Dipole and Quadrupole Electroexcitations of the Isovector $T = 1$ Particle-Hole States in ^{12}C *

F. A. Majeed[1,†], R. A. Radhi[2]

[1] *Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, 21941-972 Rio de Janeiro, RJ, Brazil and*

[2] *Department of Physics, College of Science, Baghdad University, Baghdad, IRAQ*

(Dated: February 3, 2008)

Electroexcitations of the dominantly $T = 1$ particle-hole states of ^{12}C are studied in the framework of the harmonic oscillator shell model. All possible $T = 1$ single-particle-hole states of all allowed angular momenta are considered in a basis including single-particle states up to the $1f$-$2p$ shell. The Hamiltonian is diagnoalized in this space in the presence of the modified surface delta interaction. Correlation in the ground state wave functions by mixing more than one configuration is considered and shows a major contribution that leads to enhance the calculations of the form factors. A comparison with the experiment shows that this model is able to fit the location of states and a simple scaling of the results give a good fit to the experimental form factors.

PACS numbers: 25.30.Dh, 13.40.G, 21.60.Cs

Nuclear structure models can be successfully tested by comparing the calculated and measured electron scattering form factors. The success of such a model reveals a valuable information about the charge and current distributions of nuclei.

The transverse electric form factor of the 2^+ level at 16.107 MeV $T = 1$ of ^{12}C were measured by Flanz *et al.*,[1] by means of 180° electron scattering over a momentum-transfer range from q=0.51 to 2.05 fm^{-1}. Deutschmann *et al.*,[2] studied the inelastic electron scattering cross section for the M1 transition to the 15.11 MeV (1^+,$T = 1$) level and for M2 transition to the 16.58 MeV (2^-,$T = 1$) level in ^{12}C in the momentum-transfer region (q=0.4–3.0 fm^{-1}). Electron scattering at 200 MeV on ^{12}C and ^{13}C, have been studied by T. Sato *et al.*, [3] to investigate magnetic dipole M1 and electric quadrupole E2 nuclear form factors. The effect of higher configurations wave functions are investigated by Bennhold *et al.*, [4]. Donnelly [5] studied the $T = 1$ single-particle-hole states of ^{12}C on the basis of the harmonic oscillator (HO) shell model in the particle-hole formalism developed by Lewis and Walecka, configuration mixing is included via a Serber–Yukawa residual interaction.

In this Letter, we study the isovector ($T = 1$) transition in ^{12}C which connects the (J^π =0^+, T=0) ground state with the (J^π =2^-, 3^- and 3^+) isovector states. The ground state is taken to have closed $1s_{1/2}$ and $1p_{1/2}$ shells. The states expected to be most strongly excited from closed-shell nuclei are linearly combination of a configurations in which one nucleon has been raised to a higher shell, forming pure single-particle-hole state [6]. This approximation is called Tamm-Dancoff approximation (TDA)[7]. The dominant dipole and quadrupole $T = 1$ single particle-hole states of ^{12}C are considered with the framework of the harmonic oscillator (HO) shell model. The Hamiltonian is diagnoalized in the space of the single-particle hole states, in the presence of the modified surface delta interaction (MSDI) [8]. The space of the single-particle-hole states include all shells up to $2p_{1/2}$ shell. Admixture of higher configurations is also considered. A comparison of the calculated form factors using this model with the available experimental data for the dominantly $T = 1$ states are discussed.

The ground state of ^{12}C is taken to have closed $1s_{1/2}$ and $1p_{3/2}$ shells, and is represented by Ψ_0. A state formed by the promotion of one particle from the shell-model ground state is called a particle-hole state. The particle-hole state of the total Hamiltonian is represented by $\Phi_{JM}(ab^{-1})$ with labels (a) for particles with quantum numbers $(n_a\ell_a j_a)$ and (b) for holes with quantum numbers $(n_b\ell_b j_b)$. The state $\Phi_{JM}(ab^{-1})$ indicating that a particle was vacated from j_b and promoted to j_a.

