Hydraulic Jumps

Hydraulic jumps mark the flow transition from supercritical to subcritical flow. When subcritical flow accelerates into the supercritical state the transition often is smooth with gradually increasing velocity and decreasing depth bringing about a smooth drop in the water surface until the alternate depth is achieved. Any disturbance to the water surface is smoothed out by the surface or gravity wave propagation mechanism. In these circumstances energy losses are not great and the Bernoulli equation does a credible job of describing the changes to the flow. When supercritical flow changes to subcritical flow, however, there is no smoothing of the water surface upstream of the transition because the high downstream velocity prevents upstream diffusion of the water-surface deformation. As a result the transition to subcritical flow is sudden and marked by an abrupt discontinuity, or hydraulic jump, in the water surface (Figure1). The greater the difference between the alternate depths the more severe the hydraulic jump.

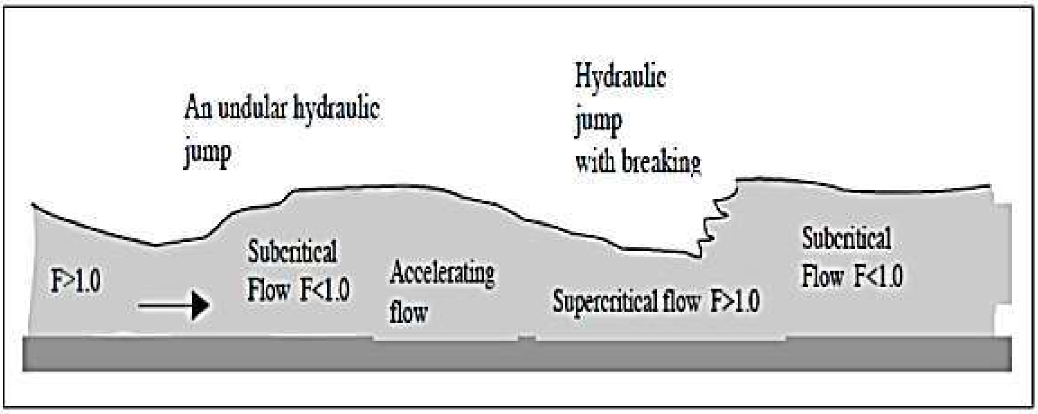


Figure (1) Undular and breaking hydraulic jumps in open-channel flow

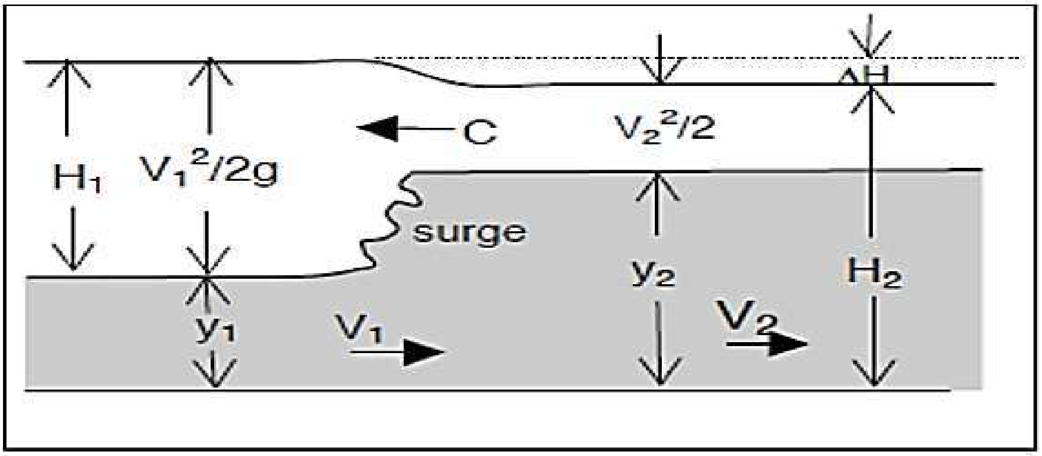


Figure (2) Flow characteristics at hydraulic jump

y_1: the initial depth of the jump (depth of water before the jump)

y_2: the sequent depth of the initial depth (depth of water after the jump).

