On Neutrosophic Crisp Generalized Alpha Generalized Closed Sets

Murtadha M. Abdulkadhim[1], Qays H. Imran[2*], Ali H. M. Al-Obaidi[3], Said Broumi[4]

[1]Department of Science, College of Basic Education, Al-Muthanna University, Samawah, Iraq
[2]Department of Mathematics, College of Education for Pure Science, Al-Muthanna University, Samawah, Iraq
[3]Department of Mathematics, College of Education for Pure Science, University of Babylon, Hillah, Iraq
[4]Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Morocco
Emails: murtadha_moh@mu.edu.iq; qays.imran@mu.edu.iq; aalobaidi@uobabylon.edu.iq; broumisaid78@gmail.com
*Correspondence: qays.imran@mu.edu.iq

Abstract

This study reveals new concepts of neutrosophic crisp closed sets named neutrosophic crisp g-closed sets, neutrosophic crisp αg-closed sets, neutrosophic crisp $g\alpha$-closed sets, and neutrosophic crisp $g\alpha g$-closed sets. Furthermore, their ultimate features in neutrosophic crisp topological spaces are examined. Moreover, the consequent new concepts are introduced, such as neutrosophic crisp $g\alpha g$-closure and neutrosophic crisp $g\alpha g$-interior, and finding some of their characteristics.

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1. Introduction

Salama et al. [1] presented the idea of neutrosophic crisp topological space (or simply Neu^{CTS}). Salama [2] give some concepts of neutrosophic crisp nearly open sets. Al-Hamido et al. [3] provided the interpretation of neutrosophic crisp semi-α-closed sets. Al-Obaidi et al. [4,5] provided new concepts of weakly neutrosophic crisp open functions and weakly neutrosophic crisp closed functions. PAGE et al. [6] presented the view of neutrosophic generalized homeomorphism. Imran et al. [7-9] provided neutrosophic generalized alpha generalized continuity, different kinds of weakly neutrosophic crisp continuity, and other concepts of neutrosophic crisp open sets. This paper aims to establish the notion of neutrosophic crisp $g\alpha g$-closed sets and survey their vital elements in neutrosophic crisp topological space. Furthermore, we identify neutrosophic crisp $g\alpha g$-closure and neutrosophic crisp $g\alpha g$-interior and achieve several of their highlights.

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2. Preliminaries

All through this work, $(\mathfrak{U}, \psi)$ (or simply $\mathfrak{U}$) frequently implies Neu^{CTS}. For a neutrosophic crisp set $\mathfrak{D}$ in a $Neu^{CTS}(\mathfrak{U}, \psi)$, we symbolize the neutrosophic crisp closure of $\mathfrak{D}$ by $Neu^{C}cl(\mathfrak{D})$, the neutrosophic crisp interior of $\mathfrak{D}$ by $Neu^{C}int(\mathfrak{D})$, and the neutrosophic crisp complement of $\mathfrak{D}$ by $\underline{\mathfrak{D}} = \mathfrak{U}_{Neu} - \mathfrak{D}$, respectively.

Definition 2.1: [1]
Presume that non-empty certain fixed space $\mathfrak{U}$ has mutually exclusive subsets $\mathfrak{D}_1$, $\mathfrak{D}_2$ & $\mathfrak{D}_3$. A neutrosophic crisp set (or simply Neu^{C}S) $\mathfrak{D}$ with form $\mathfrak{D} = \langle \mathfrak{D}_1, \quad \mathfrak{D}2, \mathfrak{D}_3 \rangle$ is called an object.

Definition 2.2: [1]
A neutrosophic crisp topology (in rapid, Neu^{CT}) on $\mathfrak{U} \neq \phi$ is a collection ψ of Neu^{C}Ss in $\mathfrak{U}$ comforting the rules given below:
(i) ϕ_{Neu}, $\mathfrak{U}_{Neu} \in \psi$,
(ii) $\mathfrak{D}_1 \sqcap \mathfrak{D}_2 \in \psi$ being $\mathfrak{D}_1$, $\mathfrak{D}_2 \in \psi$,
(iii) $\sqcup \mathfrak{D}_k \in \psi$ for arbitrary family $\{\mathfrak{D}_k | k \in \Delta\} \sqsubseteq \psi$.
In this situation the ordered pair $(\mathfrak{U}, \psi)$ is termed as Neu^{CTS} and each Neu^{C}S in ψ is named as a neutrosophic crisp open set (in short, Neu^{C}OS). The complement $\underline{\mathfrak{D}}$ of a Neu^{C}OS $\mathfrak{D}$ in $\mathfrak{U}$ is recognized as neutrosophic crisp closed set (briefly, Neu^{C}CS) in $\mathfrak{U}$.

Definition 2.3: [2]
A neutrosophic crisp subset $\mathfrak{D}$ of a $Neu^{CTS}(\mathfrak{U}, \psi)$ is stated to be a neutrosophic crisp α-open set (in brief $Neu^{C\alpha}$OS) if $\mathfrak{D} \sqsubseteq Neu^{C}int(Neu^{C}cl(Neu^{C}int(\mathfrak{D})))$ and a neutrosophic crisp α-closed set (in short $Neu^{C\alpha}$CS) if $Neu^{C}cl(Neu^{C}int(Neu^{C}cl(\mathfrak{D}))) \sqsubseteq \mathfrak{D}$. The neutrosophic crisp α-closure of $\mathfrak{D}$ of a $Neu^{CTS}(\mathfrak{U}, \psi)$ is the overlapping of the whole $Neu^{C\alpha}$CSs that include $\mathfrak{D}$ and it is symbolized by $Neu^{C\alpha}cl(\mathfrak{D})$.

Proposition 2.4: [2,10]
Every Neu^{C}OS (resp. Neu^{C}CS) is a $Neu^{C\alpha}$OS (resp. $Neu^{C\alpha}$CS).