The excited state wave function can be constructed as a linear combinations of pure basis $\Phi^{,s}$ as [6]

$$\Psi^n_{JM} = \sum_{ab} \chi^J_{ab^{-1}} \Phi_{JM}(ab^{-1}), \quad (1)$$

where the amplitude $\chi^J_{ab^{-1}}$ can be determined from a diagonalization of the residual interaction. By including the isospin T, [7] one now has to solve the secular equation

$$\sum_{ab} [\langle \acute{a}\acute{b}^{-1}|H|ab^{-1}\rangle_{JMTT_z} - E_n \delta_{\acute{a}\acute{b}^{-1},ab^{-1}}]\, \chi^{JT}_{ab^{-1}} = 0. \quad (2)$$

The matrix element of the Hamiltonian is given by [8]

$$\langle \acute{a}\acute{b}^{-1}|H|ab^{-1}\rangle_{JMTT_z} = (e_{\acute{a}} - e_{\acute{b}})\, \delta_{a\acute{a},b\acute{b}} + \langle \acute{a}\acute{b}^{-1}|V|ab^{-1}\rangle_{JMTT_z}, \quad (3)$$

* Support by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (Brazil), and the Third World Academy of Science (TWAS) (Italy) for under grant of the scheme (TWAS-CNPq exchange programs for postdoctoral researchers).

† Permenant address: Department of Physics, College of Science, Al-Nahrain University, Baghdad, IRAQ. Email:fouad@if.ufrj.br

where $e_{\acute{a}}$-$e_{\acute{b}}$ is the unperturbed energy of the particle-hole pair obtained from energies in nuclei with A±1 particles.

The matrix element of the residual interaction V is given by the modified surface delta interaction (MSDI) with the strength parameters A_0=0.8 MeV, A_1=1.0 MeV, B=0.7 MeV and C=−0.3 MeV [8]

$$\langle \acute{a}\acute{b}^{-1}|V|ab^{-1}\rangle_{JMTT_z} = -\sum_{\acute{J}\acute{T}}(2\acute{J}+1)(2\acute{T}+1) \times \left\{ \begin{array}{ccc} j_{\acute{a}} & j_b & \acute{J} \\ j_a & j_{\acute{b}} & J \end{array} \right\} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & \acute{T} \end{array} \right\} \langle \acute{a}\, b|V|a\acute{b}\rangle_{\acute{J}\acute{T}}. \quad (4)$$

The matrix elements of the multipole operators T_J are given in terms of the single particle matrix elements by [6]

$$\langle \Psi_J \| T_{Jt_z} \| \Psi_0 \rangle = \sum_{ab} \chi^{Jt_z}_{ab^{-1}} \, \langle a \| T_{Jt_z} \| b \rangle \,, \quad (5)$$

where t_z=1/2 for protons and -1/2 for neutrons. The amplitudes $\chi^{Jt_z}_{ab^{-1}}$ can be written in terms of the amplitudes $\chi^{JT}_{ab^{-1}}$ in isospin space as [8]

$$\chi^{Jt_z}_{ab^{-1}} = (-1)^{T_f - T_i} \left[\left(\begin{array}{ccc} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{array} \right) \sqrt{2}\, \frac{\chi^{JT=0}_{ab^{-1}}}{2} + 2t_z \left(\begin{array}{ccc} T_f & 0 & T_i \\ -T_z & 0 & T_z \end{array} \right) \sqrt{6}\, \frac{\chi^{JT=1}_{ab^{-1}}}{2} \right], \quad (6)$$

where

$$T_z = \frac{Z-N}{2} \quad (7)$$

The single particle matrix elements of the electron scattering operator T^{η}_{J} are those of Ref.[9] with η selects the longitudinal (L), transverse electric $(E\ell)$ and transverse magnetic (M) operators, respectively. Electron scattering form factors involving angular momentum transfer J is given by [9]

