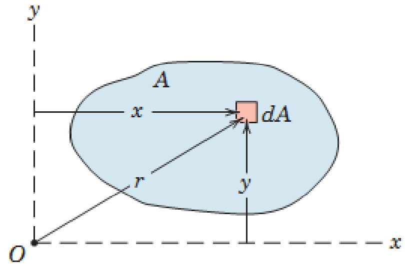
Area Moments of Inertia

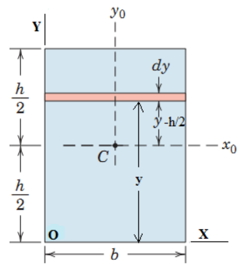
$$I_x = \int y^2 dA$$
$$I_y = \int x^2 dA$$



SAMPLE PROBLEM A/1

Determine the moments of inertia of the rectangular area about the centroidal x_0- and y_0-axes.

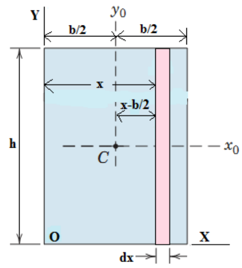
Solution: a horizontal strip of area b*dy is chosen so that all elements of the strip have the same y-coordinate.



$$I_{x0} = \int_0^h (y-\tfrac{h}{2})^2 (bdy) = b\int_0^h (y^2 - hy + \tfrac{h^2}{4})dy = b\left[\tfrac{y^3}{3} - \tfrac{hy^2}{2} + \tfrac{h^2}{4}y\right]_0^h$$

$$= b\left[\left(\tfrac{h^3}{3} - \tfrac{hh^2}{2} + \tfrac{h^2}{4}h\right) - 0\right] = \frac{bh^3}{12} \text{ about centroidal axis}$$

$$I_x = \int_0^h y^2 (bdy) = b\left[\tfrac{y^3}{3}\right]_0^h = \frac{bh^3}{3} \text{ about base axis}$$



$$I_{y0} = \int_0^b (x-\tfrac{b}{2})^2 (hdx) = h\int_0^b (x^2 - bx + \tfrac{b^2}{4})dx = h\left[\tfrac{x^3}{3} - \tfrac{bx^2}{2} + \tfrac{b^2}{4}x\right]_0^b$$

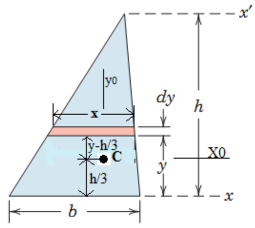
$$= b\left[\left(\tfrac{b^3}{3} - \tfrac{bb^2}{2} + \tfrac{b^2}{4}b\right) - 0\right] = \frac{hb^3}{12} \text{ about centroidal axis}$$

$$I_y = \int_0^b x^2 (hdx) = h\left[\tfrac{x^3}{3}\right]_0^b = \frac{hb^3}{3} \text{ about base axis}$$

SAMPLE PROBLEM A/2

Determine the moments of inertia of the triangular area about its base and about parallel axes through its centroid and vertex

Solution:



$$\frac{x}{h-y} = \frac{b}{h} \Rightarrow x = \frac{b}{h}(h-y)$$

$$I_{x0} = \int_0^h \left(y - \tfrac{h}{3}\right)^2 (xdy) = \int_0^h \left(y^2 - \tfrac{2}{3}hy + \tfrac{h^2}{9}\right)\frac{b}{h}(h-y)dy$$

$$= \frac{b}{h}\int_0^h (hy^2 - \tfrac{2}{3}h^2y + \tfrac{h^3}{9} - y^3 + \tfrac{2}{3}hy^2 - \tfrac{h^2y}{9})dy = \frac{b}{h}\left[\tfrac{hy^3}{3} - \tfrac{2}{3}h^2\tfrac{y^2}{2} + \tfrac{h^3}{9}y - \tfrac{y^4}{4} + \tfrac{2}{3}h\tfrac{y^3}{3} - \tfrac{h^2y^2}{9*2}\right]_0^h$$

