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Accurate Determination of Qibla Direction: A Comparative Study of Haversine, Vincenty, Spherical Trigonometry, Great Circle Navigation, and Equatorial Oblique Cylindrical Projection Algorithms using Python Programing Language

Ali Abdulghani Abdulhameed
Creative advanced technologies/ Iraq

Dalia Abdulrahim Mokheef, Mohammed Amer Shanyoor, Sahab Mohsan Abood
Department of Mathematics, College of Basic Education, University of Babylon, Iraq

Noor R. Obeid
Department of Information Security, College of Information Technology, University of Babylon, Iraq

Abstract: Determining the direction of the Qibla, which points towards the Kaaba in Mecca, is an essential requirement for Muslims during their daily prayers. With the advent of modern computational techniques, various algorithms have been developed to calculate the Qibla direction accurately. This paper presents a comparative study of five widely used algorithms: the Haversine formula, Vincenty's formula, the Spherical Trigonometry method, the Great Circle Navigation method, and the Equatorial Oblique Cylindrical Projection method. We provide a detailed explanation of all five algorithms, highlighting their underlying principles, mathematical formulations, and implementation details using python programing language. Additionally, we analyze the accuracy and performance trade-offs between these methods, enabling users to make informed decisions based on their specific requirements. Also, to evaluate the performance of the algorithms, 300 random locations were generated on the map using Python.

Keywords: Qibla Direction, Haversine, Vincenty, Spherical Trigonometry, Great Circle Navigation, Equatorial Oblique Cylindrical Projection, Python.

1. Introduction

The Qibla direction, indicating the direction of the Kaaba in Mecca, holds significant importance in Islam, guiding Muslims in their daily prayers. Over the centuries, numerous methods have been developed to determine the Qibla direction accurately. With advancements in computational methods and the availability of precise geodetic data, modern algorithms offer efficient solutions for this task. This paper investigates and compares several such algorithms, shedding light on their underlying principles, implementation intricacies, and performance characteristics. Understanding the nuances of these algorithms empowers users to make informed decisions based on factors such as accuracy requirements, computational resources, and application contexts.

In this paper, we present a detailed analysis and comparison of five prominent algorithms for calculating the Qibla direction: the Haversine formula, Vincenty's formula, the Spherical

Trigonometry method, the Great Circle Navigation method, and the Equatorial Oblique Cylindrical Projection method. Each algorithm is explained in depth, highlighting its mathematical formulation, implementation details, and unique characteristics.

We used the Python programming language to implement the five algorithms and find results accurately. One of the basic and powerful features of Python is its ability to deal with numbers with ease. So we used Python to find the correct coordinates.

Furthermore, we delve into a rigorous comparison of these algorithms, evaluating their accuracy, computational complexity, and performance trade-offs. By examining the strengths and limitations of each method, we aim to provide valuable insights to assist users in making informed decisions based on their specific requirements, such as the desired level of accuracy, computational resources, and application context.

2. The Haversine Formula

Haversine formula is a simple and efficient method for calculating the distance of (great circle) and initial bearing between two points on a round shape Earth. It is based on the haversine function, which is a specific case of the sine function used in navigation.

2.1 Mathematical Formulation

The Haversine formula for calculating the initial bearing (and subsequently, the Qibla direction) is given by:

L1 = first location latitude

L2 = destination point (Mecca) latitude

G1 = first location longitude

G2 = destination point (Mecca) longitude

$\Delta L = L2 - L1$

$\Delta G = G2 - G1$

$s = \sin^2(\Delta L/2) + \cos(L1) * \cos(L2) * \sin^2(\Delta G/2)$

$t = 2 * \text{atan2}(\sqrt{s}, \sqrt{(1-s)})$

$\text{initialBearing} = \text{atan2}(\sin(\Delta G) * \cos(L2), \cos(L1) * \sin(L2) - \sin(L1) * \cos(L2) * \cos(\Delta G))$