3. Neutrosophic Crisp Generalized αg-Closed Sets

In this part, we characterize and analyze the neutrosophic crisp generalized $\boldsymbol{\alpha g}$-closed sets and several of their features.

Definition 3.1:
Let $\mathfrak{D}$ be a neutrosophic crisp subset, and let $\mathfrak{M}$ be a Neu^{C}OS in a $Neu^{CTS}(\mathfrak{U}, \psi)$ where $\mathfrak{D} \sqsubseteq \mathfrak{M}$ then $\mathfrak{D}$ is called:
(i) a neutrosophic crisp generalized closed set (in a word, Neu^{Cg}CS) if $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$ and the complement of a Neu^{Cg}CS is a Neu^{Cg}OS in $(\mathfrak{U}, \psi)$.
(ii) a neutrosophic crisp αg-closed set (in a word, $Neu^{C\alpha g}$CS) if $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$ and the complement of a $Neu^{C\alpha g}$CS is a $Neu^{C\alpha g}$OS in $(\mathfrak{U}, \psi)$.
(iii) a neutrosophic crisp $g\alpha$-closed set (in a word, $Neu^{Cg\alpha}$CS) if $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$ and the complement of a $Neu^{Cg\alpha}$CS is a $Neu^{Cg\alpha}$OS in $(\mathfrak{U}, \psi)$.

Theorem 3.2:
In a $Neu^{CTS}(\mathfrak{U}, \psi)$, the subsequent arguments are valid:
(i) Each Neu^{C}CS is a Neu^{Cg}CS.
(ii) Each Neu^{Cg}CS is a $Neu^{C\alpha g}$CS.
(iii) Each $Neu^{C\alpha}$CS is a $Neu^{Cg\alpha}$CS.
(iv) Each $Neu^{Cg\alpha}$CS is a $Neu^{C\alpha g}$CS.
Proof:
(i) Let $\mathfrak{D} \sqsubseteq \mathfrak{R}$ and $\mathfrak{R}$ be a Neu^{C}OS in $Neu^{CTS}(\mathfrak{U}, \psi)$. Since $\mathfrak{D}$ is a Neu^{C}CS, $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{D}$. That is $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{D} \sqsubseteq \mathfrak{R}$. Hence $\mathfrak{D}$ is a Neu^{Cg}CS.

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(ii) Let $\mathfrak{D}$ be a Neu^{Cg}CS. Then $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{R}$ whenever $\mathfrak{D} \sqsubseteq \mathfrak{R}$ and $\mathfrak{R}$ is a Neu^{C}OS in $Neu^{CTS}(\mathfrak{U}, \psi)$. But every Neu^{C}OS is a $Neu^{C\alpha}$OS, which implies $\mathfrak{R}$ is a $Neu^{C\alpha}$OS. Also $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{R}$. Hence $\mathfrak{D}$ is a $Neu^{C\alpha g}$CS.
(iii) Let $\mathfrak{D}$ be a $Neu^{C\alpha}$**CS** and let $\mathfrak{D} \sqsubseteq \mathfrak{R}$ where $\mathfrak{R}$ is a $Neu^{C\alpha}$**OS** in $Neu^{CTS}(\mathfrak{U}, \psi)$. Since $\mathfrak{D}$ is a $Neu^{C\alpha}$**CS**, then $Neu^{C}cl(Neu^{C}int(Neu^{C}cl(\mathfrak{D}))) \sqsubseteq \mathfrak{D} \sqsubseteq \mathfrak{R}$, since $Neu^{C\alpha}cl(\mathfrak{D})$ is the smallest $Neu^{C\alpha}$**CS** containing $\mathfrak{D}$, so, $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq \mathfrak{D} \sqcup Neu^{C}cl(Neu^{C}int(Neu^{C}cl(\mathfrak{D}))) \sqsubseteq \mathfrak{D} \sqcup \mathfrak{R} \sqsubseteq \mathfrak{R}$. Hence $\mathfrak{D}$ is a $Neu^{Cg\alpha}$**CS**.
(iv) Let $\mathfrak{D} \sqsubseteq \mathfrak{R}$ and $\mathfrak{R}$ is a Neu^{C}**OS** in $Neu^{CTS}(\mathfrak{U}, \psi)$. Then $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq \mathfrak{R}$ as $\mathfrak{D}$ is a $Neu^{Cg\alpha}$**CS**. Since every Neu^{C}**OS** is a $Neu^{C\alpha}$**OS**. That is $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq \mathfrak{R}$ where $\mathfrak{R}$ is a $Neu^{C\alpha}$**OS**. Hence $\mathfrak{D}$ is a $Neu^{C\alpha g}$**CS**. ▪
As shown in the next examples, the opposite of the beyond theorem is not valid.

Example 3.3:
Let $\mathfrak{U} = \{r_1, r_2, r_3, r_4\}$ and let $\psi = \{\phi_{Neu}, \langle\{r_1\}, \phi, \phi\rangle, \langle\{r_3, r_4\}, \phi, \phi\rangle, \langle\{r_1, r_3, r_4\}, \phi, \phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{r_1, r_2, r_3\}, \phi, \phi\rangle$ is a Neu^{Cg}CS but not Neu^{C}CS.

Example 3.4:
Let $\mathfrak{U} = \{r_1, r_2, r_3\}$ and let $\psi = \{\phi_{Neu}, \langle\{r_1\}, \phi, \phi\rangle, \langle\{r_3\}, \phi, \phi\rangle, \langle\{r_1, r_3\}, \phi, \phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{r_3\}, \phi, \phi\rangle$ is a $Neu^{C\alpha g}$CS but not Neu^{Cg}CS.

