Quasi-Centroids Of Dendriform Algebras

Mohammed Abdul Daim Zoba[1]

College of Engineering\ Al-Musayab, Energy Engineering Department, University of Babylon
met.moh.abdaldaaem@uobabylon.edu.iq

ABSTRACT

The current study concentrated on the perception of quasi-centroids for dendriform algebras as well as their few properties. Further, we determined the two dimensional study algebra's quasi centroid & provided their classification. We focused on the two dimensions of the quasi-centroids of dendriform algebras. Thus, the usage of the outcomes of classification of dendriform algebras, we offered an explanation of two dimensional quasi-centriods of dendriform algebras.

Keywords: Dendriform algebra; Derivation; Quasi-centroid; Centroid.

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1 Introduction

The principal aim of this study is based on the centroids of Dendriform algebras. Thus, the $\Omega(E)$ centroid of a 'L' Lie algebra represents the space of $L-$module homomorphism known as ω on L : $\omega([a,b]) = [a,\omega(b)]$ for all $a,b \in L$.

Moreover, the problems of classification of the dendriform algebra by utilizing the geometric and algebraic techniques provoked a concern in the centroids of algebras. The dendriform algebra introduced by Loday [2] with a motivation provide dual of dialgebras, and have an effective computing of other algebra classes' quasi-centroids. Afterwards, this method is applied to observe the 2-dimensional quasi-centroid.

2 Preliminaries

This section reviews the different elementary feats on the centroids of algebras.

Definition 2.1. This definition deals with a dendriform algebra .It *exist as a vector space* E *compromised of maps* $\prec: E \times E \rightarrow E$ *and* $\succ: E \times E \longrightarrow E$. *It fulfills the axioms:*

$$(a \succ b) \succ c = a \succ (b \succ c + b \succ c) \quad (1)$$

been additionally considered with acquaintances to numerous sections in physics and mathematics comprising quantum field, combinatorics, Leibniz & Lie algebras, hopf algebra, homology and operads.

One of the significant outcome has indicated that whole scalar extension of a simple if & merely if centroids consist of scalars in their base field specifically the simple associative, finite dimensional algebras. The centroid is crucial in investigation of divisional algebras, Brauer groups

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The centroids of nilpotent are determined by Melville & Benkarta, and researchers can further study the papers [1], [4], [5], [17] and [16] along with other references provided within these research papers for further information. Hence, our major concern is about attaining these results on centroids for the Leibniz algebra.

In current research paper, the concentration is on the intricate field of 2 dimensional dendriform algebra's quasi centroid. Such method is established to define centroids as well as to make

$$(a \prec b) \succ c = a \prec (b \succ c) \quad (2)$$

$$a \prec (b \succ c) = (a \succ b + a \prec b) \succ c \quad (3)$$

for all a, b *and* $c \in E$.

Let $S \subseteq E$. The subset

$$Z_E(S) = \{a \in E | a * S = S * a = 0\}$$

is called centralizer.

Next Definition which is 2.2 deals with *a dendriform algebra derivation E , a linear transfiguration* $d : E \to E$ *Satisfying*

$$d(a * b) = d(a) * b + a * d(b)$$

for all a, b ∈ *E.*

The entire set of dendriform algebra 's derivation E is a subsequence of $End_K(E)$. Thus, this subspace is well fortified with bracket $[d_1, d_2] = d_1 \text{ o } d_2 - d_2 \text{ o } d_1$, & hence designated by $Der(E)$. Such perception of nilpotent algebra is presented by [7] observations and the advance of the current concept development. And readers can see [10], [11], [12], [13], [14], [15],(also see [9], and [8]).

Definition 2.3. *Here, E is presented as an arbitrary dendriform algebra above a field* K.

$$\Omega_K^*(E) = \{\omega \in End_K(E) | \omega(a * b) = a * \omega(b) = \omega(a) * b \text{ for all } a, b \in E\},$$

where the $*$ is $\prec$ and $\succ$, respectively.

We will transcribe $\Omega_K^*(A)$ for $\Omega^*(E)$) in case, it is significant to accentuate the dependency on *K*. The centroid of the associative dendriform algebra *A* is defined as Ω(*A*) = $\Omega^{\prec}(E) \cap \Omega^{\succ}(E)$.

Also we can define the set

$$Q\Gamma_K(E) == \{\omega \in End_K(E) | x * \omega(b) = \omega(a) * b \text{ for all } a, b \in E\},$$

and this termed as the Leibniz algebra L quasi-centroid.

Next definition which is 2.4. *when* E *is a dendriform algebra and* $\omega \in End(E)$. *Then* ω *would be a central derivation, if* $\omega(E) \subseteq Z(E)$ *and* $\omega(E^2) = 0$.

Thus, the left & right centroids $\Omega^{\prec}(E)$ *and* $\Omega^{\succ}(E)$ *of E are the spaces of* K*-linear transformations on E given by*

The total central derivations related to E set can be presented using *C*(*E*). This is a mere statement to perceive that *C*(*E*) ⊆ Ω(*E*). In reality, *C*(*E*) is a model of Ω(*E*).