$$|F^{\eta}_J(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \; |\langle \Psi_{J_f} \| T^{\eta}_{Jt_z} \| \Psi_{J_i} \rangle \times |F_{c.m}(q)|^2 \; |F_{f.s}(q)|^2 \quad (8)$$

where J_i= 0 and J_f=J for closed shell nuclei and q is the momentum transfer. The last two terms in Eq. (8) are the correction factors for the c.m. and the finite nucleon size $(f.s.)$[9]. The total inelastic electron scattering form factor is defined as [7]

$$|F_J(q,\theta)|^2 = |F^L_J(q)|^2 + \left[\frac{1}{2} + \tan^2(\theta/2)\right] |F^{Tr}_J(q)|^2, \quad (9)$$

where $|F^{Tr}_J(q)|^2$ is the transverse electric or transverse magnetic form factors.

The resulting particle-hole states are listed in Table I and Table II together with the values of J^{π}, for the positive and negative parity states, respectively. The configuration energies $e_a - e_b$ of these states obtained from the spectra of ^{13}C and ^{11}C [10] are also given in these tables.

TABLE I: The unperturbed energies of the particle-hole positive parity states and the possible J and $T=1$ values in ^{12}C.

Particle-hole configuration $\|a\, b^{-1}\rangle$	Particle-hole configuration energies $(e_a - e_b)$ MeV	J
$(1p_{1/2})\,(1p_{3/2})^{-1}$	13.77	1, 2
$(1d_{5/2})\,(1s_{1/2})^{-1}$	33.90	2, 3
$(2s_{1/2})\,(1s_{1/2})^{-1}$	33.14	0, 1
$(1d_{1/2})\,(1s_{1/2})^{-1}$	38.38	1, 2
$(1f_{7/2})\,(1p_{3/2})^{-1}$	25.74	2, 3, 4, 5
$(2p_{3/2})\,(1p_{3/2})^{-1}$	27.37	0, 1, 2, 3
$(1f_{5/2})\,(1p_{3/2})^{-1}$	34.17	1, 2, 3, 4
$(2p_{1/2})\,(1p_{3/2})^{-1}$	33.37	1, 2

TABLE II: The unperturbed energies of the particle-hole negative parity states and the possible J and $T=1$ values in ^{12}C.

Particle-hole configuration $\|a\, b^{-1}\rangle$	Particle-hole configuration energies $(e_a - e_b)$ MeV	J
$(1p_{1/2})\,(1s_{1/2})^{-1}$	30.05	0, 1
$(1d_{5/2})\,(1p_{3/2})^{-1}$	17.62	1, 2, 3, 4
$(1d_{3/2})\,(1p_{3/2})^{-1}$	22.11	0, 1, 2, 3
$(2s_{1/2})\,(1p_{3/2})^{-1}$	16.86	1, 2
$(1f_{7/2})\,(1s_{1/2})^{-1}$	42.02	3, 4
$(2p_{3/2})\,(1s_{1/2})^{-1}$	43.65	1, 2
$(1f_{5/2})\,(1s_{1/2})^{-1}$	50.45	2, 3
$(2p_{1/2})\,(1s_{1/2})^{-1}$	49.65	0, 1

Higher configurations are included in the calculations when the ground state is considered as a mixture of the $|(1s_{1/2})^4\,(1p_{3/2})^8\rangle$ and $|(2s_{1/2})^4\,(2p_{3/2})^8\rangle$ configurations, such that the ground state wave function becomes

$$|\Psi_{00}\rangle = \gamma|\Psi_{00}(1s_{1/2})^4(1p_{3/2})^8\rangle + \delta|\Psi_{00}(2s_{1/2})^4(2p_{3/2})^8\rangle \quad (10)$$

with $\gamma^2+\delta^2=1$, $\chi^{JT}_{ab_1^{-1}}=\gamma\chi^{JT}_{ab^{-1}}$ and $\chi^{JT}_{ab_2^{-1}}=\delta\chi^{JT}_{ab^{-1}}$

