8. Unit Hydrograph (UH)

8.1 Components of the unit hydrographs.

*UH: The hydrograph of direct runoff resulting from one unit depth (**1 cm**) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hrs).*

- The term ***unit*** here refers to ***a unit depth of rainfall excess*** which is usually taken as *1 cm.*
- The duration, being a very important characteristic, is used as a prefix to a specific UH. Thus, one has a 6-hr. unit hydrograph (UH), 12-hr. - UH, etc. and in general a D-hr. UH applicable to a given catchment.

Two basic assumptions constitute the foundations for the UH theory:

(i) the time invariance, and

(ii) the linear response.

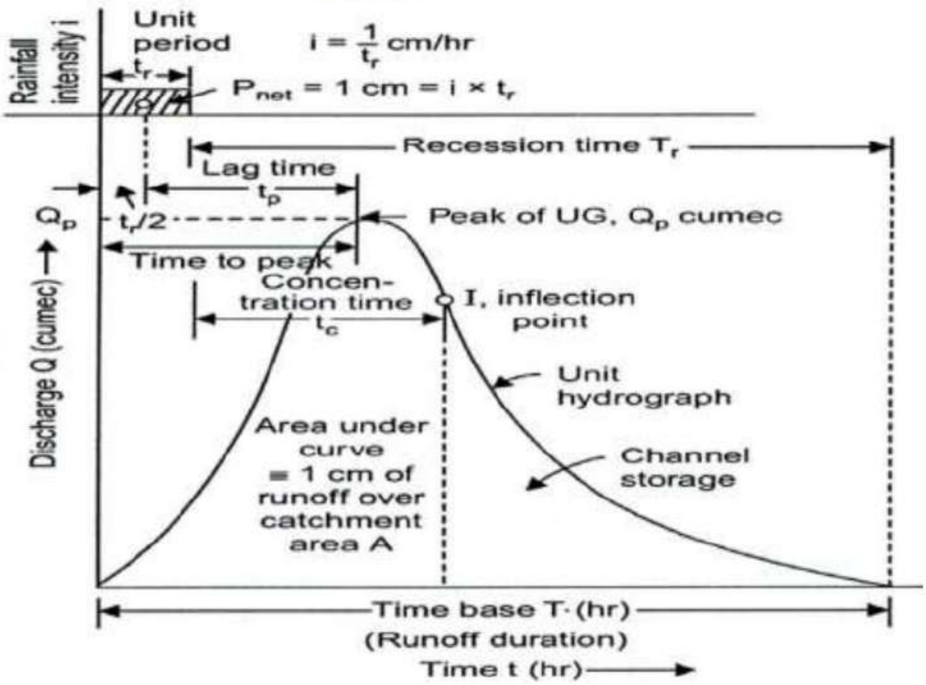

Fig. 8.1 Elements of Unit Hydrograph.

$$\mathbf{T = t_r + T_r}$$

Where T = Time base of UH;

t_r = rainfall duration;

T_r = Time of recession after the rain;

$$\mathbf{Q_P = C\,I\,A}$$

Where Q_P = peak discharge;

C = runoff coefficient;

I = rainfall intensity;

A = Area of basin.

8.2 Derivation of the unit hydrographs.

The following steps are adopted to derive a UH from an observed flood hydrograph (Fig. 8.2).

(1) Select from the records single-peaked intense storms, which occurring uniformly over the basin have produced flood hydrographs with appreciable runoff (>1 cm, say, 8 to 16 cm). The unit period selected should be such that the excess rainfall (i.e., P_{net}) occurs fairly uniformly over the entire drainage basin.

(2) Select a flood hydrograph, which has resulted from a unit storm chosen in item (1) above.

(3) Separate the base flow (BF) from the total flow (TF);

(4) From the ordinates of the TF hydrograph (at regular time intervals) deduct the corresponding ordinates of BF, to obtain the ordinates of DR;

(5) $P_{net} = \frac{\text{volume of DR}}{A}$;

(6) Divide each of the ordinates of DR by the P_{net} depth to obtain the ordinates of the UH.

(7) Plot the ordinates of the UH against time since the beginning of DR.

This will give the UH for the basin, for the duration of the unit storm (producing the flood hydrograph) selected in item (1) above.