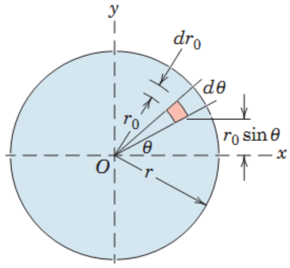
$$= \frac{bh^3}{36} \text{ about centroidal axis}$$

$$I_x = \int_0^h y^2 (xdy) = \int_0^h y^2\frac{b}{h}(h-y)dy = \frac{b}{h}\int_0^h (y^2h - y^3)dy = \frac{b}{h}\left[\frac{y^3}{3}h - \frac{y^4}{4}\right]_0^h$$

$$= \frac{bh^3}{12} \text{ about base axis}$$

SAMPLE PROBLEM A/3

Calculate the moments of inertia of the area of a circle about a diametral axis



Solution:

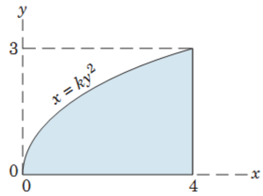
$$I_x = \int y^2(dA) = \int_0^{2\pi}\int_0^{r} (r_0 \sin\theta)^2 (r_0 d\theta dr_0) = \int_0^{2\pi}\int_0^{r} {r_0}^3 (\sin\theta)^2 d\theta dr_0$$

$$= \int_0^{2\pi} \left[\frac{{r_0}^4}{4}\right]_0^r (\sin\theta)^2 d\theta = \int_0^{2\pi} \frac{r^4}{4}(\sin\theta)^2 d\theta = \frac{r^4}{4}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta = \frac{r^4}{8}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{\pi r^4}{4}$$

$$I_x = I_y = \frac{\pi r^4}{4}$$

SAMPLE PROBLEM A/4

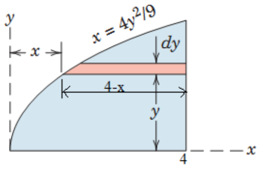
Determine the moment of inertia of the area under the parabola about the x-axis. Solve by using (a) a horizontal strip of area and (b) a vertical strip of area.



Solution:

$x = ky^2 \Rightarrow 4 = k3^2 \Rightarrow k = \frac{4}{9}$

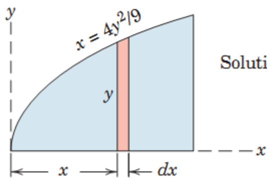
$x = \frac{4}{9}y^2$



1. Horizontal Strip

$I_x = \int_0^3 y^2(4-x)dy = \int_0^3 y^2(4-\frac{4}{9}y^2)dy = \int_0^3(4y^2 - \frac{4}{9}y^4)dy =$

$\left[\frac{4y^3}{3} - \frac{4}{9*5}y^5\right]_0^3 = 14.4\ (units)^4$

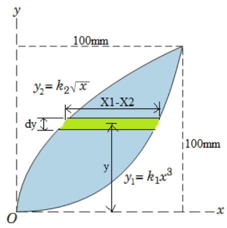


2. Vertical Strip

$$I_x = \int_0^4 \frac{y^3}{3}dx = \frac{1}{3}\int_0^4 \left[\left(\frac{9x}{4}\right)^{\frac{1}{2}}\right]^3 dx = \frac{27}{24}\left[\frac{2x^{\frac{5}{2}}}{5}\right]_0^4 = 14.4\ (units)^4$$

Prob. A/32

Calculate the moments of inertia of the shaded area about the x- and y-axes, and find the polar moment of inertia about point O.



Solution:

$y_2 = k_2\sqrt{x} \Rightarrow 100 = k_2\sqrt{100} \Rightarrow k_2 = 10$

$y_1 = k_1x^3 \Rightarrow 100 = k_1 100^3 \Rightarrow k_1 = \frac{1}{10000}$

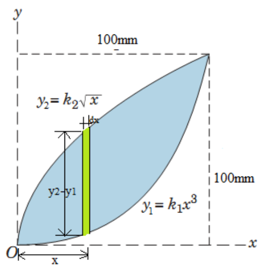
$y_1 = \frac{1}{10000}x^3,\ y_2 = 100\sqrt{x}$

$$I_x = \int_0^{100} y^2(x_1 - x_2)dy = \int_0^{100} y^2\left(\sqrt[3]{\frac{y}{k_1}} - \frac{y^2}{{k_2}^2}\right)dy = \left[\frac{3y^{\frac{10}{3}}}{10\sqrt[3]{k_1}} - \frac{y^5}{5{k_2}^2}\right]_0^{100} = 1 * 10^7 mm^4$$

$I_y = \int_0^{100}\frac{{x_1}^3 dy}{3} - \int_0^{100}\frac{{x_2}^3 dy}{3} = \frac{1}{3}\int_0^{100}({x_1}^3 - {x_2}^3)dy =$