Example 3.5:
Let $\mathfrak{U} = \{s_1, s_2, s_3, s_4\}$ and let $\psi = \{\phi_{Neu}, \langle\{s_1\}, \phi, \phi\rangle, \langle\{s_2, s_4\}, \phi, \phi\rangle, \langle\{s_1, s_2, s_4\}, \phi, \phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{s_2, s_3\}, \phi, \phi\rangle$ is a $Neu^{Cg\alpha}$CS but not $Neu^{C\alpha}$CS.

Example 3.6:
Let $\mathfrak{U} = \{r_1, r_2, r_3, r_4\}$ and let $\psi = \{\phi_{Neu}, \langle\{r_1\}, \phi, \phi\rangle, \langle\{r_2, r_4\}, \phi, \phi\rangle, \langle\{r_1, r_2, r_4\}, \phi, \phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{r_1\}, \phi, \phi\rangle$ is a $Neu^{C\alpha g}$CS but not $Neu^{Cg\alpha}$CS.

Theorem 3.7:
In a $Neu^{CTS}(\mathfrak{U}, \psi)$, the subsequent arguments are valid:
(i) Each Neu^{C}OS is a Neu^{Cg}OS.
(ii) Each Neu^{Cg}OS is a $Neu^{C\alpha g}$OS.
(iii) Each $Neu^{C\alpha}$OS is a $Neu^{Cg\alpha}$OS.
(iv) Each $Neu^{Cg\alpha}$OS is a $Neu^{C\alpha g}$OS.
Proof:
(i), (ii), (iii) and (iv) are understandable. ▪

Definition 3.8:
Let $\mathfrak{D}$ be a neutrosophic crisp subset, and $\mathfrak{M}$ be a $Neu^{C\alpha g}$OS in a $Neu^{CTS}(\mathfrak{U}, \psi)$ where $\mathfrak{D} \sqsubseteq \mathfrak{M}$, then $\mathfrak{D}$ is titled a neutrosophic crisp generalized αg-closed set (in brief, $Neu^{Cg\alpha g}$CS) if $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$ and the family of the whole $Neu^{Cg\alpha g}$CSs of a $Neu^{CTS}(\mathfrak{U}, \psi)$ is symbolized by $Neu^{Cg\alpha g}C(\mathfrak{U})$.

Theorem 3.9:
In a $Neu^{CTS}(\mathfrak{U}, \psi)$, the subsequent arguments are valid:
(i) Each Neu^{C}CS is a $Neu^{Cg\alpha g}$CS.
(ii) Each $Neu^{Cg\alpha g}$CS is a Neu^{Cg}CS.
(iii) Each $Neu^{Cg\alpha g}$CS is a $Neu^{C\alpha g}$CS.
(iv) Each $Neu^{Cg\alpha g}$CS is a $Neu^{Cg\alpha}$CS.
Proof:
(i) Let $\mathfrak{D}$ be a Neu^{C}CS and $\mathfrak{M}$ be any $Neu^{C\alpha g}$OS in a $Neu^{CTS}(\mathfrak{U}, \psi)$ including $\mathfrak{D}$. Then $Neu^{C}cl(\mathfrak{D}) = \mathfrak{D} \sqsubseteq \mathfrak{M}$. Therefore, $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS.
(ii) Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS and $\mathfrak{M}$ be any Neu^{C}OS in a $Neu^{CTS}(\mathfrak{U}, \psi)$ including $\mathfrak{D}$. According to theorem (3.7), $\mathfrak{M}$ is a $Neu^{C\alpha g}$OS in $(\mathfrak{U}, \psi)$. Because $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, we get $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$. Therefore, $\mathfrak{D}$ is a Neu^{Cg}CS.

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(iii) Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS and $\mathfrak{M}$ be any Neu^{C}OS in a $Neu^{CTS}(\mathfrak{U},\psi)$ including $\mathfrak{D}$. According to theorem (3.7), $\mathfrak{M}$ is a $Neu^{C\alpha g}$OS in $(\mathfrak{U},\psi)$. Because $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, we get $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$. Therefore, $\mathfrak{D}$ is a $Neu^{C\alpha g}$CS.
(iv) Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS and $\mathfrak{M}$ be any $Neu^{C\alpha}$OS in a $Neu^{CTS}(\mathfrak{U},\psi)$ including $\mathfrak{D}$. According to theorem (3.7), $\mathfrak{M}$ is a $Neu^{C\alpha g}$OS in $(\mathfrak{U},\psi)$. Because $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, we get $Neu^{C\alpha}cl(\mathfrak{D}) \sqsubseteq Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$. Therefore, $\mathfrak{D}$ is a $Neu^{Cg\alpha}$CS. ▪
The next examples show that the reverse of the beyond theorem is not reasonable.

Example 3.10:
Let $\mathfrak{U} = \{s_1, s_2, s_3, s_4\}$ and let $\psi = \{\phi_{Neu}, \langle\{s_1\},\phi,\phi\rangle, \langle\{s_2,s_4\},\phi,\phi\rangle, \langle\{s_1,s_2,s_4\},\phi,\phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{s_1,s_2,s_3\},\phi,\phi\rangle$ is a $Neu^{Cg\alpha g}$CS but not Neu^{C}CS.

Example 3.11:
Let $\mathfrak{U} = \{s_1, s_2, s_3, s_4, s_5\}$ and let $\psi = \{\phi_{Neu}, \langle\{s_4\},\phi,\phi\rangle, \langle\{s_1,s_2\},\phi,\phi\rangle, \langle\{s_1,s_2,s_4\},\phi,\phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{s_1,s_3,s_4\},\phi,\phi\rangle$ is a Neu^{Cg}CS but not $Neu^{Cg\alpha g}$CS.