The excited states is also assumed as a mixture of the particle-hole configurations, $|a_1\, b_1^{-1}\rangle$, $|a_2\, b_2^{-1}\rangle$, $|a_2\, b_1^{-1}\rangle$ and $|a_1\, b_2^{-1}\rangle$, where $|a_1\rangle=|a\rangle=|n_a\,\ell_a\,j_a\rangle$, $|a_2\rangle=|a\rangle=|n_a+1\,\ell_a\,j_a\rangle$, $|b_1\rangle=|b\rangle=|n_b\,\ell_b\,j_b\rangle$ and $|b_2\rangle=|b\rangle=|n_b+1\,\ell_b\,j_b\rangle$. The configurations which include the higher configurations is called the extended space configurations.

The matrix element given in Eq. (5) is called the model space matrix element, where a and b are defined by the amplitudes given in Table III and Table IV, respectively.

TABLE III: Energies and amplitudes χ^{JT} for J^+ $T=1$ states.

Particle-hole configuration $\vert a\, b^{-1}\rangle$	E(1^+)=15.54 MeV χ^{11}	E(2^+)=15.49 MeV χ^{21}
$(1p_{1/2})\,(1p_{3/2})^{-1}$	0.99671	0.99774
$(1d_{5/2})\,(1s_{1/2})^{-1}$	0.00000	0.00321
$(2s_{1/2})\,(1s_{1/2})^{-1}$	0.03900	0.00000
$(1d_{1/2})\,(1s_{1/2})^{-1}$	0.01378	0.03724
$(1f_{7/2})\,(1p_{3/2})^{-1}$	0.00000	0.00892
$(2p_{3/2})\,(1p_{3/2})^{-1}$	0.05340	-0.01953
$(1f_{5/2})\,(1p_{3/2})^{-1}$	0.01823	0.03449
$(2p_{1/2})\,(1p_{3/2})^{-1}$	0.04104	0.03828

TABLE IV: Energies and amplitudes χ^{JT} for 2^- $T=1$ state.

Particle-hole configuration $\vert a\, b^{-1}\rangle$	E(2^-)=18.19 MeV χ^{21}
$(1p_{1/2})\,(1p_{3/2})^{-1}$	0.00000
$(1d_{5/2})\,(1s_{1/2})^{-1}$	-0.54940
$(2s_{1/2})\,(1s_{1/2})^{-1}$	-0.03147
$(1d_{1/2})\,(1s_{1/2})^{-1}$	0.83484
$(1f_{7/2})\,(1p_{3/2})^{-1}$	0.00000
$(2p_{3/2})\,(1p_{3/2})^{-1}$	0.01049
$(1f_{5/2})\,(1p_{3/2})^{-1}$	0.02690
$(2p_{1/2})\,(1p_{3/2})^{-1}$	0.00000

The extended space matrix element becomes

$$\begin{aligned}\langle \Psi_J \| T_{Jt_z} \| \Psi_0 \rangle = & \sum_{a_1 b_1} \chi^{Jt_z}_{a_1 b_1^{-1}} \langle a_1 \| T_{Jt_z} \| b_1 \rangle \\ & + \sum_{a_1 b_2} \chi^{Jt_z}_{a_1 b_2^{-1}} \langle a_1 \| T_{Jt_z} \| b_2 \rangle \\ & + \sum_{a_2 b_1} \chi^{Jt_z}_{a_2 b_1^{-1}} \langle a_2 \| T_{Jt_z} \| b_1 \rangle \\ & + \sum_{a_2 b_2} \chi^{Jt_z}_{a_2 b_2^{-1}} \langle a_2 \| T_{Jt_z} \| b_2 \rangle \,, \end{aligned} \qquad (11)$$

where

$$\begin{aligned} \chi^{Jt_z}_{a_1 b_1^{-1}} &= C_1\, \chi^{Jt_z}_{ab^{-1}}, \\ \chi^{Jt_z}_{a_1 b_2^{-1}} &= C_2\, \chi^{Jt_z}_{ab^{-1}}, \\ \chi^{Jt_z}_{a_2 b_1^{-1}} &= C_3\, \chi^{Jt_z}_{ab^{-1}}, \\ \chi^{Jt_z}_{a_2 b_2^{-1}} &= C_4\, \chi^{Jt_z}_{ab^{-1}}, \end{aligned} \qquad (12)$$