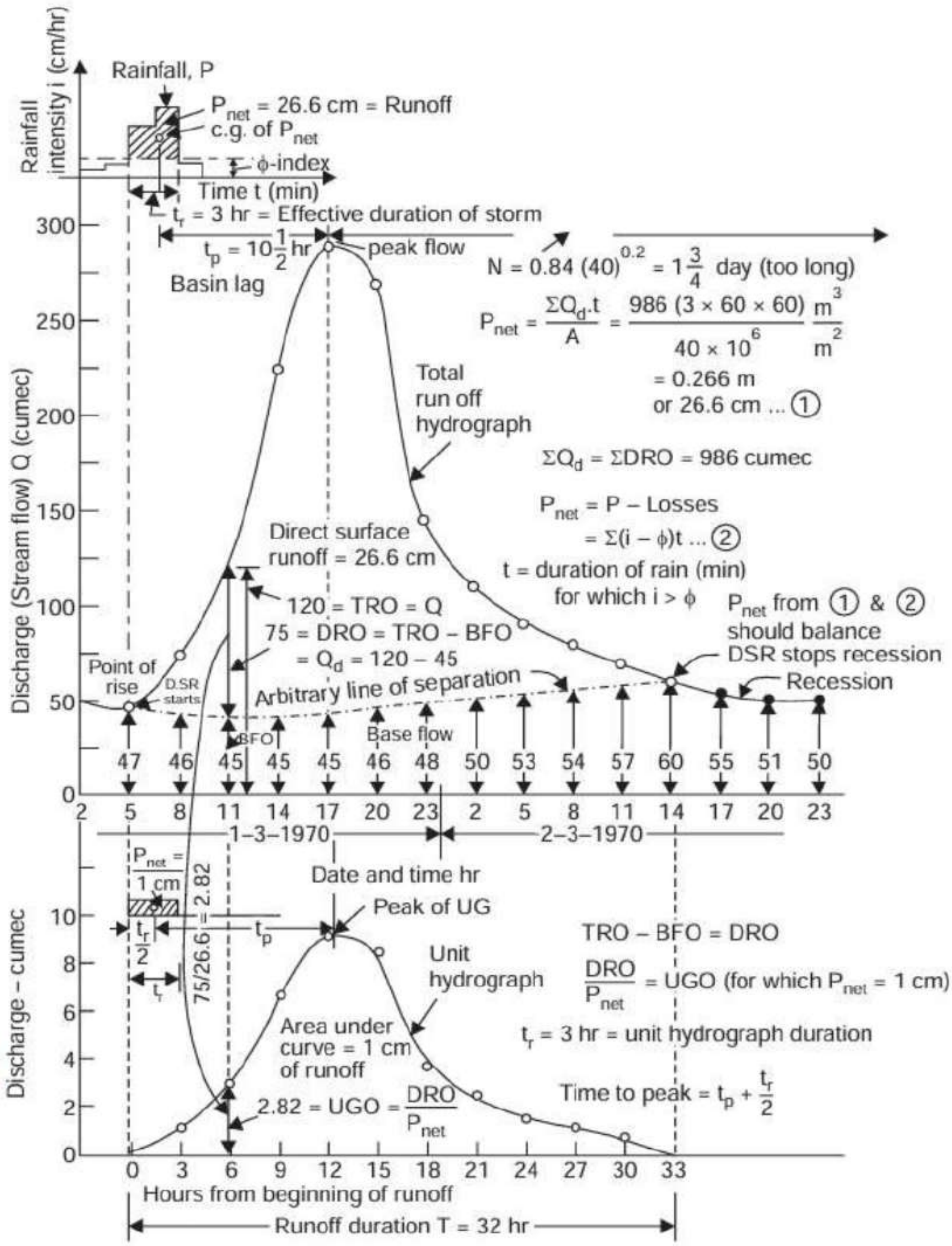

Fig. 8.2 Derivation of UH

8.3 Average unit hydrograph

It is better if several UHs are derived for different isolated (single peaked) uniform intensity storms. If the durations of storms are different, the UH_s may be altered to the same duration. From several UH_s for the same duration, so obtained, an average UH can be sketched by computing the average of the peak flows and times to peak and sketching a median line, so that the area under the graph is equal to a runoff volume of 1 cm, Fig. 8.4.