Example 3.12:
Let $\mathfrak{U} = \{r_1, r_2, r_3, r_4\}$ and let $\psi = \{\phi_{Neu}, \langle\{r_1\},\phi,\phi\rangle, \langle\{r_2,r_4\},\phi,\phi\rangle, \langle\{r_1,r_2,r_4\},\phi,\phi\rangle, \mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then the Neu^{C}S $\langle\{r_1,r_3\},\phi,\phi\rangle$ is a $Neu^{Cg\alpha}$CS. Therefore, it is $Neu^{C\alpha g}$CS, but it is not $Neu^{Cg\alpha g}$CS.

Definition 3.13:
A neutrosophic crisp subset $\mathfrak{D}$ of a $Neu^{CTS}(\mathfrak{U},\psi)$ is termed a neutrosophic crisp generalized αg-open set (in brief, $Neu^{Cg\alpha g}$OS) iff $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS. The collection of the whole $Neu^{Cg\alpha g}$OSs of a $Neu^{CTS}(\mathfrak{U},\psi)$ is signified by $Neu^{Cg\alpha g}O(\mathfrak{U})$.

Proposition 3.14:
Let $(\mathfrak{U},\psi)$ be a Neu^{CTS}. If any Neu^{C}OS in $(\mathfrak{U},\psi)$, then it is a $Neu^{Cg\alpha g}$OS.
Proof:
Let $\mathfrak{D}$ be a Neu^{C}OS in a Neu^{CTS} $(\mathfrak{U},\psi)$, then $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a Neu^{C}CS in $(\mathfrak{U},\psi)$. According to theorem (3.9), point (i), $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS. Therefore, $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$OS in $(\mathfrak{U},\psi)$. ▪

Proposition 3.15:
Let $(\mathfrak{U},\psi)$ be a Neu^{CTS}. If any $Neu^{Cg\alpha g}$OS is in $(\mathfrak{U},\psi)$, then it is a Neu^{Cg}OS.
Proof:
Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$OS in a Neu^{CTS} $(\mathfrak{U},\psi)$, then $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS in $(\mathfrak{U},\psi)$. According to theorem (3.9), point (ii), $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a Neu^{Cg}CS. Therefore, $\mathfrak{D}$ is a Neu^{Cg}OS in $(\mathfrak{U},\psi)$. ▪

Theorem 3.16:
In a $Neu^{CTS}(\mathfrak{U},\psi)$, the next arguments are valid:
(i) Each $Neu^{Cg\alpha g}$OS is a $Neu^{C\alpha g}$OS.
(ii) Each $Neu^{Cg\alpha g}$OS is a $Neu^{Cg\alpha}$OS.
Proof:
Analogous to the beyond proposition. ▪

Proposition 3.17:
If $\mathfrak{D}$ and $\mathfrak{R}$ are $Neu^{Cg\alpha g}$CSs in a $Neu^{CTS}(\mathfrak{U},\psi)$, then $\mathfrak{D}\sqcup\mathfrak{R}$ is a $Neu^{Cg\alpha g}$CS.
Proof:
Let $\mathfrak{D}$ and $\mathfrak{R}$ be $Neu^{Cg\alpha g}$CSs in a $Neu^{CTS}(\mathfrak{U},\psi)$ and let $\mathfrak{M}$ be any $Neu^{C\alpha g}$OS including $\mathfrak{D}$ and $\mathfrak{R}$. Then $\mathfrak{D}\sqcup\mathfrak{R} \sqsubseteq \mathfrak{M}$. Then $\mathfrak{D} \sqsubseteq \mathfrak{M}$ and $\mathfrak{R} \sqsubseteq \mathfrak{M}$. Since $\mathfrak{D}$ and $\mathfrak{R}$ are $Neu^{Cg\alpha g}$CSs, $Neu^{C}cl(\mathfrak{D}) \sqsubseteq \mathfrak{M}$ and $Neu^{C}cl(\mathfrak{R}) \sqsubseteq \mathfrak{M}$. Now, $Neu^{C}cl(\mathfrak{D}\sqcup\mathfrak{R}) = Neu^{C}cl(\mathfrak{D})\sqcup Neu^{C}cl(\mathfrak{R}) \sqsubseteq \mathfrak{M}$ and so $Neu^{C}cl(\mathfrak{D}\sqcup\mathfrak{R}) \sqsubseteq \mathfrak{M}$. Hence, $\mathfrak{D}\sqcup\mathfrak{R}$ is a $Neu^{Cg\alpha g}$CS. ▪

Proposition 3.18:
If $\mathfrak{D}$ and $\mathfrak{R}$ are $Neu^{Cg\alpha g}$OSs in a $Neu^{CTS}(\mathfrak{U},\psi)$, then $\mathfrak{D}\sqcap\mathfrak{R}$ is a $Neu^{Cg\alpha g}$OS.

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Proof:

Let $\mathfrak{D}$ and $\mathfrak{R}$ be $Neu^{Cg\alpha g}$OSs in a $Neu^{CTS}(\mathfrak{U},\psi)$. Then $\mathfrak{U}_{Neu}-\mathfrak{D}$ and $\mathfrak{U}_{Neu}-\mathfrak{R}$ are $Neu^{Cg\alpha g}$CSs. According to proposition (3.17), $(\mathfrak{U}_{Neu}-\mathfrak{D})\sqcup(\mathfrak{U}_{Neu}-\mathfrak{R})$ is a $Neu^{Cg\alpha g}$CS. Subsequently, $(\mathfrak{U}_{Neu}-\mathfrak{D})\sqcup(\mathfrak{U}_{Neu}-\mathfrak{R})=\mathfrak{U}_{Neu}-(\mathfrak{D}\sqcap\mathfrak{R})$. Therefore, $\mathfrak{D}\sqcap\mathfrak{R}$ is a $Neu^{Cg\alpha g}$OS. ▪

Proposition 3.19:

If a Neu^{C}S $\mathfrak{D}$ is $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$, then $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$ includes no non-empty Neu^{C}CS in $(\mathfrak{U},\psi)$.