$\frac{1}{3}\int_0^{100}\left(\left(\sqrt[3]{\frac{y}{k_1}}\right)^3 - \left(\frac{y^2}{{k_2}^2}\right)^3\right)dy = \frac{1}{3}\int_0^{100}\left(\frac{y}{k_1} - \frac{y^6}{{k_2}^6}\right)dy = 11.9 * 10^6 mm^4$

OR



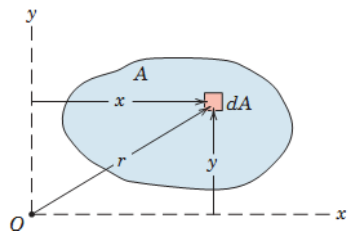
$$I_y = \int_0^{100} x^2(y_2 - y_1)dx = \int_0^{100} x^2(k_2\sqrt{x} - k_1x^3)dx = 11.9 * 10^6 mm^4$$

Polar Moments of Inertia:

The moment of inertia of dA about the pole O (z-axis) is

$$I_z = \int r^2 dA = \int \underbrace{(x^2 + y^2)}_{r^2} dA = \int x^2 dA + \int y^2 dA = I_y + I_x$$

Radius of Gyration,k



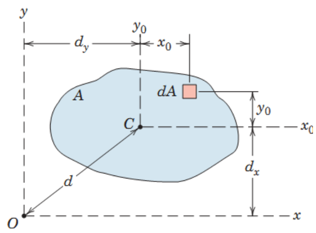
Is a measure of the distribution of the area from the axis in question.

$$k = \sqrt{\frac{I}{A}}$$

$$k_x = \sqrt{\frac{I_x}{A}}, \quad k_y = \sqrt{\frac{I_y}{A}}, \quad k_z = \sqrt{\frac{I_z}{A}}$$

$$I_z = I_x + I_y \Rightarrow {k_z}^2 A = {k_x}^2 A + {k_y}^2 A \Rightarrow {k_z}^2 = {k_x}^2 + {k_y}^2$$

Transfer of Axes(parallel-axis theorems)



C: the centroid of the area.

$$I_x = \int (y_0 + d_x)^2 dA = \int ({y_0}^2 + 2y_0 d_x + {d_x}^2) dA$$

$$= \underbrace{\int ({y_0}^2) dA}_{I_{x0}} + \underbrace{\int (2y_0 d_x) dA}_{zero} + \underbrace{{d_x}^2 \int dA}_{{d_x}^2 A}$$

The second integral is zero, since $\int (y_0) dA = A\bar{y}_0$ and $\bar{y}_0$is automatically zero with the centroid on the x_0-axis.

$$I_x = I_{x0} + {d_x}^2 A$$

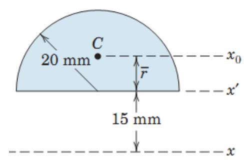
$$I_y = I_{y0} + {d_y}^2 A$$

$$I_z = I_x + I_y = I_{x0} + {d_x}^2 A + I_{y0} + {d_y}^2 A = \underbrace{I_{x0} + I_{y0}}_{I_{z0}} + A \underbrace{({d_x}^2 + {d_y}^2)}_{=d^2} = I_{z0} + Ad^2$$

Two points in particular should be noted.

1. The axes between which the transfer is made must be parallel.
2. One of the axes must pass through the centroid of the area.

SAMPLE PROBLEM A/5



Find the moment of inertia about the x-axis of the semicircular area.

Solution.

$$\bar{r} = \frac{4r}{3\pi} = \frac{4*20}{3\pi} = \frac{80}{3\pi}$$

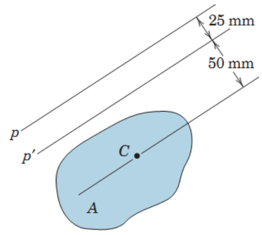
$$I_{\bar{x}} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{1}{2}\left(\frac{\pi 20^4}{4}\right) = 62832\ mm^4$$

$$I_{\bar{x}} = I_{x_0} + A * (\bar{r})^2 \Rightarrow 62832 = I_{x_0} + \frac{\pi 20^2}{2} * \left(\frac{80}{3\pi}\right)^2 \Rightarrow I_{x_0} = 17561\ mm^4$$

$$\text{OR } I_{x_0} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) 20^4 = 17561\ mm^4$$

$$I_x = I_{x_0} + A * (\bar{r} + 15)^2 = 17561 + \frac{\pi 20^2}{2} * \left(\frac{80}{3\pi} + 15\right)^2 = 36.4 * 10^4\ mm^4$$

Prob. A/5



The moments of inertia of the area A about the parallel p- and p' -axes differ by $15*10^6$ mm^4. Compute the area A, which has its centroid at C.