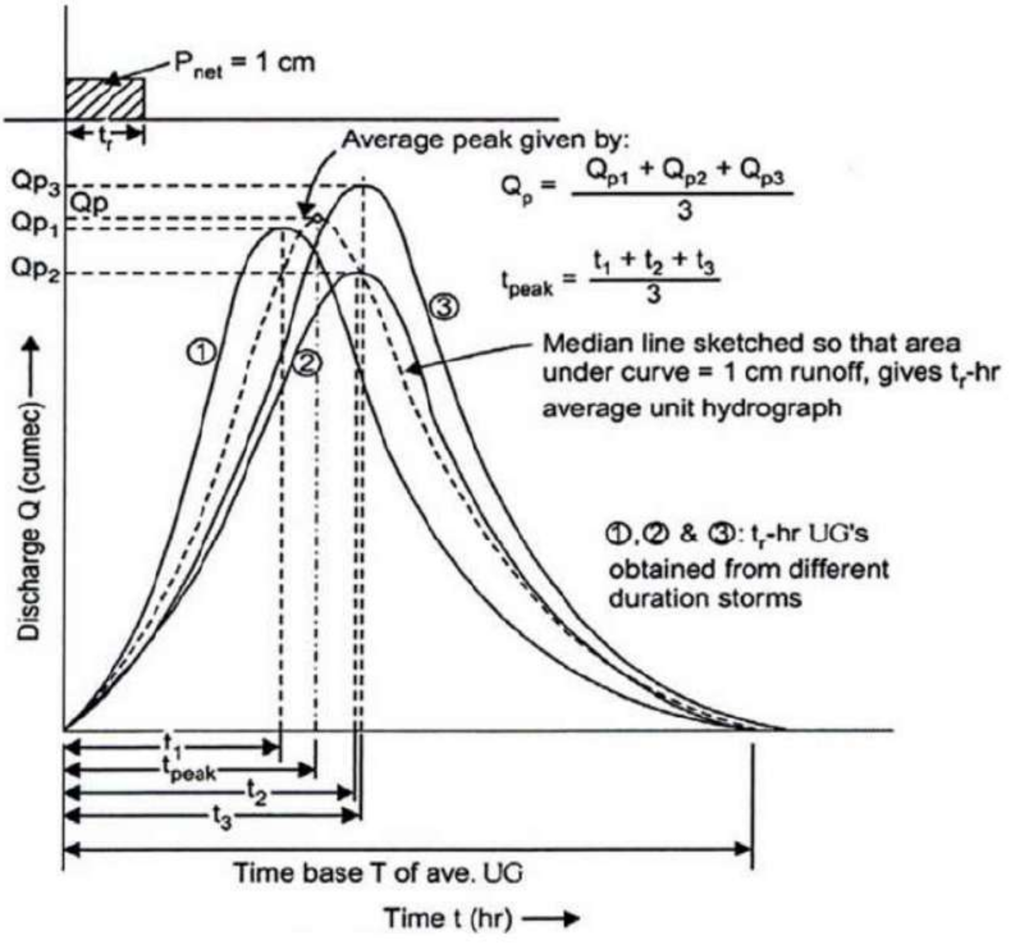

Fig. 8.3 Average unit hydrograph

Example 1: The table below gives the recorded flow from a storm of **3-hrs**. duration on a stream with **a basin area equal 2231 km^2**. Derive the UH for the basin if the BF is 600m^3/s.

Time (hr.)	Day 1	Day 2	Day 3
3 AM	600	4600	1700
6 AM	550	4000	1500
9 AM	6000	3500	1300
Noon	9500	3100	1100
3 PM	8000	2700	900
6 PM	7000	2400	800
9 PM	6100	2100	700
Mid night	5300	1900	600

Solution:

Col.1	Col.2	Col.3	**Col.4 = Col.2 –Col. 3**	**5= 4/29.82**
Time (day)	Flow (m^3/s)	Base Flow (m^3/s)	**Direct Flow (m^3/s)**	**3-hr UH (m^3/s)**
3	600	600	0	0
6	550	600	0	0
9	6000	600	5400	181.09
12	9500	600	8900	298.46
15	8000	600	7400	248.16
18	7000	600	6400	214.62
21	6100	600	5500	184.44
24	5300	600	4700	157.61
27	4600	600	4000	134.14
30	4000	600	3400	114.02
33	3500	600	2900	97.25
36	3100	600	2500	83.84
39	2700	600	2100	70.42
42	2400	600	1800	60.36
45	2100	600	1500	50.30
48	1900	600	1300	43.60
51	1700	600	1100	36.89
54	1500	600	900	30.18

57	1300	600	700	23.47
60	1100	600	500	16.77
63	900	600	300	10.06
66	800	600	200	6.71
69	700	600	100	3.35
72	600	600	0	0
			Sum = 61600	Sum = 2065.728

$$\text{Depth} = \frac{\text{Volume}}{A} = \frac{61600 \times 3 \times 3600}{2231 \times 10^6} \times 100 = 29.82 \text{ cm}$$

To check ……. $\text{depth} = \frac{2065.74 \times 3 \times 3600}{2231 \times 10^6} \times 100 = 1 \text{ cm}$ must be

Example 2: Two storms each of **6-hrs** duration and having rainfall excess values of **(3 and 2 cm)**, respectively, occur successively. Determine the resulting direct runoff hydrograph if the **6-hrs UH** for the basin is tabulated below:

Time (hr)	6-hr-UH (m^3/s)
0	0
6	25
12	50
18	85
24	125
30	160
36	185
42	160
48	110
54	60
60	36
66	25
72	16
78	8
84	0
90	0

Solution:

Time (hr)	6-hr-UH (m^3/s)	DRH **of (3 cm)** (m^3/s)	DRH **of (2 cm)** (m^3/s)	Total DRH (m^3/s)
0	0	0	0	0
6	25	75	0	75
12	50	150	50	200
18	85	255	100	355
24	125	375	170	545
30	160	480	250	730
36	185	555	320	875
42	160	480	370	850
48	110	330	320	650
54	60	180	220	400
60	36	108	120	228
66	25	75	72	147
72	16	48	50	98
78	8	24	32	56
84	0	0	16	16
90	0	0	0	0

8.3 Unit Hydrograph From Complex Storms

UH_s from complex storms, involving varying intensities of rain can be obtained by considering the complex storm as successive unit storms of different intensities and the runoff hydrograph, due to complex storm, as the result of superposition of the successive storm hydrographs. The ordinates of each storm hydrograph are obtained as "the storm intensity times the corresponding ordinate of the UH" as shown in Fig. 8.4.

The UH ordinates:

u_1, u_2, ... are thus obtained by writing a series of eqs for each of the ordinates Q_1, Q_2, ... of the runoff hydrograph (due to complex storm) and successively solving them. In Fig. 5.3,

Number of equations can be write to calculate (Q_n) by (n-m+1) unknown values of UHG;

Where m = number of the rainfall excess, (R).

n = number of the pulses in the DRH, (Q).

n – m + 1 = number of the pulses in the UH, (U).

$$Q_1 = x\,u_1 + 0 + 0 + \cdots \ldots\ldots\ldots\ldots\ldots. \qquad \lll\lll\lll \qquad \therefore\ u_1 = \frac{Q_1}{x}$$

$$Q_2 = x\,u_2 + \ y\,u_1 + 0 + \cdots \ldots\ldots\ldots. \qquad \lll\lll\lll \qquad \therefore\ u_2 = \frac{Q_2\ - y\,u_1}{x}$$

$$Q_3 = x\,u_3 + y\,u_2 + \ z\,u_1 + 0 + \cdots. \qquad \lll\lll\lll \qquad \therefore\ u_3 = \frac{Q_3\ - y\,u_2 -\ z\,u_1}{x}$$

