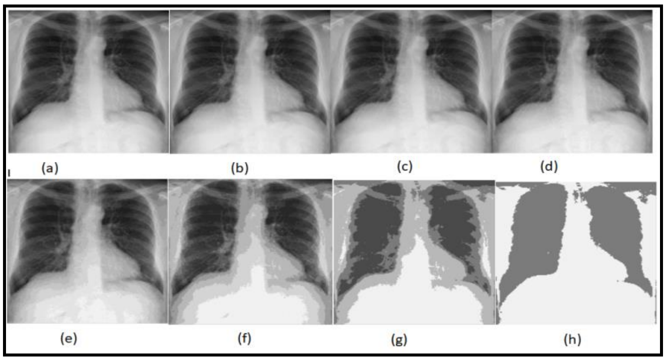

Fig. 4 Origin and quantized X-ray image using proposed method: (a) origin image, (b) 128 gray levels, (c) 64 gray levels, (d) 32 gray levels, (e) 16 gray levels, (f) 8 gray levels, (g) 4 gray levels, (h) 2 gray levels

Fig. 5 Origin and quantized Lena image using proposed method: (a) origin image, (b) 128 gray levels, (c) 64 gray levels, (d) 32 gray levels, (e) 16 gray levels, (f) 8 gray levels, (g) 4 gray levels, (h) 2 gray levels

TABLE I: PSNR RESULTS BETWEEN ORIGINAL AND QUANTIZED LINA IMAGE USING UNIFORM QUANTIZATION (UNIFORM), PAPAMARKO [20], AND PROPOSED METHOD (OUR) FOR DIFFERENT GRAY LEVELS VALUES.

L	Uniform	Papamarko	our
2	11.37	18.7341	**24.591**
4	16.649	24.7890	**25.284**
8	22.712	29.5320	**31.171**
16	28.867	32.2695	**35.479**
32	35.479	33.5525	**39.506**
64	42.451	34.0097	**44.613**

TABLE II : COMPARISON OF PSNR RESULTS USING UNIFORM QUANTIZATION, K-MEAN CLUSTERING AND PROPOSED METHOD BETWEEN THE ORIGINAL AND THE QUANTIZED LENA, X-RAY AND BABOON IMAGES WITH DIFFERENT GRAY LEVELS.

	X-Ray			Baboon			Brain tumor		
L	*Uniform*	*K-mean*	*Our method*	*Uniform*	*K-mean*	*Our method*	*Uniform*	*K-mean*	*Our method*
4	16.371	**24.721**	24.439	16.332	**26.573**	25.50	15.973	24.958	24.163
8	21.631	**30.665**	30.62	22.646	**32.101**	31.909	22.326	31.115	29.082
16	28.099	**36.111**	35.979	28.92	**37.138**	36.115	28.488	35.6832	35.095
32	34.652	**38.417**	38.032	35.337	39.142	**39.879**	34.937	37.398	**37.723**
64	41.736	43.079	**44.379**	42.196	43.638	**45.342**	41.794	42.428	**42.997**
128	51.160	45.780	**52.142**	51.143	46.743	**52.441**	51.124	46.504	**52.627**

TABLE III : COMPARISON OF SSIM METRIC USING UNIFORM QUANTIZATION, K-MEAN CLUSTERING AND PROPOSED METHOD BETWEEN THE ORIGINAL AND THE QUANTIZED X-RAY, BABOON AND BRAIN TUMOR IMAGES WITH DIFFERENT GRAY LEVELS.

	X-Ray			Baboon			Brain tumor		
L	*Uniform*	*K-mean*	*Our*	*Uniform*	*K-mean*	*Our*	*Uniform*	*K-mean*	*Our*
4	0.546	0.6539	**0.7725**	0.597	0.8059	**0.8292**	0.5391	0.7745	**0.835**
8	0.7516	0.8117	**0.8373**	0.8317	0.9209	**0.9264**	0.748	0.9244	**0.9402**
16	0.8883	0.9258	**0.9268**	0.9428	0.9724	**0.9751**	0.9014	0.9684	**0.9855**
32	0.9662	0.9518	**0.976**	0.9842	0.9829	**0.9927**	0.9714	0.9798	**0.9912**
64	0.9915	0.9826	**0.9932**	0.9961	0.9931	**0.9978**	0.9922	0.9935	**0.9969**
128	0.9983	0.9911	**0.998**	0.9992	0.9970	**0.9994**	0.9981	0.9975	**0.9995**

