Probability (الاحتمالية)

Probability refers to the likelihood of an event happening when we're uncertain about the outcome. It's a key concept in daily life, helping us predict how likely something is to occur.

Probability values range from 0 to 1:

- 0 means the event is impossible.
- 1 means the event is certain.

For an experiment with sample space S and an event E, the probability of E is:

$$P(E) = \frac{n(E)}{n(S)},$$

where:

- $n(E)$ is the number of outcomes in event E.
- $n(S)$ is the total number of outcomes in the sample space S.

Example 1: Statistics on blood types in Iraq based on the 2024 population census are as follows:

Blood type	O^+	A^+	B^+	AB^+	O^-	A^-	B^-	AB^-
Number of persons	14,804,132	11,529,698	11,806,411	3,412,791	1,660,277	1,245,209	1,245,206	415,069

What is the probability that a randomly selected person has B^+ blood type?

Solution:

$$P(B^+) = \frac{n(B^+)}{n(S)} = \frac{11,806,411}{46,118,793} = 0.256$$

Elementary Properties of Probabilities

The elementary properties of probabilities dictate that the likelihood of any event E is a number $P(E)$ such that $0 \le P(E) \le 1$.

For a sample space S with n events, $S = \{E_1, E_2, \cdots, E_n\}$. These properties are:

1. The probability of any event is always non-negative: $P(E) \ge 0, \forall E \in S$.
2. The probability of the sample space S is 1: $P(S) = 1$.
3. The probability of an event that cannot occur is 0: $P(\emptyset) = 0$.

4. The probability of a complement E is: $P(\overline{E}) = 1 - P(E)$.
5. For events that are disjoint E_1 and E_2 (i.e., cannot occur at the same time), That is $E_1 \cap E_2 = \emptyset$, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.
6. For any two events E_1 and E_2, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
7. If event E_1 is a subset of event E_2 $(E_1 \subseteq E_2)$, then $P(E_1) \leq P(E_2)$.
8. Events A and B are statistically independent if and only if $P(A \cap B) = P(A)P(B)$.

Example 2: Let $S = \{1,2,3,4,5,6\}$. If an element is selected from S,

1. What is the probability that it is an odd number and divisible by 3?
2. What is the probability that it is an odd number or divisible by 3?

Solution: The event that the number is odd: $A = \{1,3,5\}$

The event that the number divisible by 3: $B = \{3,6\}$

1. $P(A \cap B) = P(A)P(B) = \frac{3}{6} \times \frac{2}{6} = \frac{1}{6}$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{2}{3}$.

Probability Distributions

Probability distributions are key in statistical analysis since they show all possible values of a random variable and their associated probabilities. This lets researchers summarize data in a way that makes it easy to make objective decisions based on samples from the populations these distributions represent.

Often, researchers prefer dealing with numerical values tied to sample points rather than the points themselves, especially when those points are qualities or names that are hard to work with mathematically. To handle this, descriptive values get converted into numerical values, known as random variable values. Random variables express random experiment outcomes and events as numbers instead of names or qualities.

For example, when a coin is tossed three times, the sample space is:

S = {HHH,HHT,HTH, THH,HTT, THT, TTH, TTT}.

If we're interested in the number of times H appears, regardless of other details, that's a random variable x_i with values $x_i = 0,1,2,3$.

Random variables come in types: discrete and continuous.

Discrete Probability Distributions

A discrete probability distribution is a table or rule listing all values of a discrete random variable with their associated probabilities, denoted by $P(x)$. It satisfies:

1- $P(x) \geq 0$.

2- $\sum P(x_i) = 1$.

$P(x)$ is the probability density function (pdf).

Example 3: In 2014, 50 students from the College of Science at the University of Babylon donated blood to the wounded of the Ali al-Akbar Brigade, who were injured during the operations to liberate Talafar from Daesh gangs. The hemoglobin levels in their blood were summarized in the following table:

Hemoglobin levels	12.8-13.8	13.9-14.9	15.0-16.0	16.1-17.1	17.2-18.2	18.3-19.3
Number of students	4	7	12	13	8	6

Create a table listing all values of a discrete random variable and their corresponding probabilities.

Solution:

Callses	12.8-13.8	13.9-14.9	15.0-16.0	16.1-17.1	17.2-18.2	18.3-19.3	Sum
x_i	13.3	14.4	15.5	16.6	17.7	18.8	
f_i	4	7	12	13	8	6	50
$P(x_i)$	$\frac{4}{50} = 0.08$	$\frac{7}{50} = 0.14$	$\frac{12}{50} = 0.24$	$\frac{13}{50} = 0.26$	$\frac{8}{50} = 0.16$	$\frac{6}{50} = 0.12$	1

Mean and Variance of Discrete Probability Distributions

The mean μ and variance σ^2 of a discrete probability distribution can be calculated using specific formulas:

$$\mu = \sum x_i P(x_i)$$

$$\sigma^2 = \sum (x_i - \mu)^2 P(x_i)$$

The standard deviation σ is just the positive square root of the variance.

Question: Calculate the mean and variance for Example 3.

Example 3: Pain levels were assessed post-surgery in 60 Zionist patients injured by Iranian missiles. Here's the outcome:

Pain levels	Very mild	Mild	Moderate	Severe	Very severe
Number of Zionist	6	9	12	15	18

Calculate the coefficient of variation $C.V$ and standard error SE.

Solution: Let's assign numerical values to pain levels and set up the table to simplify the solution.

x_i	1	2	3	4	5	Sum
f_i	6	9	12	15	18	60
$P(x_i)$	$\frac{6}{60} = 0.1$	$\frac{9}{60} = 0.15$	$\frac{12}{60} = 0.2$	$\frac{15}{60} = 0.25$	$\frac{18}{60} = 0.3$	1
$x_i P(x_i)$	0.1	0.3	0.6	1	1.5	$\mu = 3.5$
$x_i - \mu$	-2.5	-1.5	-0.5	0.5	1.5	
$(x_i - \mu)^2$	6.25	2.25	0.25	0.25	2.25	
$(x_i - \mu)^2 P(x_i)$	0.625	0.3375	0.05	0.0625	0.675	$\sigma^2 = 1.75$

$$C.V = \frac{\sigma}{\mu} \times 100\% = \frac{\sqrt{1.75}}{3.5} \times 100\% = 37.8\%$$

$$SE = \frac{\sigma}{\sqrt{\sum f_i}} = \frac{\sqrt{1.75}}{\sqrt{60}} = 0.17$$

H.W. A Zionist enemy camp was targeted by an Iranian Kheiber rocket, resulting in 75 casualties, including:

Injury Type	Mild	Moderate	Severe	Critical	Dead
Number of Casualties	9	12	18	21	15

Calculate the standard deviation, coefficient of variation, and standard error of the given data.