Definition 2.5. *Let consider E as an indecomposable of dendriform algebra. However, the* $\Omega(E)$ *is small if* $\Omega(E)$ *is produced by the scalars and the central derivations.*

In the preceding, we offer a small number of prior outcomes on few properties of centroids of dendriform algebras that indicate the association amongst centroid and derivation. Thus, the evidence of few of the facts specified below can be observed in [3].

Theorem 2.1. Considering $(E, \prec, \succ)$ as a dendriform algebra. Then

$i)$ $\Omega(E)Der(E) \subseteq Der(E)$.

$ii)$ $[\Omega(E), Der(E)] \subseteq \Omega(E)$.

$iii)$ $[\Omega(E), \Omega(E)](E) \subseteq C(E)$ and $[\Omega(E), \Omega(E)](E^2) = 0$

Theorem 2.2. *Let* $(E, \prec, \succ)$ *be a dendriform algebra. Then for any* $d \in Der(E)$ *and* $\omega \in \Omega(E)$ *one has the following.*

(a) *The composition of* $d\omega$ *is in* $\Omega(E)$ *in case that* ωd *is the E's central derivation;*

(b) *The composition* $d\omega$ *can be described as derivation of* E. *It is merely if* $[E, \omega]$ *is a* 's central derivation.

3 Process involved in calculating Quasi-Centroid

In this segment the particulars of quasi-centroid related to dendriform algebras in two dimensions over the intricate field $\mathbb{C}$. is provided. Hence, $\{r_1, r_2, r_3, \ldots, r_n\}$ *is selected as* the fundamental of an $n-$ dimensional dendriform algebra E. Therefore, the basis element's product is given as

$$r_i \prec r_j = \sum_{k=1}^{n} \gamma_{ij}^{k} r_k = 0 \text{ and } r_s \succ r_t = \sum_{l=1}^{n} \delta_{st}^{l} r_l, \quad i, j, s, t, = 1,2, \ldots, n.$$

Meanwhile, the $\Omega(E)$ quasi centroid represents the vector space's (L) linear transformation, whereas, $\omega \in \Omega(E)$ is an element which is signified in matrix form of $[a_{ij}]_{i,j=1,2,\ldots,n}$, i.e. $\omega(r_i) = \sum_{k=1}^{n} a_{ji} r_j, \; i = 1,2, \ldots, n.$ Conferring to the centroid's definition, entries a_{ij} $i, j = 1,2, \ldots, n$ of the matrix$[a_{ij}]_{i,j=1,2,\ldots,n}$, we get:

$$\sum_{t=1}^{n} \gamma_{it}^{k} a_{tj} - a_{ti}\gamma_{tj}^{k} = 0$$

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hus, in the quasi-centroid's dendriform algebra , the L, under contemplation could be termed by solving the equation system above thea_{ij} $i, j = 1,2, \ldots, n$, as soon as the constants $\{\gamma_{ij}^{k}\}$ of L are provided. Hence, we utilized the classification outcomes of the complex algebras of two dimensional dendriform from the [6].

$$\sum_{k=1}^{n} (\gamma_{ij}^{k} a_{kt} - a_{tj}\gamma_{it}^{k}) = 0$$

and

$$\sum_{t=1}^{n} (\delta_{st}^{k} a_{kt} - a_{sl}\delta_{st}^{k}) = 0$$

The classification of all two-dimensional dendriform algebras has been given by [6]. Therefore taking into account the classification result on associative algebras.

Theorem 3.1. Some of 2-dimensional dendriform algebra could be incorporated in one of the algebras' classes given below:

$Dend_2^1(\alpha):\; r_1 \prec r_1 = r_2, \quad r_1 \succ r_1 = \alpha r_2, \quad \alpha \in \mathbb{C};$

$Dend_2^2$: $r_1 \prec r_1 = r_1$, $r_1 \succ r_2 = r_2$;

$Dend_2^3$: $r_1 \prec r_1 = r_1$, $r_2 \prec r_1 = r_2$, $r_1 \succ r_2 = r_2$, $r_2 \succ r_1 = -r_2$;

$Dend_2^4$: $r_2 \prec r_1 = r_2$, $r_1 \succ r_1 = r_1$;

$Dend_2^5$: $r_1 \prec r_2 = -r_2$, $r_2 \prec r_1 = r_2$, $r_1 \succ r_1 = r_1$, $r_1 \succ r_2 = r_2$;

$Dend_2^6$: $r_1 \prec r_1 = r_1$, $r_2 \succ r_2 = r_2$;

$Dend_2^7$: $r_1 \prec r_2 = -r_2$, $r_2 \prec r_2 = r_2$, $r_1 \succ r_1 = r_1$, $r_1 \succ r_2 = r_2$;

$Dend_2^8$: $r_1 \prec r_1 = r_1 + r_2$, $r_1 \prec r_2 = -r_2$, $r_2 \prec r_2 = r_2$, $r_1 \succ r_1 = -r_2$; $r_1 \succ r_2 = r_2$;