C. Time Complexity Analysis

In the proposed method, the search process for the optimum PSNR is applied on desired gray levels M that determined by the user. However, the search process adapts the divide and conquers strategy that divides the search space into two ranges in every iteration. Therefore, The time complexity of finding optimum PSNR is $O(\log M)$. Each search process requires PSNR computing and compares it with the original image. This process scans the entire image with size A×B. Therefore, the time complexity for the proposed method is $O(A\times B\times \log M)$. Fig. 6 and fig. 7 illustrate the changing time with different gray levels values and different images used in experiment results and two big datasets mentioned in section IV.A. The time complexity is computed using 2.2 core i7 8750H and 16G RAM computer. As shown in fig. 6, the time consuming is much closed for an equal size X-Ray and Brain images and very different for different size images as Baboon and Lina images. That means the proposed method time complexity depends on the size of the image. Also, we observed that the time is increased as the number of quantized gray levels is increased. That means the number of quantized gray levels effects on the time consuming. From fig. 7, we observed that the proposed method can be applied on big datasets with acceptable time consuming.

V. CONCLUSIONS

The paper introduces a new, efficient, simple, and not complex method for reducing the gray levels of the image that can be incorporated with other image processing applications. This method can be applied easily on different types of images and have acceptable results compared with other standard methods. The proposed method finds the optimum gray level values using the Divide and Conquer strategy and maximizing the PSNR for the chosen gray levels. The results show that the performance of the proposed method exceeds the uniform quantization and gets very good results with respect to PSNR and SSIM. According to the proposed method evaluation, the PSNR is computed for the quantized image with gray levels 2, 4, 8, 16, 32, 64, and 128 and improves the different image types' quality significantly as shown in table I and table II. Also, from the PSNR results observation, we find that when the number of desired gray levels is low like 4 or 8, the difference in PSNR between the proposed method and the k-mean clustering method will be small. Moreover, as the number of gray levels increases like 32, 64, or 128 the PSNR of our method is superior to the k-mean method as shown in table II. That means, our method is considered as a quantization or multi-thresholding method more than as to be a clustering method. Also, as shown in table III and according to the SSIM results, the proposed method is efficient in conserving the general structure of the image and outperforms the other methods in all experiments according to the SSIM quality measure. The time complexity of the proposed method is considered sufficient to apply the method on big datasets and it's depending on number of quantized gray levels and image size.

ACKNOWLEDGMENT

I would like to thanks the supervisor of Tawfiq A. Al-Assadi, University of Babylon, and the staff of the College of Information technology\ Software department for their support and assistance they give to me to accomplishes this paper

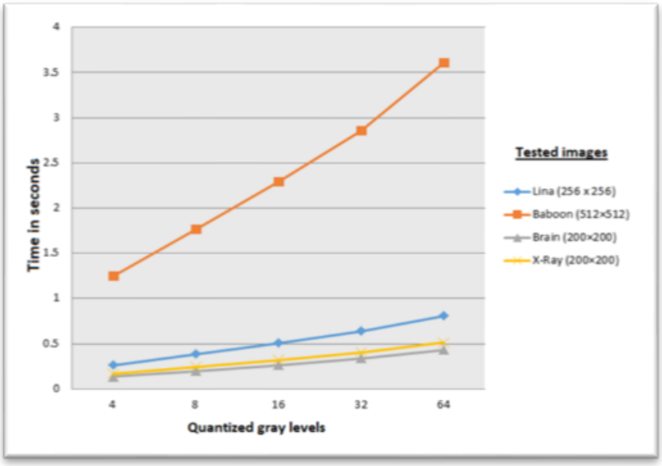

Fig. 6 Time consuming for apply proposed method on Lina, Baboon, X-Ray, and Brain images with different quantized gray levels

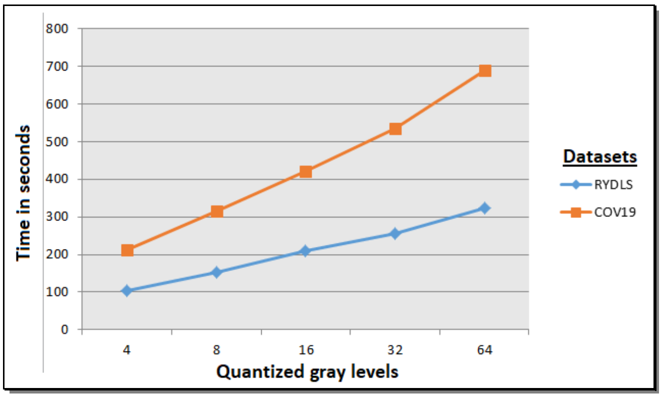

Fig. 7 Time consuming for apply proposed method on RYDLS and COV19 datasets with different quantized gray levels

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