The following relations can be used for the calculations of hydraulic jump:

$$\frac{V_1}{\sqrt{gy_1}} = F_1 = \sqrt{\frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)}$$

$$\frac{V_1^2}{gy_1} = \frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{8F_1^2+1}-1\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{8F_2^2+1}-1\right)$$

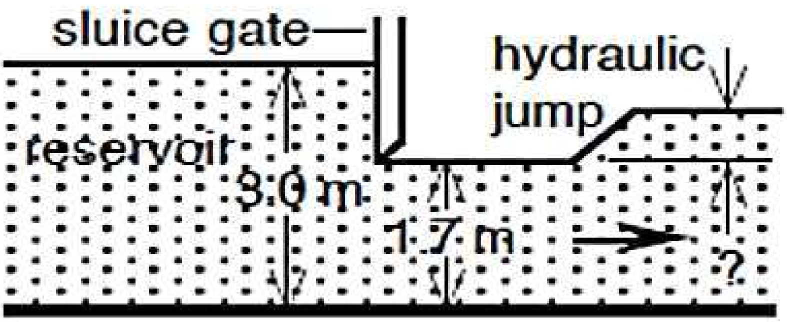
Problem (1): Water flows along a 10-m wide rectangular channel and through a hydraulic jump. If the flow depth just before the jump is 2.0 m, and 3.0 m after it, what is the discharge through the channel?

We get $\frac{V_1^2}{9.806y_1} = \frac{1}{2}\, x\, \frac{3}{2}\left(\frac{3}{2}+1\right)$

So that $V_1^2 = 36.7725$ and $V_1 = 6.064\ ms^{-1}$

Thus discharge, $Q = wV_1y_1 = 10x6.064\ x\ 2 = 121.3\ m^3s^{-1}$

Problem (2): A sluice gate at the base of a large reservoir is raised 1.7 m, as shown opposite, and the water discharges through this 5.0 m-wide rectangular orifice into a rectangular channel of the same width. If a hydraulic jump forms in the channel, what will be its height?



Solution:

Since the reservoir is large, we can assume that the water depth will not change rapidly and that the depth of water in the tank represents the total head E1 = 3.0 m. Thus we can write the Bernoulli equation for this case as

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

or

$$3 = 1.7 + \frac{V_1^2}{2x\ 9.8} \quad \text{Therefore,} \quad V1 = 5.049\ \frac{m}{sec}$$

The flow discharging from under the sluice gate is supercritical since $F = \frac{5.049}{\sqrt{9.806 \times 1.70}} = 1.237$

$$\frac{y2}{y1} = \frac{1}{2}\left(\sqrt{(8 \times 1.237^2) + 1} - 1\right) = 1.319$$

Since $y_1 = 1.70$ m, $y_2 = 2.242$ m so that the hydraulic jump must be 0.542 m high

Energy Loss in Hydraulic Jumps

To obtain the energy loss across a hydraulic jump in a rectangular channel, we have to compute E1-E2 when M1 = M2. It will be found that unless F12 and y2/y1 are quite large, the difference E1-E2 is substantially smaller than either E1 or E2, making it difficult to calculate the difference with reasonable accuracy .This is a common problem in computing work; the obvious but clumsy remedy is to calculate E1and E2 to a higher degree of precision than is required for the difference E1-E2. A much better way out of this difficulty, one suggested by the influential river engineer professor Frank Henderson (1966), is outlined below. He recommended rearranging the algebra so that E1-E2 is expressed in a form involving products rather than differences. We have

$$E_1\text{-}E_2 = (y_1 + \frac{v_1^2}{2g}) - (y_2 + \frac{v_2^2}{2g}) = (y_1 - y_2) + \frac{q^2}{2g}\left[\frac{1}{y_1^2} - \frac{1}{y_2^2}\right]$$