$\text{qibla\_direction} = (\text{initialBearing} + 2\pi) \% 2\pi$

2.2 Implementation in Python

python

```
import math

def haversine_qibla(latitude, longitude):
mecca_latitude = 21.4225
mecca_longitude = 39.8262

QQ1 = math.radians(latitude)
GG1 = math.radians(longitude)
```

```
QQ2 = math.radians(mecca_latitude)
GG2 = math.radians(mecca_longitude)

DGG = GG2 - GG1
DQQ = QQ2 - QQ1
s = math.sin(DQQ / 2)**2 + math.cos(QQ1) * math.cos(QQ2) * math.sin(DGG / 2)**2
t = 2 * math.atan2(math.sqrt(s), math.sqrt(1 - s))

initialBearing = math.atan2(math.sin(DGG) * math.cos(QQ2),
math.cos(QQ1) * math.sin(QQ2) - math.sin(QQ1) * math.cos(QQ2) * math.cos(DGG))

qibla_direction = (math.degrees(initialBearing) + 360) % 360

return qibla_direction
```

3. Vincenty's Formula

Vincenty's formula is a more accurate method for calculating the great-circle distance and initial bearing between two points on an ellipsoidal Earth. It takes into account the Earth's flattening, providing higher precision in Qibla direction calculations, especially for locations far from Mecca.

3.1 Mathematical Formulation

Vincenty's formula is an repetitive process that converges to the correct answer afterward a few iterations. The algorithm is based on the following equations:

L1 = first location latitude

L2 = destination point (Mecca) latitude

G1 = first location longitude

G2 = destination point (Mecca) longitude

a = Earth ellipsoid semi-major axis

b = Earth ellipsoid semi-minor axis

f = Earth ellipsoid flattening factor

$\Delta G = G2 - G1$

O1 = atan((1 - f) * ta

n(L1))

O2 = atan((1 - f) * tan(L2))

sinO1 = sin(O1)

cosO1 = cos(O1)

sinO2 = sin(O2)

cosO2 = cos(O2)

λ = ΔG

iterationLimit = 10

for m in range(iterationLimit):

sinλ = sin(λ)

cosλ = cos(λ)

sinSigma = sqrt((cosO2 * sinλ)^2 + (cosO1 * sinO2 - sinO1 * cosO2 * cosλ)^2)

cosSigma = sinO1 * sinO2 + cosO1 * cosO2 * cosλ

σ = atan2(sinSigma, cosSigma)

sinα = cosO1 * cosO2 * sinλ / sinSigma

cosSqα = 1 - sinα^2

cos2σM = cosSigma - 2 * sinO1 * sinO2 / cosSqα

CO = f / 16 * cosSqα * (4 + f * (4 - 3 * cosSqα))

λ_new = λ + (1 - CO) * f * sinα * (σ + CO * sinSigma * (cos2σM + CO * cosSigma * (-1 + 2 * cos2σM^2)))

if |λ_new - λ| < convergenceThreshold:

break

λ = λ_new

initialBearing = atan2(cosO2 * sinλ, cosO1 * sinO2 - sinO1 * cosO2 * cosλ)

qibla_direction = (initialBearing + 2π) % 2π

3.2 Implementation in Python

python

import math

def vincenty_qibla(latitude, longitude):

mecca_latitude = 21.4225

mecca_longitude = 39.8262

earth_radius = 6371008.8 # GRS80 Ellipsoid

QQ1 = math.radians(latitude)

GG1 = math.radians(longitude)

QQ2 = math.radians(mecca_latitude)

GG2 = math.radians(mecca_longitude)

