

# Lecture 5: De Morgan's Theorem

## De Morgan's Theorem

### 1. De Morgan's Theorem

DeMorgan, a mathematician, proposed two theorems that are an important part of Boolean algebra. In practical terms, DeMorgan's theorems provide mathematical verification of the equivalency of the NAND and negative-OR gates and the equivalency of the NOR and negative-AND gates.

#### 1.1 DeMorgan's first theorem

The complement of a product of variables is equal to the sum of the complements of the variables.

Stated another way,

The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

- Verifying DeMorgan's First Theorem using Truth Table

Inputs		Truth Table Outputs For Each Term				
B	A	A.B	$\overline{A \cdot B}$	$\overline{A}$	$\overline{B}$	$\overline{\overline{A} + \overline{B}}$
0	0	0	1	1	1	1
0	1	0	1	0	1	1
1	0	0	1	1	0	1

1	1	1	0	0	0	0
---	---	---	---	---	---	---

We can also show that  $\overline{A \cdot B} = \overline{A} + \overline{B}$  using logic gates as shown.

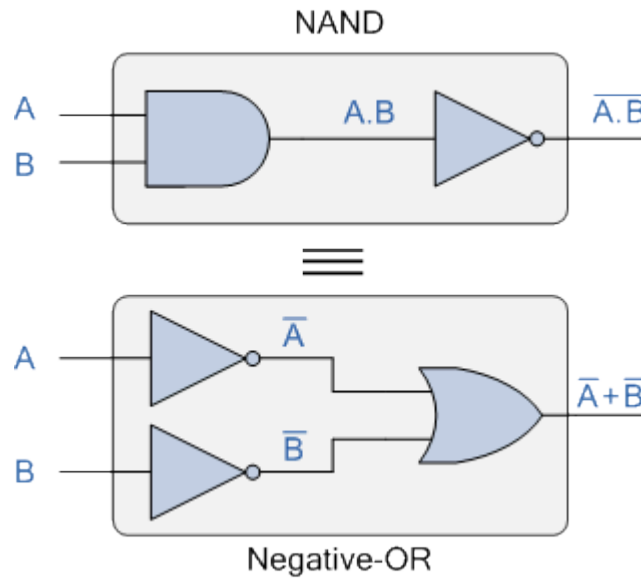


Fig. 1 DeMorgan's First Law Implementation using Logic Gates

### 1.2 DeMorgan's second theorem

The complement of a sum of variables is equal to the product of the complements of the variables.

Stated another way,

The complement of two or more OR'ed variables is equivalent to the AND of the complements of the individual variables.

The formula for expressing this theorem for two variables is

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

• Verifying DeMorgan’s Second Theorem using Truth Table

Inputs		Truth Table Outputs For Each Term				
B	A	A+B	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	0	1	1	1	1
0	1	1	0	0	1	0
1	0	1	0	1	0	0
1	1	1	0	0	0	0

We can also show that  $\overline{A+B} = \overline{A} \cdot \overline{B}$  using the following logic gates example.

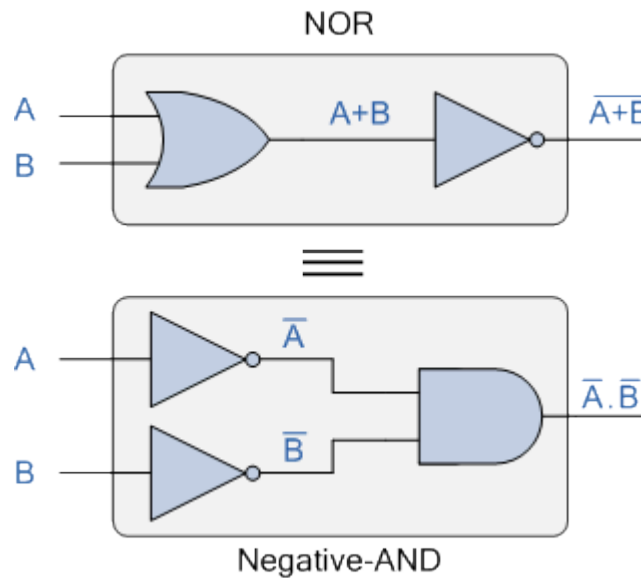


Fig. 2 DeMorgan’s Second Law Implementation using Logic Gates

- ❖ Although we have used DeMorgan’s theorems with only two input variables A and B, they are equally valid for use with three, four or more input variable expressions, for example:

❖ For a 3-variable input

$$\overline{A.B.C} = \overline{A} + \overline{B} + \overline{C}$$

and also

$$\overline{A+B+C} = \overline{A}. \overline{B}. \overline{C}$$

❖ For a 4-variable input

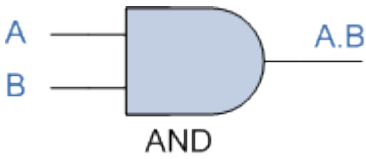
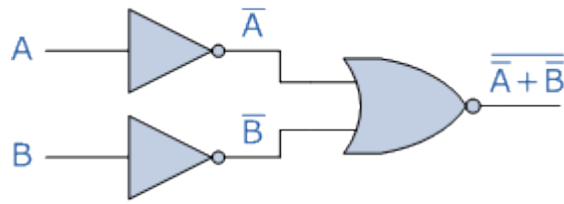
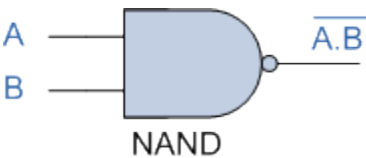
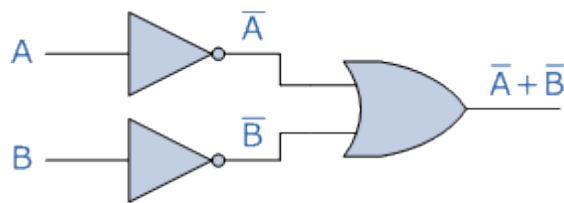
$$\overline{A.B.C.D} = \overline{A} + \overline{B} + \overline{C} + \overline{D}$$