$Dend_2^9$: $r_1 \prec r_1 = r_1$, $r_2 \prec r_1 = r_2$, $r_2 \succ r_1 = -r_2$ $r_2 \succ r_2 = r_2$;

$Dend_2^{10}$: $r_1 \prec r_1 = -r_2$, $r_2 \prec r_1 = r_2$, $r_1 \succ r_1 = r_1 + r_2$; $r_2 \succ r_1 = -r_2$

$r_2 \succ r_2 = r_2$;

$Dend_2^{11}$: $r_2 \prec r_1 = r_2$, $r_1 \succ r_1 = r_1$, $r_1 \succ r_2 = r_2$;

$Dend_2^{12}$: $r_1 \prec r_1 = r_1$, $r_2 \prec r_1 = r_2$, $r_1 \succ r_2 = r_2$.

Theorem 3.2. *The complex two dimensional Dendriform algebras' Quasi-centroids are specified in the following table:*

Table 1: Description of Quasi-centroids of two dimensional Dendriform algebras

Isomorphism Classes	**Quasi-Centroid** $\Omega(E)$	**Dim**
$Dend_2^1(\alpha)$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	1
$Dend_2^2$	$\begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{11} \end{pmatrix}$	2

$Dend_2^3$	$\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{11} \end{pmatrix}$	2
$Dend_2^4$	$\begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix}$	3
$Dend_2^5$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	1
$Dend_2^6$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	1
$Dend_2^7$	$\begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{11} \end{pmatrix}$	2
$Dend_2^8$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	1
$Dend_2^9$	$\begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}$	3
$Dend_2^{10}$	$\begin{pmatrix} a_{11} & 0 \\ 0 & a_{11} \end{pmatrix}$	1
$Dend_2^{11}$	$\begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{11} \end{pmatrix}$	2
$Dend_2^{12}$	$\begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{11} \end{pmatrix}$	2

References

[1] Benkarta G., Neherb E., The centroid of extended affine and root graded Lie algebras, *Journal of Pure and Applied Algebra*, 205(1), 2006, 117- 145. de Lie: les de Leibniz. *L'Ens. Math.*, 39, 1993, 269-293.

[2] Loday J.-L., Frabetti A., Chapoton F., Goichot F., *Dialgebras and Related Operads*, Lecture Notes in Math., 1763, Springer, Berlin, 2001.

[3] Mohammed Abdul Daim Zoba, Centroids Of Dendriform Algebras, *International Journal of Pure and Applied Mathematics*, 119(15), 2018,427-435.

[4] Melville D.J., Centroids of nilpotent Lie algebras, *Comm. Algebra*, 20(12), 1992, 3649-3682.

[5] Melville D.J., Centroids and Cartan algebras, *Comm. Algebra*, 21(8), 1993, 2791-2818.

[6] Rikhsiboev, I. M., Rakhimov, I. S. and Basri, W., The description of dendriform algebra structures on two-dimensional complex space, *Journal of Algebra,Number*

Theory: Advances and Applications, 4 (1),2010,118.

[7] Jacobson N., A note on automorphisms and derivations of Lie algebras, *Proc. Amer. Math. Soc*., 6, 281-283, (1955).

[8] Rakhimov I.S., Al-Nashri Al-Hossain, On Derivations of lowdimensional complex Leibniz algebras, *JP Journal of Algebra, Number Theory and Applications*, 21(1), 69-81, (2011).

[9] Rakhimov I.S. and Al-Hossain N. On derivations of some classes of Leibniz algebras,*Journal Generalized Lie theory and Applications*, 6, Article ID G120501, doi 10:4303/jglta/G120501, (2012)

[10] Ancochea J.M., Commpoamor R., Characteristically nilpotent Lie algebras: a survey, arXiv:1103224v1.[math.RA], (2000)

[11] Dixmier J. and Lister W.G. Derivations of nilpotent Lie algebras, *Proc. Amer. Soc*., 8, 155-158, (1957).

[12] Khakimdjanov Yu.B., Characteristically nilpotent Lie algebras, *Math. USSR Sbornik*, 70(1), (1990).

[13] Khakimdjanov Yu.B., On characteristically nilpotent Lie algebras, *Soviet Math. Dokl*., 40(1), (1990).

[14] Leger G.F., Togo S., Characteristically nilpotent Lie algebras, *Duke Math. J*., 26, 623-628, (1959).

[15] Ravisankar T. S., Characteristically nilpotent algebras, *Canadian J. Math.*, 23, 222-235, (1971).

[16] Rakhimov, I. S., and Fiidow, M. A., Centroids of finite dimensional associative dialgebras. *Far East Journal of Mathematical Sciences*, 98(4), 427-443, (2015).

[17] M.A. Fiidow, I.S. Rakhimov, S.K. Said Hussain, Derivations and Centroids of Associative Algebras. *IEEE Proceedings of International Conference on Research and Education in Mathematics* (ICREM7), 227-232, (2015)