Proof:

Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$ and let $\mathfrak{F}$ be any Neu^{C}CS in $(\mathfrak{U},\psi)$ s.t. $\mathfrak{F}\sqsubseteq Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$. Since $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, we have $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{U}_{Neu}-\mathfrak{F}$. This implies $\mathfrak{F}\sqsubseteq\mathfrak{U}_{Neu}-Neu^{C}cl(\mathfrak{D})$. Then $\mathfrak{F}\sqsubseteq Neu^{C}cl(\mathfrak{D})\sqcap(\mathfrak{U}_{Neu}-Neu^{C}cl(\mathfrak{D}))=\phi_{Neu}$. Thus, $\mathfrak{F}=\phi_{Neu}$. Hence, $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$ contains no non-empty Neu^{C}CS in $(\mathfrak{U},\psi)$. ▪

Proposition 3.20:

A Neu^{C}S $\mathfrak{D}$ is $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$ iff $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$ includes no non-empty $Neu^{C\alpha g}$CS in $(\mathfrak{U},\psi)$.

Proof:

Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$ and let $\mathfrak{S}$ be any $Neu^{C\alpha g}$CS in $(\mathfrak{U},\psi)$ s.t. $\mathfrak{S}\sqsubseteq Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$.
Since $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, we have $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{U}_{Neu}-\mathfrak{S}$. This implies $\mathfrak{S}\sqsubseteq\mathfrak{U}_{Neu}-Neu^{C}cl(\mathfrak{D})$. Then $\mathfrak{S}\sqsubseteq Neu^{C}cl(\mathfrak{D})\sqcap(\mathfrak{U}_{Neu}-Neu^{C}cl(\mathfrak{D}))=\phi_{Neu}$. Thus, $\mathfrak{S}$ is empty.
On the other hand, imagine that $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$ includes no non-empty $Neu^{C\alpha g}$CS in $(\mathfrak{U},\psi)$. Allow $\mathfrak{D}\sqsubseteq\mathfrak{M}$ and $\mathfrak{M}$ is $Neu^{C\alpha g}$OS. If $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{M}$ then $Neu^{C}cl(\mathfrak{D})\sqcap(\mathfrak{U}_{Neu}-\mathfrak{M})$ is non-empty. Because $Neu^{C}cl(\mathfrak{D})$ is Neu^{C}CS and $\mathfrak{U}_{Neu}-\mathfrak{M}$ is $Neu^{C\alpha g}$CS, we get $Neu^{C}cl(\mathfrak{D})\sqcap(\mathfrak{U}_{Neu}-\mathfrak{M})$ as non-empty $Neu^{C\alpha g}$CS of $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$, which is a conflict. Consequently, $Neu^{C}cl(\mathfrak{D})\not\sqsubseteq\mathfrak{M}$. Therefore, $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS. ▪

Theorem 3.21:

If $\mathfrak{D}$ is a $Neu^{C\alpha g}$OS and a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$, then $\mathfrak{D}$ is a Neu^{C}CS in $(\mathfrak{U},\psi)$.

Proof:

Assume that $\mathfrak{D}$ is a $Neu^{C\alpha g}$OS and a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$ then $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{D}$ and since $\mathfrak{D}\sqsubseteq Neu^{C}cl(\mathfrak{D})$. Therefore, $Neu^{C}cl(\mathfrak{D})=\mathfrak{D}$. Hence, $\mathfrak{D}$ is a Neu^{C}CS. ▪

Theorem 3.22:

If $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$ and $\mathfrak{D}\sqsubseteq\mathfrak{R}\sqsubseteq Neu^{C}cl(\mathfrak{D})$, then $\mathfrak{R}$ is a $Neu^{Cg\alpha g}$CS in $(\mathfrak{U},\psi)$.

Proof:

Suppose that $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$. Let $\mathfrak{M}$ be a $Neu^{C\alpha g}$OS in $(\mathfrak{U},\psi)$ s.t. $\mathfrak{R}\sqsubseteq\mathfrak{M}$. Then $\mathfrak{D}\sqsubseteq\mathfrak{M}$. Since $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS, it follows that $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{M}$. Now, $\mathfrak{R}\sqsubseteq Neu^{C}cl(\mathfrak{D})$ implies $Neu^{C}cl(\mathfrak{R})\sqsubseteq Neu^{C}cl(Neu^{C}cl(\mathfrak{D}))=Neu^{C}cl(\mathfrak{D})$. Thus, $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{M}$. Hence, $\mathfrak{R}$ is a $Neu^{Cg\alpha g}$CS. ▪

Theorem 3.23:

If $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$OS in a $Neu^{CTS}(\mathfrak{U},\psi)$ and $Neu^{C}int(\mathfrak{D})\sqsubseteq\mathfrak{R}\sqsubseteq\mathfrak{D}$, then $\mathfrak{R}$ is a $Neu^{Cg\alpha g}$OS in $(\mathfrak{U},\psi)$.

Proof:

Suppose that $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$OS in a $Neu^{CTS}(\mathfrak{U},\psi)$ and $Neu^{C}int(\mathfrak{D})\sqsubseteq\mathfrak{R}\sqsubseteq\mathfrak{D}$. Then $\mathfrak{U}_{Neu}-\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS and $\mathfrak{U}_{Neu}-\mathfrak{D}\sqsubseteq\mathfrak{U}_{Neu}-\mathfrak{R}\sqsubseteq Neu^{C}cl(\mathfrak{U}_{Neu}-\mathfrak{D})$. Then $\mathfrak{U}_{Neu}-\mathfrak{R}$ is a $Neu^{Cg\alpha g}$CS corresponding to theorem (3.22). Hence, $\mathfrak{R}$ is a $Neu^{Cg\alpha g}$OS. ▪

Theorem 3.24:

A Neu^{C}S $\mathfrak{D}$ is $Neu^{Cg\alpha g}$OS iff $\mathfrak{P}\sqsubseteq Neu^{C}int(\mathfrak{D})$ where $\mathfrak{P}$ is a $Neu^{Cg\alpha g}$CS and $\mathfrak{P}\sqsubseteq\mathfrak{D}$.