Solution:

$$I_{\bar{p}} = I_0 + A * d^2 = I_0 + A * 50^2$$

$$I_p = I_0 + A * d^2 = I_0 + A * 75^2$$

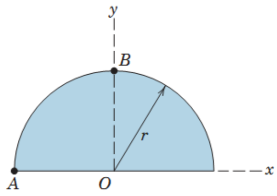
$$I_p - I_{\bar{p}} = (I_0 + A * 75^2) - (I_0 + A * 50^2) = 75^2 A - 50^2 A$$

$$75^2 A - 50^2 A = 15 * 10^6 \Rightarrow A = 4800\ mm^2$$

Prob. A/7

Determine the polar moments of inertia of the semi-circular area about points A and B.

Solution:



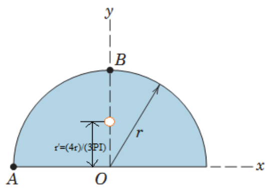
$$I_x = I_y = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{\pi r^4}{8}$$

$$I_x = I_{x_0} + Ad^2 \Rightarrow \frac{\pi r^4}{8} = I_{x_0} + \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2 \Rightarrow I_{x_0} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

<u>Point B</u>

$$I_{\bar{x}} = I_{x_0} + Ad^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^2}{2}\left(r - \frac{4r}{3\pi}\right)^2 = \left(\frac{5}{8}\pi - \frac{4}{3}\right) r^4$$

$$I_z = I_{\bar{x}} + I_y = \left(\frac{5}{8}\pi - \frac{4}{3}\right) r^4 + \frac{\pi r^4}{8} = \left(\frac{3}{4}\pi - \frac{4}{3}\right) r^4$$



<u>Point A</u>

$$I_{\bar{y}} = I_{y_0} + Ad^2 = \frac{\pi r^4}{8} + \frac{\pi r^2}{2} r^2 = \frac{5}{8}\pi r^4$$

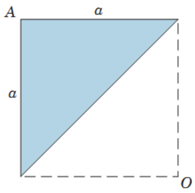
$$I_z = I_x + I_{\bar{y}} = \frac{\pi r^4}{8} + \frac{5}{8}\pi r^4 = \frac{3}{4}\pi r^4$$

Prob. A/9

Determine the polar radii of gyration of the triangular area about points O and A.

Solution:

<u>Point A</u>

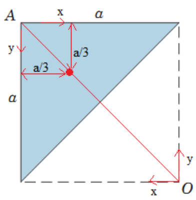


$$I_x = I_y = \frac{aa^3}{12}$$

$$I_z = I_x + I_y = \frac{a^4}{12} + \frac{a^4}{12} = \frac{a^4}{6}$$

$$A = \frac{a}{2} * a = \frac{a^2}{2}$$

$$k_A = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{a^4/6}{a^2/2}} = \frac{a}{\sqrt{3}}$$



<u>Point O</u>

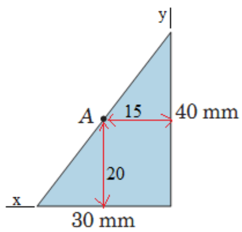
$$I_x = I_y = \frac{aa^3}{3} - \frac{aa^3}{12} = \frac{a^4}{4} \quad \text{OR } I_x = I_y = I_{y_0} + Ad^2 = \frac{aa^3}{36} + \frac{a^2}{2} * \left(\frac{2}{3}a\right)^2$$

$$I_z = I_x + I_y = \frac{a^4}{4} + \frac{a^4}{4} = \frac{a^4}{2}$$

$$k_O = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{a^4/2}{a^2/2}} = a$$

Prob. A/13

Determine the radius of gyration about a polar axis through the midpoint A of the hypotenuse of the right-triangular area. (Hint: Simplify your calculation by observing the results for a 30*40-mm rectangular area.)