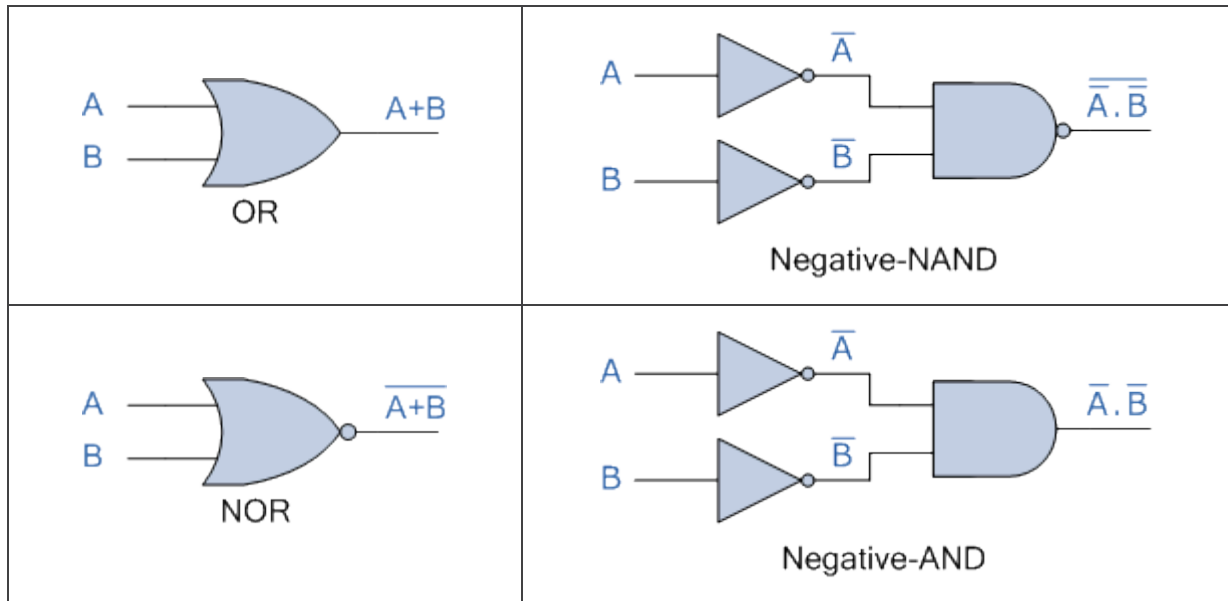
and also

$$\overline{A+B+C+D} = \overline{A}. \overline{B}. \overline{C}. \overline{D}$$

and so on.

## 2. DeMorgan's Equivalent Gates

Standard Logic Gate	DeMorgan's Equivalent Gate
 <p>AND</p>	 <p>Negative-NOR</p>
 <p>NAND</p>	 <p>Negative-OR</p>



Let's take some examples in which we take some expressions and apply DeMorgan's theorems.

**Example 1:** Apply DeMorgan's theorems to the expressions  $(A . B . C)'$  or  $\overline{A . B . C}$

**Sol :**  $(A . B . C)' = A' + B' + C'$

**Or**  $\overline{A . B . C} = \bar{A} + \bar{B} + \bar{C}$

**Example 2:** Apply DeMorgan's theorems to the expressions  $(A + B + C)'$

**Sol:**  $(A + B + C)' = A' . B' . C'$

**Or**  $\overline{A + B + C} = \bar{A} . \bar{B} . \bar{C}$

**Example3:** Prove that  $A+AB = A$

**Sol.**

$$A + AB = A (1 + B)$$

$$= A . 1 = A$$

**Example 4:** Prove that  $A + \bar{A}B = A + B$

Sol.

$$\begin{aligned} A + \bar{A}B &= (A + ((\bar{A}B)) + \bar{A}B) \\ &= A + B(\bar{A} + A) \\ &= A + B.1 \\ &= A + B \end{aligned}$$

**Example 5:**  $(\bar{A}B'.(A + C))' + A'B.(A + B' + C)'$

$$\begin{aligned} &\overline{\bar{A}B.(A+C)} + \overline{A'B.(A+B'+C)} \\ &\overline{\bar{A}B.(A+C)} + \overline{A'B.(A+B'+C)} = \overline{\bar{A}B} + \overline{A+C} + \overline{A'B} + \overline{A+B'+C} \\ &\overline{\bar{A}B} + \overline{A+C} + \overline{A'B} + \overline{A+B'+C} = (\bar{A} + \bar{B}) + \bar{A} + \bar{C} + \bar{A} + \bar{A}'B + B + C \\ &\overline{\bar{A}B} + \overline{A+C} + \overline{A'B} + \overline{A+B'+C} = \bar{A} + B + \bar{A} + \bar{C} + \bar{A} + \bar{A}'B + B + C \\ &\bar{A} + B + \bar{A} + \bar{C} + \bar{A} + \bar{A}'B + B + C = \bar{A}(1 + \bar{C}) + B + \bar{A}'B + C \\ &\bar{A}(1 + \bar{C}) + B + \bar{A}'B + C = \bar{A} + B + \bar{A}'B + C \\ &\bar{A} + B + \bar{A}'B + C = \bar{A} + B(1 + \bar{A}') + C \\ &\bar{A} + B(1 + \bar{A}') = \bar{A} + B \end{aligned}$$

**HW:**

1-Prove that  $(A + B)(A + C) = A + BC$

2- Apply DeMorgan's theorems to the following expressions:

$$1. \overline{ABC} + \overline{(D + E)} \quad 2. \overline{(A + B)C} \quad 3. \overline{A + B + C} + \overline{DE}$$