Proof:

Suppose that $\mathfrak{P}\sqsubseteq Neu^{C}int(\mathfrak{D})$ where $\mathfrak{P}$ is a $Neu^{Cg\alpha g}$CS and $\mathfrak{P}\sqsubseteq\mathfrak{D}$. Then $\mathfrak{U}_{Neu}-\mathfrak{D}\sqsubseteq\mathfrak{U}_{Neu}-\mathfrak{P}$ and $\mathfrak{U}_{Neu}-\mathfrak{P}$ is a $Neu^{C\alpha g}$OS by theorem (3.16). Now, $Neu^{C}cl(\mathfrak{U}_{Neu}-\mathfrak{D})=\mathfrak{U}_{Neu}-Neu^{C}int(\mathfrak{D})\sqsubseteq\mathfrak{U}_{Neu}-\mathfrak{P}$. Then $\mathfrak{U}_{Neu}-\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS. Hence, $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$OS.

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Conversely, let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$OS and $\mathfrak{P}$ be a $Neu^{Cg\alpha g}$CS and $\mathfrak{P} \sqsubseteq \mathfrak{D}$. Then $\mathfrak{U}_{Neu} - \mathfrak{D} \sqsubseteq \mathfrak{U}_{Neu} - \mathfrak{P}$. Since $\mathfrak{U}_{Neu} - \mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS and $\mathfrak{U}_{Neu} - \mathfrak{P}$ is a $Neu^{C\alpha g}$OS, we have $Neu^{C}cl(\mathfrak{U}_{Neu} - \mathfrak{D}) \sqsubseteq \mathfrak{U}_{Neu} - \mathfrak{P}$. Then $\mathfrak{P} \sqsubseteq Neu^{C}int(\mathfrak{D})$. ▪

Remark 3.25:
The later chart illustrates the virtual among the separate sorts of Neu^{C}CS:

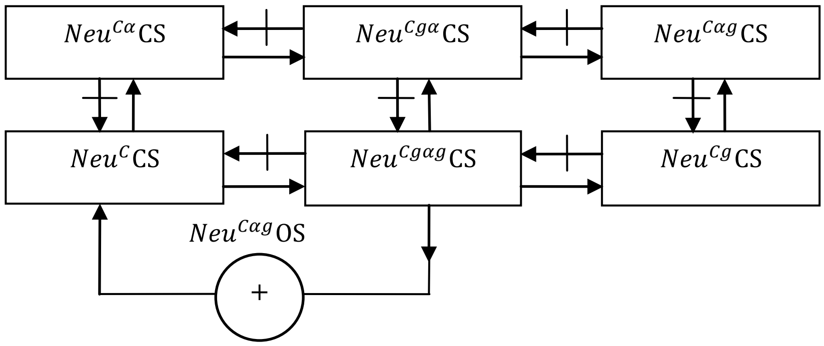

Figure 1: The later

4. Neutrosophic Crisp $g\alpha g$-Closure and Neutrosophic Crisp $g\alpha g$-Interior

We stand for neutrosophic crisp $g\alpha g$-closure and neutrosophic crisp $g\alpha g$-interior and achieve several of their highlights in this sector.

Definition 4.1:
The overlapping of the whole $Neu^{Cg\alpha g}$CSs in a $Neu^{CTS}(\mathfrak{U}, \psi)$ including $\mathfrak{D}$ is termed neutrosophic crisp $g\alpha g$-closure of $\mathfrak{D}$, and it is symbolized by $Neu^{Cg\alpha g}cl(\mathfrak{D})$.

Definition 4.2:
The union of all $Neu^{Cg\alpha g}$OSs in a $Neu^{CTS}(\mathfrak{U}, \psi)$ included in $\mathfrak{D}$ is termed neutrosophic crisp $g\alpha g$-interior of $\mathfrak{D}$, and it is symbolized by $Neu^{Cg\alpha g}int(\mathfrak{D})$.

Theorem 4.3:
Let $\mathfrak{D}$ be any Neu^{C}S in a $Neu^{CTS}(\mathfrak{U}, \psi)$. Then the next issues stand:
(i) $Neu^{Cg\alpha g}int(\mathfrak{D}) = \mathfrak{D}$ iff $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$OS.
(ii) $Neu^{Cg\alpha g}cl(\mathfrak{D}) = \mathfrak{D}$ iff $\mathfrak{D}$ is a $Neu^{Cg\alpha g}$CS.
(iii) $Neu^{Cg\alpha g}int(\mathfrak{D})$ is the biggest $Neu^{Cg\alpha g}$OS included in $\mathfrak{D}$.
(iv) $Neu^{Cg\alpha g}cl(\mathfrak{D})$ is the fewest $Neu^{Cg\alpha g}$CS including $\mathfrak{D}$.
Proof:
(i), (ii), (iii) and (iv) are obvious. ▪