Solution:

$$I_{x0} = \frac{30*40^3}{36} = 53333\, mm^4,\ I_{y0} = \frac{40*30^3}{36} = 30000\, mm^4,\ A = \frac{40*30}{2} = 600 mm^2$$

<u>Poi nt A</u>

$$I_x = I_{x_0} + Ad^2 = 53333 + 600 * \left(20 - \frac{40}{3}\right)^2 = 80000 mm^4$$

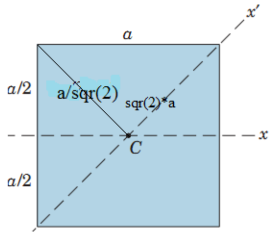
$$I_y = I_{y_0} + Ad^2 = 30000 + 600 * \left(15 - \frac{30}{3}\right)^2 = 45000 mm^4$$

$$I_z = I_x + I_y = 80000 + 45000 = 125000 mm^4$$

$$k_A = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{125000}{600}} = 14.43 mm$$

Prob. A/17

In two different ways show that the moments of inertia of the square area about the x- and x'-axes are the same.



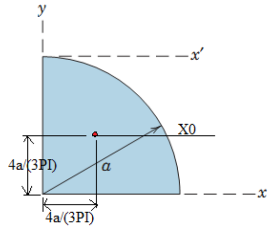
Solution:

$$I_x = \frac{aa^3}{12} = \frac{a^4}{12}$$

$$I_{\bar{x}} = 2\left[\frac{\sqrt{2}a(\frac{a}{\sqrt{2}})^3}{12}\right] = \frac{a^4}{12}$$

Prob. A/23

Determine the moment of inertia of the quarter-circular area about the tangent x' -axis.



Solution:

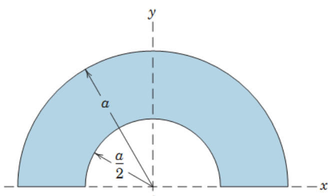
$$I_x = \frac{\pi r^4}{16} = \frac{\pi a^4}{16}$$

$$I_x = I_{x_0} + Ad^2 \Rightarrow \frac{\pi a^4}{16} = I_{x_0} + \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right)^2 \Rightarrow I_{x_0} = a^4(\frac{\pi}{16} - \frac{4}{9\pi})$$

$$I_{\bar{x}} = I_{x_0} + Ad^2 = a^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) + \frac{\pi a^2}{4}\left(a - \frac{4a}{3\pi}\right)^2 = 0.315a^4$$

Prob. A/33

By the methods of this article, determine the rectangular and polar radii of gyration of the shaded area about the axes shown.



Solution:

For circular area, $I_x = I_y = \frac{\pi r^4}{4}$

For half-circular area, $I_x = I_y = \frac{\pi r^4}{8} = \frac{\pi\left[a^4 - (\frac{a}{2})^4\right]}{8} = \frac{15}{128}\pi a^4$

$$I_z = I_x + I_y = \frac{15}{128}\pi a^4 * 2 = \frac{15}{64}\pi a^4$$

$$k_z = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{15}{64}\pi a^4}{\frac{\pi\left[a^2 - (\frac{a}{2})^2\right]}{2}}} = 0.79a$$