The values of the parameters C^{,s} are given in Table V. Experimentally the states 1^+, 2^+ and 2^- are found at

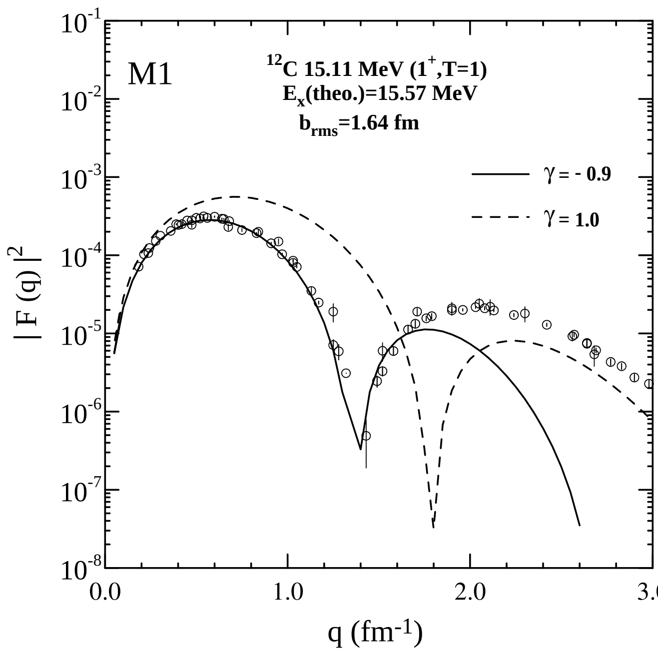

FIG. 1: Form factor for the M1 transition to the (1^+, 1) 15.57 MeV state compared with the experimental data taken from Ref. [10].

15.11 MeV, 16.11 MeV and 16.58 MeV, respectively [11]. We obtain the values 15.54 MeV, 15.52 MeV and 18.19 MeV for the states 1^+, 2^+ and 2^-, respectively.

The 1^+ (15.11 MeV), M1 form factor is shown in Fig. 1. The dashed curve represents the calculations with the amplitudes $\chi^{,s}$ reduced by a factor 2, to agree with the low q data [6]. The single-particle matrix elements are calculated with the harmonic oscillator wave functions (HO) with oscillator parameter $b = 1.64$ fm to agree with the elastic form factor determination [12]. A best fit to the M1 form factor data is obtained by Hicks *et al.* [11] with $b = 1.67$ fm.

Karataglidis *et al.* [13] adopted the value 1.7 fm in describing the form factors for the $0 \rightarrow 2^+$ (4.44 MeV and 16.11 MeV) transitions in ^{12}C. The low momentum transfer data for the 15.11 MeV 1^+ state are fitted with $b = 1.9$ fm [14].

This b value is adopted also by Donnelly [6], to describe the M1 form factors up to momentum transfer 1.4 fm^{-1}. In our work we fix the value of $b = 1.64$ fm and trying to find other contributions that can describe the experimental form factors. When the M1 form factor is calculated with the extended space configurations, the result is shown by the solid curve in Fig. 1. The data are very well described up to momentum-transfer 1.7 fm^{-1} , and the form factor is shifted to agree with the location of the experimental diffraction minimum. The calculated form factor decreases too steeply compared to the data at high q values. All previous theoretical treatments shows this behaviour. Our results are consistent with those of Sato *et al.* [12], where the meson exchange current is