Proposition 4.4:
Let $\mathfrak{D}$ be any Neu^{C}S in a $Neu^{CTS}(\mathfrak{U}, \psi)$. Then the next issues stand:
(i) $Neu^{Cg\alpha g}int(\mathfrak{U}_{Neu} - \mathfrak{D}) = \mathfrak{U}_{Neu} - (Neu^{Cg\alpha g}cl(\mathfrak{D}))$,
(ii) $Neu^{Cg\alpha g}cl(\mathfrak{U}_{Neu} - \mathfrak{D}) = \mathfrak{U}_{Neu} - (Neu^{Cg\alpha g}int(\mathfrak{D}))$.
Proof:
(i) By definition, $Neu^{Cg\alpha g}cl(\mathfrak{D}) = \sqcap\{\mathfrak{R}: \mathfrak{D} \sqsubseteq \mathfrak{R}, \mathfrak{R} \text{ is a } Neu^{Cg\alpha g}\text{CS}\}$
$\mathfrak{U}_{Neu} - (Neu^{Cg\alpha g}cl(\mathfrak{D})) = \mathfrak{U}_{Neu} - \sqcap\{\mathfrak{R}: \mathfrak{D} \sqsubseteq \mathfrak{R}, \mathfrak{R} \text{ is a } Neu^{Cg\alpha g}\text{CS}\}$
$= \sqcup\{\mathfrak{U}_{Neu} - \mathfrak{R}: \mathfrak{D} \sqsubseteq \mathfrak{R}, \mathfrak{R} \text{ is a } Neu^{Cg\alpha g}\text{CS}\}$

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$= \sqcup\{\mathfrak{M}: \mathfrak{U}_{Neu} - \mathfrak{D} \sqsupseteq \mathfrak{M}, \mathfrak{M}$ is a $Neu^{Cg\alpha g}$OS$\}$
$= Neu^{Cg\alpha g}int(\mathfrak{U}_{Neu} - \mathfrak{D})$.
(ii) The validation is the same as (i). ▪

Theorem 4.5:
Let $\mathfrak{D}$ and $\mathfrak{R}$ be two Neu^CSs in a $Neu^{CTS}(\mathfrak{U}, \psi)$. Then the following properties hold:
(i) $Neu^{Cg\alpha g}cl(\phi_{Neu}) = \phi_{Neu}$, $Neu^{Cg\alpha g}cl(\mathfrak{U}_{Neu}) = \mathfrak{U}_{Neu}$.
(ii) $\mathfrak{D} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D})$.
(iii) $\mathfrak{D} \sqsubseteq \mathfrak{R} \Longrightarrow Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(iv) $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcap \mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcap Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(v) $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R}) = Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(vi) $Neu^{Cg\alpha g}cl(Neu^{Cg\alpha g}cl(\mathfrak{D})) = Neu^{Cg\alpha g}cl(\mathfrak{D})$.
Proof:
(i) and (ii) are apparent.
(iii) By part (ii), $\mathfrak{R} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$. Since $\mathfrak{D} \sqsubseteq \mathfrak{R}$, we have $\mathfrak{D} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$. But $Neu^{Cg\alpha g}cl(\mathfrak{R})$ is a $Neu^{Cg\alpha g}$CS. Thus $Neu^{Cg\alpha g}cl(\mathfrak{R})$ is a $Neu^{Cg\alpha g}$CS containing $\mathfrak{D}$. Since $Neu^{Cg\alpha g}cl(\mathfrak{D})$ is the smallest $Neu^{Cg\alpha g}$CS containing $\mathfrak{D}$, we have $Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(iv) We know that $\mathfrak{D} \sqcap \mathfrak{R} \sqsubseteq \mathfrak{D}$ and $\mathfrak{D} \sqcap \mathfrak{R} \sqsubseteq \mathfrak{R}$. Therefore, by part (iii), $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcap \mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D})$ and $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcap \mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$. Hence $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcap \mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcap Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(v) Since $\mathfrak{D} \sqsubseteq \mathfrak{D} \sqcup \mathfrak{R}$ and $\mathfrak{R} \sqsubseteq \mathfrak{D} \sqcup \mathfrak{R}$, it follows from part (iii) that $Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R})$ and $Neu^{Cg\alpha g}cl(\mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R})$. Hence $Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R})$………. (1)
Since $Neu^{Cg\alpha g}cl(\mathfrak{D})$ and $Neu^{Cg\alpha g}cl(\mathfrak{R})$ are $Neu^{Cg\alpha g}$CSs, $Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$ is also $Neu^{Cg\alpha g}$CS by proposition (3.17). Also $\mathfrak{D} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D})$ and $\mathfrak{R} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{R})$ implies that $\mathfrak{D} \sqcup \mathfrak{R} \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$. Thus $Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$ is a $Neu^{Cg\alpha g}$CS containing $\mathfrak{D} \sqcup \mathfrak{R}$. Since $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R})$ is the smallest $Neu^{Cg\alpha g}$CS containing $\mathfrak{D} \sqcup \mathfrak{R}$, we have $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R}) \sqsubseteq Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$………. (2)
From (1) and (2), we have $Neu^{Cg\alpha g}cl(\mathfrak{D} \sqcup \mathfrak{R}) = Neu^{Cg\alpha g}cl(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}cl(\mathfrak{R})$.
(vi) Since $Neu^{Cg\alpha g}cl(\mathfrak{D})$ is a $Neu^{Cg\alpha g}$CS, we have by theorem (4.3) part (ii), $Neu^{Cg\alpha g}cl(Neu^{Cg\alpha g}cl(\mathfrak{D})) = Neu^{Cg\alpha g}cl(\mathfrak{D})$. ▪

Theorem 4.6:
Let $\mathfrak{D}$ and $\mathfrak{R}$ be two Neu^CSs in a $Neu^{CTS}(\mathfrak{U}, \psi)$. Then the following properties hold:
(i) $Neu^{Cg\alpha g}int(\phi_{Neu}) = \phi_{Neu}$, $Neu^{Cg\alpha g}int(\mathfrak{U}_{Neu}) = \mathfrak{U}_{Neu}$.
(ii) $Neu^{Cg\alpha g}int(\mathfrak{D}) \sqsubseteq \mathfrak{D}$.
(iii) $\mathfrak{D} \sqsubseteq \mathfrak{R} \Longrightarrow Neu^{Cg\alpha g}int(\mathfrak{D}) \sqsubseteq Neu^{Cg\alpha g}int(\mathfrak{R})$.
(iv) $Neu^{Cg\alpha g}int(\mathfrak{D} \sqcap \mathfrak{R}) = Neu^{Cg\alpha g}int(\mathfrak{D}) \sqcap Neu^{Cg\alpha g}int(\mathfrak{R})$.
(v) $Neu^{Cg\alpha g}int(\mathfrak{D} \sqcup \mathfrak{R}) \sqsupseteq Neu^{Cg\alpha g}int(\mathfrak{D}) \sqcup Neu^{Cg\alpha g}int(\mathfrak{R})$.
(vi) $Neu^{Cg\alpha g}int(Neu^{Cg\alpha g}int(\mathfrak{D})) = Neu^{Cg\alpha g}int(\mathfrak{D})$.
Proof:
(i), (ii), (iii), (iv), (v) and (vi) are apparent. ▪