TABLE D/3 PROPERTIES OF PLANE FIGURES

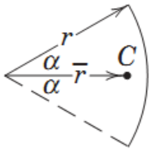
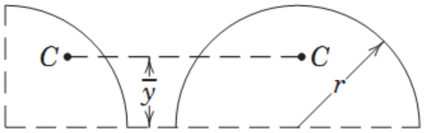
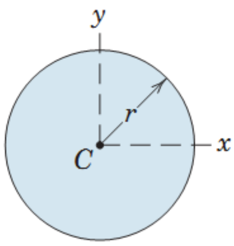
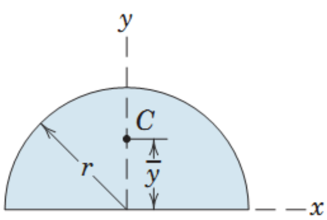
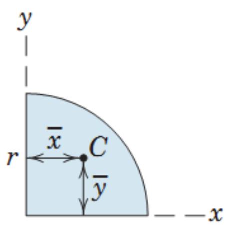
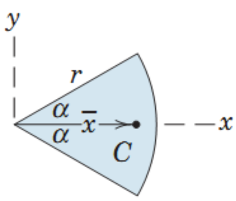
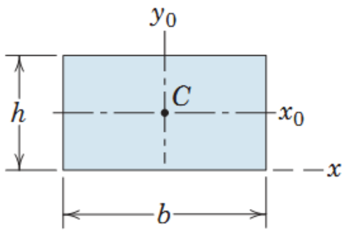
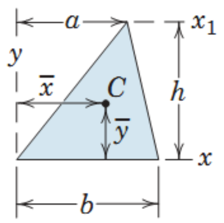
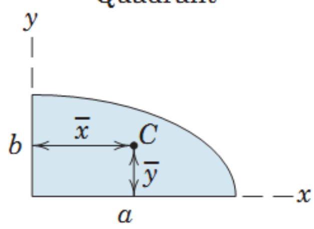
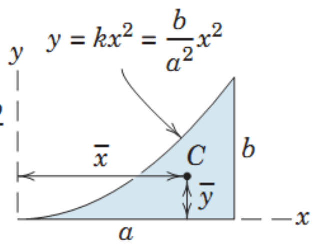
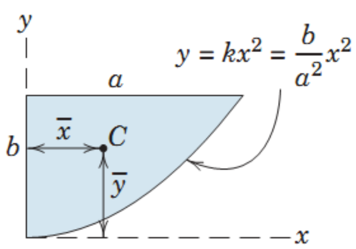
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment 	$\bar{r} = \dfrac{r \sin \alpha}{\alpha}$	—
Quarter and Semicircular Arcs 	$\bar{y} = \dfrac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \dfrac{\pi r^4}{4}$ $I_z = \dfrac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{8}$ $\bar{I}_x = \left(\dfrac{\pi}{8} - \dfrac{8}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \dfrac{4r}{3\pi}$	$I_x = I_y = \dfrac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right) r^4$ $I_z = \dfrac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \dfrac{2}{3} \dfrac{r \sin \alpha}{\alpha}$	$I_x = \dfrac{r^4}{4}\left(\alpha - \dfrac{1}{2} \sin 2\alpha\right)$ $I_y = \dfrac{r^4}{4}\left(\alpha + \dfrac{1}{2} \sin 2\alpha\right)$ $I_z = \dfrac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area  	—	$I_x = \dfrac{bh^3}{3}$ $\bar{I}_x = \dfrac{bh^3}{12}$ $\bar{I}_z = \dfrac{bh}{12}(b^2 + h^2)$
Triangular Area  	$\bar{x} = \dfrac{a+b}{3}$ $\bar{y} = \dfrac{h}{3}$	$I_x = \dfrac{bh^3}{12}$ $\bar{I}_x = \dfrac{bh^3}{36}$ $I_{x_1} = \dfrac{bh^3}{4}$
Area of Elliptical Quadrant  	$\bar{x} = \dfrac{4a}{3\pi}$ $\bar{y} = \dfrac{4b}{3\pi}$	$I_x = \dfrac{\pi ab^3}{16}$, $\bar{I}_x = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right)ab^3$ $I_y = \dfrac{\pi a^3 b}{16}$, $\bar{I}_y = \left(\dfrac{\pi}{16} - \dfrac{4}{9\pi}\right)a^3 b$ $I_z = \dfrac{\pi ab}{16}(a^2 + b^2)$
Subparabolic Area Area $A = \dfrac{ab}{3}$  	$\bar{x} = \dfrac{3a}{4}$ $\bar{y} = \dfrac{3b}{10}$	$I_x = \dfrac{ab^3}{21}$ $I_y = \dfrac{a^3 b}{5}$ $I_z = ab\left(\dfrac{a^2}{5} + \dfrac{b^2}{21}\right)$
Parabolic Area Area $A = \dfrac{2ab}{3}$  	$\bar{x} = \dfrac{3a}{8}$ $\bar{y} = \dfrac{3b}{5}$	$I_x = \dfrac{2ab^3}{7}$ $I_y = \dfrac{2a^3 b}{15}$ $I_z = 2ab\left(\dfrac{a^2}{15} + \dfrac{b^2}{7}\right)$

Moment of Inertia of Composite Areas

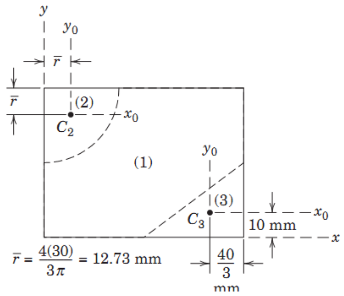
Part	Area, A	d_x	d_y	$Ad_x{}^2$	$Ad_y{}^2$	$\bar{I}_x or\ I_{x0}$	$\bar{I}_y or\ I_{y0}$
1							
2							
3							
Sums	$\sum A$			$\sum Ad_x{}^2$	$\sum Ad_y{}^2$	$\sum \bar{I}_x$	$\sum \bar{I}_y$