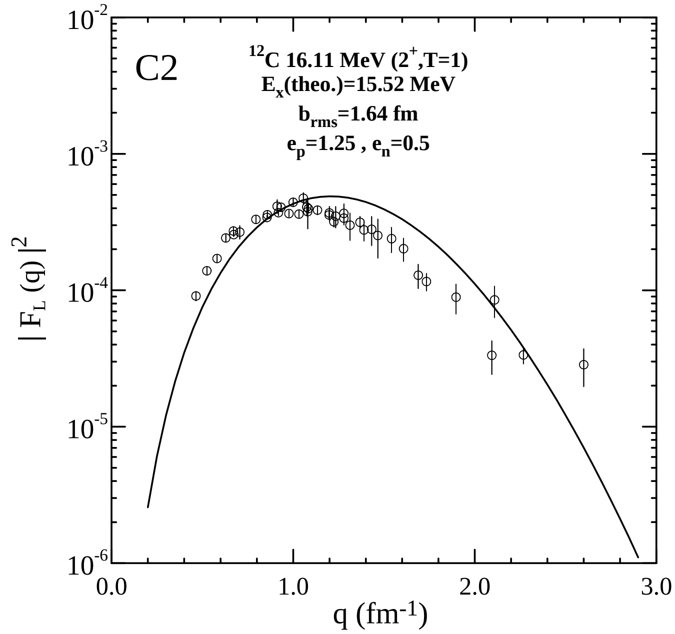

FIG. 2: Longitudinal form factor for the C2 transition to the $(2^+, 1)$ 15.52 MeV state compared with the experimental data taken from Ref. [16].

included but shows a minor contribution.

The Coulomb C2 and the transverse E2 form factors are shown in Figs. 2 and 3. The amplitudes χ have to be reduced also by a factor 2 to fit the low q data. The coulomb form factor is calculated with core-polarization effects by introducing effective charges [15] for the protons ($1.25e$) and for the neutrons ($0.5e$). With these effective charges, the experimental data are very well described for the region of the momentum transfer $q \leq 2.2$ fm^{-1}. The same agreement are obtained for the E2 form factor.

The transverse M2 form factor for the excitation to the 2^- 16.58 MeV state are shown in Fig. 4. The dashed curve represents the model space calculations with $b = 1.64$ fm, where the data can not be satisfactorily described, especially in the region of the low q values. Due to the absence of an accepted model, Hicks *et al.* [11] used a simple phenomenological procedure to interpret the data. In our work the data are well described using the extended space configurations and $b = 1.35$ fm. The amplitudes are quenched by 86%. The results are consistent with those of Hicks *et al.*,[11] a diffraction minimum is obtained at $q \cong 0.6$ fm^{-1}.

Good agreement is obtained in comparison of the form factors for group of single-particle-hole states with the experimental data. The amplitudes of the transition to the positive-parity states considered in this work have to be reduced by a factor of 2 to describe the low q data. The form factors for the odd-parity particle-hole states are in reasonable agreement with the experiment and need a reduction factor of 1.16 in the amplitudes χ to yield excellent agreement. Higher configurations are necessary to improve the agreement with the low q values of the form factors.

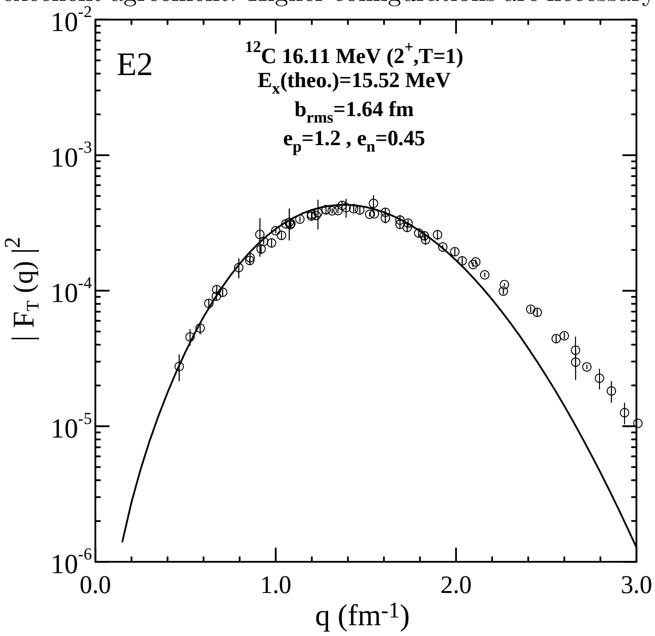

FIG. 3: Transverse electric form factor for the E2 transition to the $(2^+, 1)$ 15.52 MeV state compared with the experimental data taken from Ref. [16].