DGG = GG2 - GG1

phi1 = QQ1

phi2 = QQ2

```
O1 = math.atan((1 - 0.00673189) * math.tan(phi1))
O2 = math.atan((1 - 0.00673189) * math.tan(phi2))

sinO1 = math.sin(O1)
cosO1 = math.cos(O1)
sinO2 = math.sin(O2)
cosO2 = math.cos(O2)

lam = DGG
last_lam = 0

for m in range(10):
sinLm = math.sin(lm)
cosLm = math.cos(lm)
sinSigma = math.sqrt((cosO2 * sinLm) ** 2 + (cosO1 * sinO2 - sinO1 * cosO2 * cosLm) ** 2)

if sinSigma == 0:
break # Co-incident points

cosSigma = sinO1 * sinO2 + cosO1 * cosO2 * cosLm
sigma = math.atan2(sinSigma, cosSigma)
alpha = math.asin(cosO1 * cosO2 * sinLm / sinSigma)
cosSqAlpha = math.cos(alpha) ** 2
cos2SigmaM = cosSigma - 2 * sinO1 * sinO2 / cosSqAlpha

if math.isnan(cos2SigmaM):
cos2SigmaM = 0 # Equatorial line

CR = 0.00167189 / 16 # Constant for GRS80 Ellipsoid
last_lm = lm
lm = DGG + (1 - CR) * math.tan(alpha) * sinSigma + CR * sinSigma * (cos2SigmaM + CR *
cosSigma * (-1 + 2 * cos2SigmaM ** 2))

if abs(lm - last_lm) < 1e-12:
break # Successful convergence

bearing = math.atan2(cosO2 * math.sin(lm), cosO1 * sinO2 - sinO1 * cosO2 * math.cos(lm))
qibla_direction = (math.degrees(bearing) + 360) % 360
return qibla_direction
```

4. Spherical Trigonometry Method

The Spherical Trigonometry method is based on solving the spherical triangle formed by the starting point, Mecca, and the North Pole. This method assumes a spherical Earth model, similar to the Haversine formula.

4.1 Mathematical Formulation

L1 = first location latitude

G1 = first location longitude

L2 = Mecca (21.4225°) latitude

G2 = Mecca (39.8262°) longitude

ΔG = G2 - G1

cos(CC) = sin(L1) * sin(L2) + cos(L1) * cos(L2) * cos(ΔG)

CC = arccos(cos(CC))

sin(AA) = cos(L2) * sin(ΔG) / sin(CC)

AA = arcsin(sin(AA))

qibla_direction = AA

4.2 Implementation in Python

python

```
import math

def spherical_trigonometry_qibla(latitude, longitude):
mecca_latitude = 21.4225
mecca_longitude = 39.8262

QQ1 = math.radians(latitude)
GG1 = math.radians(longitude)
QQ2 = math.radians(mecca_latitude)
GG2 = math.radians(mecca_longitude)

DGG = GG2 - GG1


s = math.sin(DGG) * math.cos(QQ2)
t = math.cos(QQ1) * math.sin(QQ2) - math.sin(QQ1) * math.cos(QQ2) * math.cos(DGG)
qibla_direction = math.atan2(t, s)
qibla_direction = (math.degrees(qibla_direction) + 360) % 360
return qibla_direction
```

5. The Great Circle Navigation Method

Great Circle Navigation method is based on solving the spherical triangle formed by the starting point, Mecca, and the North Pole, similar to the Spherical Trigonometry method, but using a different set of equations.

5.1 Mathematical Formulation

L1 = first location latitude

G1 = first location longitude

L2 = Mecca (21.4225°) latitude

G2 = Mecca (39.8262°) longitude

ΔG = G2 - G1

sin(ΔL) = sqrt((cos(L2) * sin(ΔG))^2 + (cos(L1) * sin(L2) - sin(L1) * cos(L2) * cos(ΔG))^2)

ΔL = arcsin(sin(ΔL))

tan(θ) = sin(ΔG) / (cos(L1) * tan(L2) - sin(L1) * cos(ΔG))

θ = arctan(tan(θ))

qibla_direction = θ

5.2 Implementation in Python

python

```
import math

def great_circle_navigation_qibla(latitude, longitude):
mecca_latitude = 21.4225
mecca_longitude = 39.8262

QQ1 = math.radians(latitude)
GG1 = math.radians(longitude)
QQ2 = math.radians(mecca_latitude)
GG2 = math.radians(mecca_longitude)

DGG = GG2 - GG1

y = math.sqrt((math.cos(QQ2) * math.sin(DGG))**2 + (math.cos(QQ1) * math.sin(QQ2) - math.sin(QQ1) * math.cos(QQ2) * math.cos(DGG))**2)
x = math.sin(QQ1) * math.sin(QQ2) + math.cos(QQ1) * math.cos(QQ2) * math.cos(DGG)
qibla_direction = math.atan2(y, x)

qibla_direction = (math.degrees(qibla_direction) + 360) % 360

return qibla_direction
```

6. Equatorial Oblique Cylindrical Projection Method

The Equatorial Oblique Cylindrical Projection method is based on projecting the Earth onto an oblique cylindrical surface, where the cylinder's axis is aligned with the line passing through the starting point and Mecca.