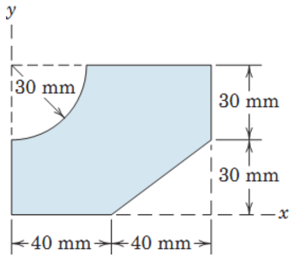
$$I_x = \sum \bar{I}_x + \sum Ad_x{}^2$$
$$I_y = \sum \bar{I}_y + \sum Ad_y{}^2$$
$$k_x = \sqrt{\frac{I_x}{A}},\quad k_y = \sqrt{\frac{I_y}{A}},\quad k_z = \sqrt{\frac{I_z}{A}}$$

SAMPLE PROBLEM A/7

Determine the moments of inertia about the x- and y-axes for the shaded area. Make direct use of the expressions given in Table D/3 for the centroidal moments of inertia of the constituent parts.

Solution:



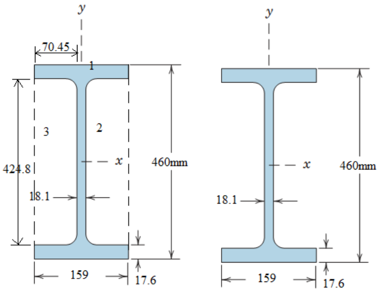


Part	**A, mm^2**	**d_x, mm**	**d_y, mm**	**$Ad_x{}^2, mm^4$**	**$Ad_y{}^2, mm^4$**	**$\bar{I}_x or\ I_{x0}, mm^4$**	**$\bar{I}_y or\ I_{y0}, mm^4$**
1.Rct.	80*60=4800	$\frac{60}{2} = 30$	$\frac{80}{2} = 40$	$4800 * 30^2 = 4.32 * 10^6$	$7.68 * 10^6$	$\frac{80*60^3}{12} = 1.44 * 10^6$	$\frac{60*80^3}{12} = 2.56 * 10^6$
2. quarter-circular	$-\frac{\pi 30^2}{4} = -707$	$60 - \frac{4*30}{3\pi} = 47.27$	$\frac{4*30}{3\pi} = 12.73$	$-1.58 * 10^6$	$-0.1146 * 10^6$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) 30^4 = -0.044 * 10^6$	$-\left(\frac{\pi}{16} - \frac{4}{9\pi}\right) 30^4 = -0.044 * 10^6$
3. triangular	$-\frac{40*30}{2} = -600$	$\frac{30}{3} = 10$	$80 - \frac{40}{3} = 66.67$	$-0.06 * 10^6$	$-2.67 * 10^6$	$\frac{40 * 30^3}{36} = -0.03 * 10^6$	$\frac{30*40^3}{36} = -0.0533 * 10^6$
Sums	3493			$2.68 * 10^6$	$4.9 * 10^6$	$1.366 * 10^6$	$2.462 * 10^6$

$$I_x = \sum I_{x0} + \sum Ad_x{}^2 = 1.366 * 10^6 + 2.68 * 10^6 = 4.046 * 10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y{}^2 = 2.462 * 10^6 + 4.9 * 10^6 = 7.36 * 10^6 mm^4$$
$$I_z = I_x + I_y = 4.046 * 10^6 + 7.36 * 10^6 = 11.406 * 10^6 mm^4$$
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.046*10^6}{3493}} = 34mm$$
$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{7.36*10^6}{3493}} = 46mm$$
$$k_z = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{11.406*10^6}{3493}} = 57mm\ OR\ k_z{}^2 = k_x{}^2 + k_y{}^2 = 34^2 + 46^2 \Rightarrow k_z = 57mm$$

Prob. A/40

The cross-sectional area of a wide-flange I-beam has the dimensions shown. Obtain a close approximation to the handbook value of by treating the section as being composed of three rectangles.