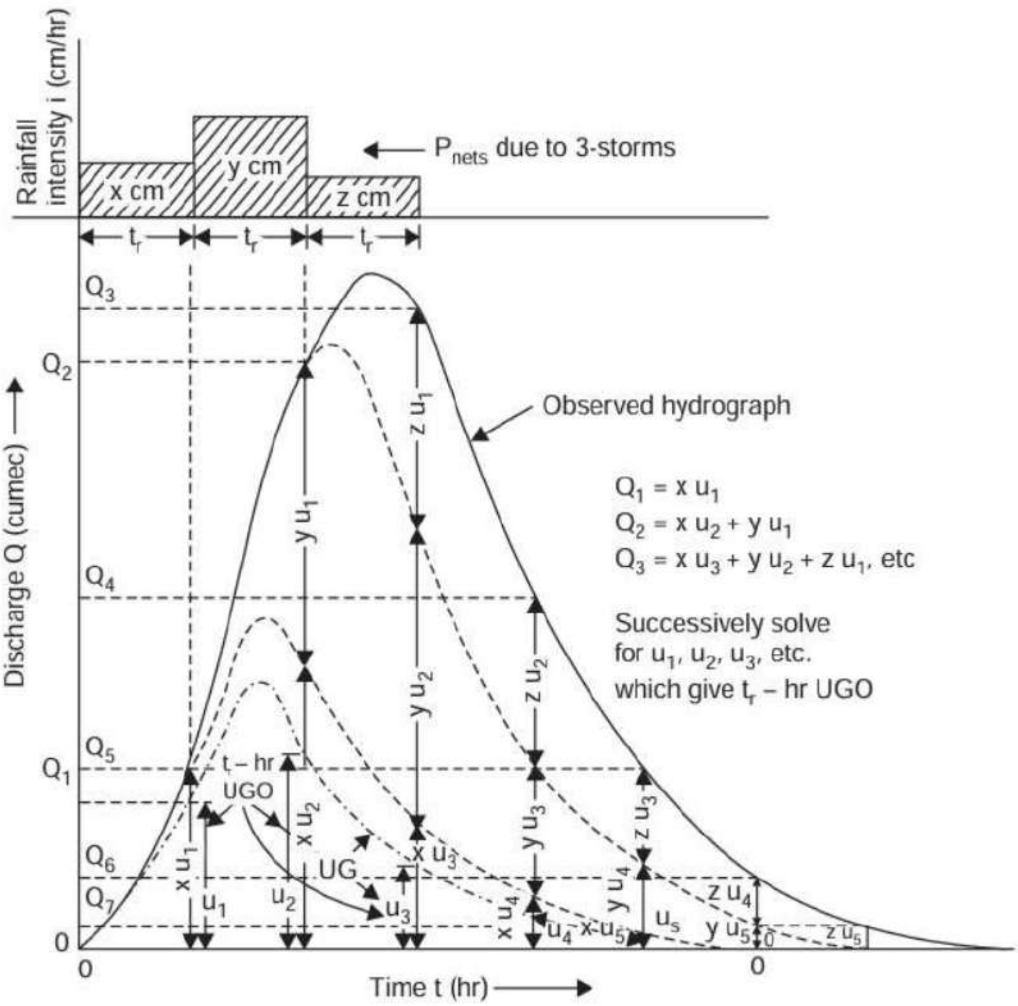

Fig. 8.4 Derivation of unit hydrograph from multi-period storms

Example 3: Derive a UH from a DRH data resulting from a storm of variable rainfall excess.

Time (hr)	1	2	3	4	5	6	7	8	9	10	11
DRH (cfs)	428	1923	5297	9131	10625	7834	3921	1846	1402	830	313
E. R. (in)	1.06	1.93	1.81								

Solution:

m = 3 number of the rainfall excess, (R).

n = 11 number of the pulses in the DRH, (Q).

n – m + 1 = 11 – 3 +1=9 number of the pulses in the UH, (U).