Definition 4.7:
A $Neu^{CTS}(\mathfrak{U}, \psi)$ is called a neutrosophic crisp $T_{\frac{1}{2}}$-space (in short, $Neu^C T_{\frac{1}{2}}$-space) if each Neu^{Cg}CS in it is a Neu^CCS.

Definition 4.8:
A $Neu^{CTS}(\mathfrak{U}, \psi)$ is called a neutrosophic crisp $T_{g\alpha g}$-space (in short, $Neu^C T_{g\alpha g}$-space) if each $Neu^{Cg\alpha g}$CS in it is a Neu^CCS.

Proposition 4.9:
Every $Neu^C T_{\frac{1}{2}}$-space is a $Neu^C T_{g\alpha g}$-space.

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Proof:
Let $(\mathfrak{U},\psi)$ be a $Neu^{C}T_{\frac{1}{2}}$-space and let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS in $\mathfrak{U}$. Then $\mathfrak{D}$ is a Neu^{Cg}CS, by theorem (3.9) part (ii). Since $(\mathfrak{U},\psi)$ is a $Neu^{C}T_{\frac{1}{2}}$-space, then $\mathfrak{D}$ is a Neu^{C}CS in $\mathfrak{U}$. Hence $(\mathfrak{U},\psi)$ is a $Neu^{C}T_{g\alpha g}$-space. ▪
The next instance reveals that the overhead proposition's converse is not reasonable.

Example 4.10:
Let $\mathfrak{U}=\{s_1,s_2,s_3\}$ and let $\psi=\{\phi_{Neu},\langle\{s_1\},\phi,\phi\rangle,\langle\{s_2,s_3\},\phi,\phi\rangle,\mathfrak{U}_{Neu}\}$ be a Neu^{CT} on $\mathfrak{U}$. Then $(\mathfrak{U},\psi)$ is a $Neu^{C}T_{g\alpha g}$-space but not $Neu^{C}T_{\frac{1}{2}}$-space.

Proposition 4.11:
For a $Neu^{CTS}(\mathfrak{U},\psi)$, the following statements are equivalent:
(i) $(\mathfrak{U},\psi)$ is a $Neu^{C}T_{g\alpha g}$-space.
(ii) Every singleton of a $Neu^{CTS}(\mathfrak{U},\psi)$ is either $Neu^{C\alpha g}$CS or Neu^{C}OS.
Proof:
(i) ⟹(ii) Assume that for some $u\in\mathfrak{U}$ the Neu^{C}S $\langle\{u\},\phi,\phi\rangle$ is not a $Neu^{C\alpha g}$CS in a $Neu^{CTS}(\mathfrak{U},\psi)$. Then the only $Neu^{C\alpha g}$OS containing $\mathfrak{U}_{Neu}-\langle\{u\},\phi,\phi\rangle$ is the space $\mathfrak{U}$ itself and $\mathfrak{U}_{Neu}-\langle\{u\},\phi,\phi\rangle$ is a $Neu^{Cg\alpha g}$CS in $(\mathfrak{U},\psi)$. By assumption $\mathfrak{U}_{Neu}-\langle\{u\},\phi,\phi\rangle$ is a Neu^{C}CS in $(\mathfrak{U},\psi)$ or equivalently $\langle\{u\},\phi,\phi\rangle$ is a Neu^{C}OS.
(ii) ⟹(i) Let $\mathfrak{D}$ be a $Neu^{Cg\alpha g}$CS in $(\mathfrak{U},\psi)$ and let $u\in Neu^{C}cl(\mathfrak{D})$. By assumption $\langle\{u\},\phi,\phi\rangle$ is either $Neu^{C\alpha g}$CS or Neu^{C}OS.
Case(1). Suppose $\langle\{u\},\phi,\phi\rangle$ is a $Neu^{C\alpha g}$CS. If $u\notin\mathfrak{D}$, then $Neu^{C}cl(\mathfrak{D})-\mathfrak{D}$ contains a non-empty $Neu^{C\alpha g}$CS $\langle\{u\},\phi,\phi\rangle$ which is a contradiction. Therefore $u\in\mathfrak{D}$.
Case(2). Suppose $\langle\{u\},\phi,\phi\rangle$ is a Neu^{C}OS. Since $u\in Neu^{C}cl(\mathfrak{D})$, $\langle\{u\},\phi,\phi\rangle\sqcap\mathfrak{D}\neq\phi_{Neu}$ and therefore $Neu^{C}cl(\mathfrak{D})\sqsubseteq\mathfrak{D}$ or equivalently $\mathfrak{D}$ is a Neu^{C}CS in a $Neu^{CTS}(\mathfrak{U},\psi)$. ▪

5. Conclusion

The sense of $Neu^{Cg\alpha g}$CS distinguished employing $Neu^{C\alpha g}$CS creates a Neu^{CT} and remains among the idea of Neu^{C}CS and the idea of Neu^{Cg}CS. The $Neu^{Cg\alpha g}$CS can be utilized to derive a new decomposition of $Neu^{Cg\alpha g}$-continuity and new $Neu^{Cg\alpha g}$-separation axioms.

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