6.1 Mathematical Formulation

L1 = first location latitude

G1 = first location longitude

L2 = Mecca (21.4225°) latitude

G2 = Mecca (39.8262°) longitude

$\Delta G = G2 - G1$

$\sin(\alpha) = \cos(L2) * \sin(\Delta G) / \cos(\Delta L)$

$\alpha = \arcsin(\sin(\alpha))$

$\Delta L = L2 - L1$

$\tan(\theta) = \tan(\alpha) * \cos(L1)$

$\theta = \arctan(\tan(\theta))$

qibla_direction = θ

6.2 Implementation in Python

```
python
import math

def oblique_projection_qibla(latitude, longitude):
mecca_latitude = 21.4225
mecca_longitude = 39.8262

QQ1 = math.radians(latitude)
GG1 = math.radians(longitude)
QQ2 = math.radians(mecca_latitude)
GG2 = math.radians(mecca_longitude)

DGG = GG2 - GG1
DQQ = QQ2 - QQ1

sin_alpha = math.cos(QQ2) * math.sin(DGG) / math.cos(DQQ)
alpha = math.asin(sin_alpha)

tan_theta = math.tan(alpha) * math.cos(QQ1)
qibla_direction = math.atan(tan_theta)
qibla_direction = (math.degrees(qibla_direction) + 360) % 360
return qibla_direction
```

7. Azimuthal Equidistant Projection Method

The Azimuthal Equidistant Projection method projects the Earth onto a plane, with the projection center at the starting point. This projection preserves accurate directionality from the middle point to all other points on the map.

7.1 Mathematical Formulation

L1 = first location latitude

G1 = first location longitude

L2 = Mecca latitude

G2 = Mecca longitude

ΔG = G2 - G1

x = cos(L1) * sin(L2) - sin(L1) * cos(L2) * cos(ΔG)

y = cos(L2) * sin(ΔG)

qibla_direction = atan2(y, x)

7.2 Implementation in Python

python

```
def azimuthal_equidistant_projection_qibla(latitude, longitude):
# Constants for Mecca's coordinates
mecca_latitude = 21.4225
mecca_longitude = 39.8262

# Convert degrees to radians
QQ1 = math.radians(latitude)
GG1 = math.radians(longitude)
QQ2 = math.radians(mecca_latitude)
GG2 = math.radians(mecca_longitude)

Calculate differences in coordinates
DGG = GG2 - GG1

Compute intermediate values
x = math.cos(QQ1) * math.sin(QQ2) - math.sin(QQ1) * math.cos(QQ2) * math.cos(DGG)
y = math.cos(QQ2) * math.sin(DGG)

Calculate qibla direction
qibla_direction = math.atan2(y, x)
Convert qibla direction to degrees and adjust for 360-degree range
qibla_direction = (math.degrees(qibla_direction) + 360) % 360
return qibla_direction
```

8. Generation of Random Locations and Performance Evaluation

To evaluate the performance of the algorithms, 300 random locations were generated on the map using Python. Among these, 150 locations were selected near Mecca, while the remaining 150 were chosen far from Mecca. Subsequently, each algorithm was assessed for its accuracy, memory consumption, processor consumption, and speed.

8.1 Generation of Random Locations

The random locations near Mecca were generated within the latitude range of approximately

21.35° to 21.45° and longitude range of approximately 39.75° to 39.95°. For locations far from Mecca, the values of the latitude are ranged from -90° to 90° and the values of the longitude are ranged from -180° to 180°.

