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Lecture-5

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Branch and Bound Method

- The **Branch and Bound (B&B) Method** is a widely used technique for solving **Integer Linear Programming (ILP)** problems.
- **Linear Programming Relaxation** is a technique used in **Integer Linear Programming (ILP)** or **Mixed Integer Programming (MIP)** problems. In these problems, some or all of the decision variables are required to take integer values, so to simplify the problem, the integer constraints are temporarily **ignored**, and the problem is treated as a **Linear Programming (LP)** problem .

Branch and Bound Method steps

Step 1: Solve the LP Relaxation:

First, ignore the integer constraints and solve the Linear Programming relaxation of the problem. This involves solving the LP as if all variables can take fractional values.

Step 2: Check for Integer Feasibility:

- If the solution from the LP relaxation is integer, this is the optimal solution, and you’re done.
- If any variable in the solution is non-integer, proceed to the next step.

Branch and Bound Method steps

Step 3: Branch:

Select a variable with a non-integer value and create two subproblems (branches):

- One subproblem fixes the variable to the largest integer smaller than the fractional value (lower bound).
- The other subproblem fixes the variable to the smallest integer larger than the fractional value (upper bound).
- This divides the original problem into two smaller subproblems.

Branch and Bound Method steps

Step 4: Bound:

- Solve each of the subproblems using the LP relaxation again.
- Keep track of the best integer solution found so far (this is called the incumbent).
- If any subproblem gives a better integer solution than the incumbent, update the incumbent.

Branch and Bound Method steps

Step 5: Prune:

- If a subproblem's solution is infeasible or its objective value is worse than the incumbent, it can be discarded (pruned) since it cannot lead to a better solution.
- Continue branching and bounding on the remaining subproblems.

Step 6: Repeat: Repeat the process until all subproblems have been pruned or solved. The incumbent is the optimal integer solution.

Branch and Bound -EXAMPLE-1

Objective:

$$\boldsymbol{Maximize\ Z = 3x + 2y}$$

Subject to:

$$\boldsymbol{x + 2y \leq 4}$$
$$\boldsymbol{4x + 3y \leq 12}$$

$$\boldsymbol{x \geq 0, y \geq 0}$$

Branch and Bound -EXAMPLE-1

Using the **Simplex Method** or any LP solver, the **LP relaxation** gives the solution:

$$\boldsymbol{x = 2}$$

$$\boldsymbol{y = 1}$$

$$\boldsymbol{Z = 8}$$

Since the LP relaxation has already provided integer values then it's the optimal integer solution .

Branch and Bound -EXAMPLE-2

Objective:

$$Maximize\ Z = 5x + 7y$$

Subject to:

$$2x + 3y \leq 12$$
$$x + y \leq 5$$

$$x \geq 0, y \geq 0$$

Branch and Bound -EXAMPLE-2

Step1: Solving this LP relaxation, we get the solution:

$$\boldsymbol{x = 3.6}$$

$$\boldsymbol{y = 1.4}$$

$$\boldsymbol{Z = 30.2}$$

Since the values of x, y, Z are not integers, we need to **branch**.

Branch and Bound -EXAMPLE-2

Step 2: Branch: We choose $\boldsymbol{x = 3.6}$ (non-integer) to branch. We create two subproblems:

1. **Subproblem 1**: $\boldsymbol{x \leq 3 \ \ (fix)}$
2. **1. Subproblem 1**: $\boldsymbol{x \geq 4 \ \ (fix)}$

.

Branch and Bound -EXAMPLE-2

Step 3: Solve Subproblems:

Subproblem-1:

$$Maximize\ Z = 5(3) + 7y$$
$$Z = 15 + 7y$$

Subject to:

$$2(3) + 3y = 12 \quad ----\rightarrow 6 + 3y = 12 ----\rightarrow y = 2$$
$$3 + y = 5 ----\rightarrow y = 2$$

So, $y = 2$ is the optimal solution for this subproblem, with:

$$Z = 15 + 7(2) = 29$$

Branch and Bound -EXAMPLE-2

Step 3: Solve Subproblems:

Subproblem-2:

$$Maximize\ Z = 5(4) + 7y$$
$$Z = 20 + 7y$$

Subject to:

$$2(4) + 3y = 12 \quad ----\rightarrow 8 + 3y = 12 ----\rightarrow y = 1.33$$
$$4 + y = 5 ----\rightarrow y = 1$$

So, $y = 1$ is the optimal solution for this subproblem, with:

$$Z = 20 + 7(1) = 27$$

Branch and Bound -EXAMPLE-2

Step 4: Select the Best Solution:

The **best integer solution** is from **Subproblem 1**, where:

$$x = 3$$

$$y = 2$$

$$Z = 29$$

.

Branch and Bound -EXAMPLE-3

Home work 4- solve this example by using branch and bound .

suppose that after Using the **Simplex Method** or any LP solver, the **LP relaxation** gives the solution:

$$x = 5$$

$$y = 1.5$$

$$Z = 29$$

Objective:

$$Maximize\ Z = 4x + 6y$$

Subject to:

$$3x + 2y \leq 18$$
$$x + 2y \leq 8$$

$$x \geq 0, y \geq 0$$

THANK YOU