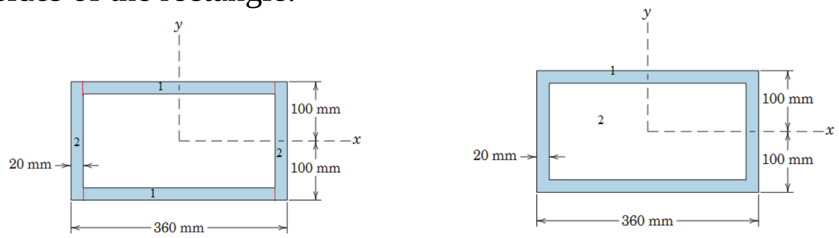


Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x^{\ 2}, m$	$Ad_y^{\ 2}, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	159*460 =73140	0	0	0	0	$\frac{159*460^3}{12}=$ $1290*10^6$	$\frac{460*159^3}{12}=$ $154.1*10^6$
2. Rct.	70.45*424.8 = -29927	0	$\frac{70.45}{2}+\frac{18.1}{2}=$ 44.275	0	$-58.66*10^6$	$\frac{70.45*424.8^3}{12}=$ $-450*10^6$	$\frac{424.8*70.45^3}{12}=$ $-12.38*10^6$
3. Rct.	70.45*424.8 = -29927	0	$\frac{70.45}{2}+\frac{18.1}{2}=$ 44.275	0	$-58.66*10^6$	$\frac{70.45*424.8^3}{12}=$ $-450*10^6$	$\frac{424.8*70.45^3}{12}=$ $-12.38*10^6$
Sums	13286			0	$-117.32*10^6$	$390*10^6$	$129.34*10^6$

$$I_x=\sum I_{x0}+\sum Ad_x^{\ 2}=390*10^6+0=390*10^6 mm^4$$
$$I_y=\sum I_{y0}+\sum Ad_y^{\ 2}=129.34*10^6-117.32*10^6=11.97*10^6 mm^4$$
$$I_z=I_x+I_y=390*10^6+11.97*10^6=402*10^6 mm^4$$
$$k_x=\sqrt{\frac{I_x}{A}}=\sqrt{\frac{390*10^6}{13286}}=171mm$$
$$k_y=\sqrt{\frac{I_y}{A}}=\sqrt{\frac{11.97*10^6}{13286}}=30mm$$
$$k_z=\sqrt{\frac{I_z}{A}}=\sqrt{\frac{402*10^6}{13286}}=174mm\ OR\ k_z^{\ 2}=k_x^{\ 2}+k_y^{\ 2}=171^2+30^2\Rightarrow k_z=174mm$$

Prob. A/41

Determine the moment of inertia of the shaded area about the x-axis in two ways. The wall thickness is 20 mm on all four sides of the rectangle.



Solution: way1

Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x^{\ 2}, mm^4$	$Ad_y^{\ 2}, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	200*360 =72000	0	0	0	0	$\frac{360*200^3}{12}=$ $240*10^6$	$\frac{200*360^3}{12}=$ $777*10^6$
2. Rct.	160*320 = -51200	0	0	0	0	$\frac{320*160^3}{12}=$ $-109*10^6$	$\frac{160*320^3}{12}=$ $-437*10^6$
Sums	20800			0	0	$131*10^6$	$340*10^6$

$$I_x=\sum I_{x0}+\sum Ad_x^{\ 2}=131*10^6+0=131*10^6 mm^4$$
$$I_y=\sum I_{y0}+\sum Ad_y^{\ 2}=340*10^6-0=340*10^6 mm^4$$

way2

Part	A, mm^2	d_x, mm	d_y, mm	$Ad_x^{\ 2}, mm^4$	$Ad_y^{\ 2}, mm^4$	$\bar{I}_x or\ I_{x0}, mm^4$	$\bar{I}_y or\ I_{y0}, mm^4$
1.Rct.	(320*20)*2 =12800	90	0	$103.7 * 10^6$	0	$\left(\frac{320*20^3}{12}\right)*2 = 0.427*10^6$	$\left(\frac{20*320^3}{12}\right)*2 = 109.2*10^6$
2. Rct.	(200*20)*2 =8000	0	170	0	$231.2*10^6$	$\left(\frac{20*200^3}{12}\right)*2 = 26.67*10^6$	$\left(\frac{200*20^3}{12}\right)*2 = 0.267*10^6$
Sums	20800			$103.7 * 10^6$	$231.2*10^6$	$27.1*10^6$	$109.47*10^6$

$$I_x = \sum I_{x0} + \sum Ad_x^{\ 2} = 27.1*10^6 + 103.7*10^6 = 131*10^6 mm^4$$
$$I_y = \sum I_{y0} + \sum Ad_y^{\ 2} = 109.47*10^6 - 231.2*10^6 = 340*10^6 mm^4$$