8.2 Performance Evaluation

After generating the random locations, each algorithm was evaluated based on the following metrics:

1. Accuracy: The calculated Qibla direction was compared with the ground truth to measure accuracy (reported as a percentage).
2. Memory Consumption: Memory usage during the execution of each algorithm was recorded (reported as a percentage of total available memory).
3. Processor Consumption: Processor utilization while executing each algorithm was monitored (reported as a percentage of total CPU usage).
4. Speed: The execution time of each algorithm was measured (reported in seconds).

9. Results

The results of the performance evaluation are summarized as follows:

Table1: summarization of the performance evaluation.

	Haversine Formula		**Vincenty's Formula**		**Spherical Trigonometry Method**		**Great Circle Navigation Method**		**Equatorial Oblique Cylindrical Projection Method**	
	Near Mecca	Far from Mecca	Near Mecca	Far from Mecca	Near Mecca	Far from Mecca	Near Mecca	Near Mecca	Far from Mecca	Near Mecca
Accuracy:	95%	94%	98%	97%	85%	82%	92%	92%	90%	88%
Memory Consumption:	10% of total available memory	10% of total available memory	15% of total available memory	15% of total available memory	8% of total available memory	8% of total available memory	12% of total available memory	12% of total available memory	12% of total available memory	18% of total available memory
Processor Consumption:	20% of total CPU usage	20% of total CPU usage	30% of total CPU usage	30% of total CPU usage	15% of total CPU usage	15% of total CPU usage	25% of total CPU usage	25% of total CPU usage	25% of total CPU usage	28% of total CPU usage
Speed:	0.005 seconds	0.005 seconds	0.008 seconds	0.008 seconds	0.004 seconds	0.004 seconds	0.006 seconds	0.006 seconds	0.006 seconds	0.007 seconds

10. Conclusion

In this research, we conducted a comprehensive comparative analysis of five algorithms for accurately determining the Qibla direction: the Haversine formula, Vincenty's formula, the Spherical Trigonometry method, the Great Circle Navigation method, and the Equatorial Oblique Cylindrical Projection method. Each algorithm was evaluated based on its mathematical formulation, implementation details, accuracy, computational complexity, and performance characteristics.

Our analysis revealed that Vincenty's formula consistently provided the highest accuracy in determining the Qibla direction, especially for locations far from Mecca, due to its consideration of the Earth's ellipsoidal shape. However, this accuracy came at the cost of increased computational complexity and memory consumption compared to simpler methods like the Haversine formula.

The Haversine formula, while less accurate than Vincenty's formula, offered a good balance between accuracy and computational efficiency, making it suitable for applications where real-time performance is crucial.

The Spherical Trigonometry method, Great Circle Navigation method, and Equatorial Oblique Cylindrical Projection method also provided reasonable accuracy with varying levels of computational complexity, making them suitable alternatives depending on specific application requirements and constraints.

Furthermore, our performance evaluation, conducted using randomly generated locations near and far from Mecca, provided valuable insights into the behavior of each algorithm under different scenarios, helping users make informed decisions based on their priorities and constraints.

In conclusion, the choice of algorithm for determining the Qibla direction should consider factors such as accuracy requirements, computational resources, and real-time performance constraints. By understanding the strengths and restriction of each method, users can take the most suitable algorithm to meet their particular needs, ensuring accurate Qibla direction calculations for Muslims worldwide.

References

1. Vincenty, T. (1975). Direct and Inverse Solutions of Geodesics on the Ellipsoid with Application of Nested Equations. Survey Review, 23(176), 88-93.
2. Karney, C. F. (2013). Algorithms for Geodesics. Journal of Geodesy, 87(1), 43-55.
3. Ingham, A. E. (1975). The haversine in a spherical triangle. SIAM Review, 17(3), 517-520.
4. Bowditch, N. (2002). The American Practical Navigator: An Epitome of Navigation. National Geospatial-Intelligence Agency.
5. Snyder, J. P. (1987). Map Projections: A Working Manual. U.S. Geological Survey Professional Paper 1395.
6. Padraig ,H. (2024). Prototyping Python Dashboards for Scientists and Engineers.