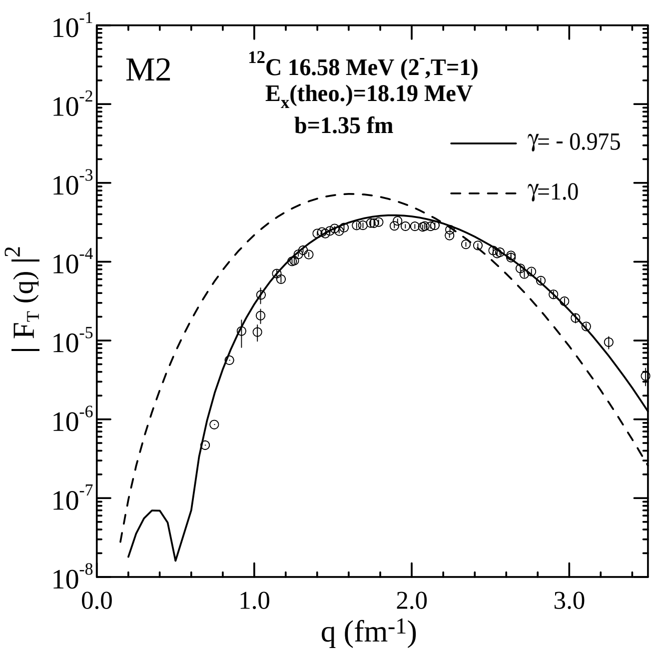

FIG. 4: Transverse magnetic form factor for the M2 transition to the $(2^-, 1)$ 18.19 MeV state compared with the experimental data taken from Ref. [10].

TABLE V: Values of the parameters $C^{,s}$ used in the extended space calculations.

J^π	C_1	C_2	C_3	C_4
1^+	0.92	-0.27	-0.27	0.078
2^-	-0.92	0.27	-0.27	0.078

[1] Flanz J B et al 1978 Phys. Rev. Lett. **41** 1642

[2] Deutschmann U et al 1983 Nucl. Phys. A **411** 337

[3] Sato T, Koshigiri K, and Ohtsubo H 1985 Z. Phys. A **320** 507

[4] Bennhold C et al 1992 Phys. Rev. C **46** 2456

[5] Donnelly T W 1970 Phys. Rev. C **1** 833

[6] Donnelly T W and Sick I 1984 Rev. Mod. Phys. Vol. **56** (3) 461

[7] deForest T, Jr and Walecka J D 1966 Adv. Phys. **15** 1

[8] Brussaard P J and Glaudemans P W M 1977*Shell-Model Applications in Nuclear Spectrscopy* (Amsterdam: North Holland)

[9] Brown B A et al 1985 Phys. Rev. C **32** 1127

[10] Deutschmann U, Lahm G, Neuhausen R and Bergstrom J C 1983 Nucl. Phys. A **411** 337

[11] Hicks R S, Huffman R L, Lindgren R A, Peterson G A, Plum M A and Button-Shafer J 1987 Phys. Rev. C **36** 485

[12] Sato T, Koshigiri K, and Ohtsubo H 1985 Z.Phys. A **320** 507

[13] Karataglidis S, Halse P and Amos K 1995 Phys. Rev. C **51** 2494

[14] Gudden F 1964 Phys.Lett. **10** 313

[15] Brown B A, Radhi R A and Wildenthal B H 1983 Phys. Rep. **101** 311

[16] Tiator L, Bennhold C and Nehausen R 1989 Nucl. Phys. A **501** 751