Time (hr)	UH (cfs)	DRH			DRH (cfs)		UH (cfs)
		1st storm (**x, 1.06 in**)	2nd storm (**y, 1.93 in**)	3rd storm (**z, 1.81 in**)			
1	U_1	1.06 x U_1	0	0	428	428 = 1.06 x U_1	**U_1 =** 404
2	U_2	1.06 x U_2	1.93 x U_1	0	1923	1923 = 1.06 x U_2 + 1. 93 x 404	**U_2 =** 1079
3	U_3	1.06 x U_3	1.93 x U_2	1.81 x U_1	5297	5297 = 1.06 x U_3 + 1. 93 x 1079 + 1.81 x 404	**U_3 =** 2343
4	U_4	1.06 x U_4	1.93 x U_3	1.81 x U_2	9131	9131 = 1.06 x U_4 + 1. 93 x 2343 + 1.81 x 1079	**U_4 =** 2506
5	U_5	1.06 x U_5	1.93 x U_4	1.81 x U_3	10625	10625 = 1.06 x U_5 + 1. 93 x 2506 + 1.81 x 2343	**U_5 =** 1460
6	U_6	1.06 x U_6	1.93 x U_5	1.81 x U_4	7834	7834 = 1.06 x U_6 + 1. 93 x 1460 + 1.81 x 2506	**U_6 =** 453
7	U_7	1.06 x U_7	1.93 x U_6	1.81 x U_5	3921	3921 = 1.06 x U_7 + 1. 93 x 453 + 1.81 x 1460	**U_7 =** 381
8	U_8	1.06 x U_8	1.93 x U_7	1.81 x U_6	1846	1846 = 1.06 x U_8 + 1. 93 x 381+ 1.81 x 453	**U_8 =** 274
9	U_9	1.06 x U_9	1.93 x U_8	1.81 x U_7	1402	1402 = 1.06 x U_9 + 1. 93 x 274 + 1.81 x 381	**U_9 =** 173
10	U_{10}	1.06 x U_{10}	1.93 x U_9	1.81 x U_8	830	830 = 1.06 x U_{10}+ 1.93 x 173 + 1.81x 274	**U_{10} =** 0

Example 4: The stream flows due to three successive storms of **2.9, 4.9, and 3.9 cm** of **6-hrs** duration each on a basin are tabulated below. The area of the basin is **118.8 km^2**. Assume a constant base flow of **20 cumec** and an average storm loss of **0.15 cm/hr**. Derive **a 6-hr UH** for the basin.

Time (hr)	0	3	6	9	12	15	18	21	24	27	30	33
Flow (cumec)	20	50	92	140	199	202	204	144	84.5	45.5	29	20

Solution: For the 1st storm (0 to 6) : $x = 2.9 - 0.15 \times 6 = 2$ cm

For the 2nd storm (6 to 12) : $y = 4.9 - 0.15 \times 6 = 4$ cm

For the 3rd storm (0 to 6) : $z = 3.9 - 0.15 \times 6 = 3$ cm

Time (hr)	UH	DRH			DRH	Equation	6-hr.-UH
		1st storm (x, 2 cm)	**2nd storm (y, 4 cm)**	**3rd storm (z, 3 cm)**		TDRH = TR - BF	
0	U_0	0	-	-	20-20=0	0 = 20 - 20	0
3	U_1	$2 \times U_1$	-	-	50-20=30	$2 \times U_1 = 30$	15
6	U_2	$2 \times U_2$	0	-	92-20=72	$2 \times U_2 = 72$	36
9	U_3	$2 \times U_3$	$4 \times U_1$	-	140-20=120	$2 \times U_3 + 4 \times U_1 = 120$	30
12	U_4	$2 \times U_4$	$4 \times U_2$	0	199-20=179	$2 \times U_4 + 4 \times U_2 + 0 = 179$	17.5
15	U_5	$2 \times U_5$	$4 \times U_3$	$3 \times U_1$	202-20=182	$2 \times U_5 + 4 \times U_3 + 3 \times U_1 = 182$	8.5
18	U_6	$2 \times U_6$	$4 \times U_4$	$3 \times U_2$	204-20=184	$2 \times U_6 + 4 \times U_4 + 3 \times U_2 = 184$	3
21	U_7	$2 \times U_7$	$4 \times U_5$	$3 \times U_3$	144-20=124	$2 \times U_7 + 4 \times U_5 + 3 \times U_3 = 124$	0
24			$4 \times U_6$	$3 \times U_4$	84.5-20=64.5	$4 \times U_6 + 3 \times U_4 = 64.5$	
27			$4 \times U_7$	$3 \times U_5$	45.5-25.520=25.5	$4 \times U_7 + 3 \times U_5 = 25.5$	
30				$3 \times U_6$	29-20=9	$3 \times U_6 = 9$	
Depth $= \frac{110 \times 3 \times 60 \times 60}{118.8 \times 10^6} = 0.01$ m, or 1 cm ok							