So, making the substitution $F_1^2 = \frac{q^2}{gy_1^3}$, we take out the factor $(y_1 - y_2)$ leading to

$$E_1 - E_2 = (y_1 - y_2)\left[1 - \frac{F_1^2}{2}\frac{y_1(y_1+y_2)}{y_2^2}\right] \ldots\ldots\ldots\ldots$$

Now we impose the condition $M_1 = M_2$ by using the hydraulic jump equation:

$F_1^2 = \frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)$. Setting $r = \frac{y_2}{y_1}$ and simplifying the expression within the bracket yields:

and finally, $$\frac{E_1 - E_2}{y_1} = \frac{(r-1)^3}{4r}$$

For example, in a hydraulic jump where $y_1 = 1.000$ and $y_2 = 1.100$ $(r = \frac{y_2}{y_1} = 1.100)$

$$E_1 - E_2 = (1.000)\frac{(1.100-1)^3}{4(1.100)} = 0.00023 \text{ m.}$$

In other words, energy loss through this mild hydraulic jump amounts to about two fifths of one millimeter of head

In the case of severe hydraulic jumps, energy losses may be very large indeed.

In a hydraulic jump where $y_1 = 1.000$ and $y_2 = 2.000$ $(r = \frac{y_2}{y_1} = 2.0)$

$$E_1 - E_2 = (1.000)\frac{(2.000-1.000)^3}{4(2.00)} = 0.125 \text{ m.}$$

Yields F_1=1.73 for these flow conditions, associated with a head loss amounting to 12.5% of the energy of the approach flow. As we noted earlier, hydraulic jumps can be very efficient dissipaters of flow energy and are sometimes designed by engineers to form in outlet flows where the approach flow might otherwise cause damaging scour to the channel bed.

Types of jump:

Depending upon the incoming Froude No. F1, the jump on horizontal floor can be classified as follows:-

No.	Froude N0.	Type of jump	Efficiency	Length
1	1	Critical flow (no jump)		
2	1 – 1.7	Undular jump	0 % - 5 %	
3	1.7 – 2.5	Weak jump	5 % - 16 %	4 – 4.9 y_2
4	2.5 – 4.5	Oscillating jump	16 % - 45 %	4.9 – 5.9 y_2
5	4.5 – 9	Well developed jump(steady jump)	45 % - 70%	5.9 – 6.1 y_2
6	9 - 20	Strong jump	70 % - 85 %	6.1 – 8.4 y_2

Note:

Type 1, 2, and 3 not useful for energy dissipation
Type 4 should be avoided
Type 5 is the best

References

Henderson, F.M., 1966, Open Channel Flow, Macmillan, New York.

Example:

In order to dissipate energy below the spillway, it is proposed to form a hydraulic jump in the stilling basin. Due to this, the depths of flow changes from 1.0 m to 3.8 m. Calculate the discharge over the spillway, the crest length of which is 110 m.

Solution:

From the equation: $\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{8F_1^2 + 1} - 1\right)$

$F_1 = 3.02$ then solve for V_1 = initial velocity of flow = $F_1\sqrt{gy_1}$

$V_1 = 9.45$ m /sec

Hence the discharge over spillway $Q = V_1\, y_1\, L = 9.45\ (1)\ (110) = 1040$ m^3/sec

Example:

Water flows at a rate of 10 m^3/sec in a rectangular channel 5.5 m wide at a depth of 30 cm. What is the total energy loss in a hydraulic jump which has occurred from this flow?

Solution:

$$V_1 = \frac{Q}{By_1} = \frac{10}{5.5\ (0.3)} = 6.06\ m/sec$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{6.06}{\sqrt{9.81\ (0.3)}} = 3.53$$

Use $\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{8F_1^2 + 1} - 1\right)$

Solve for $y_2 = 1.357$ m and $V_2 = \frac{Q}{By_2} = \frac{10}{5.5\ (1.357)} = 1.34\ m/sec$

Use $E_1 - E_2 = y_1 \frac{(r-1)^3}{4r}$

$r = \frac{y_2}{y_1} = \frac{1.357}{0.3} = 4.52$ then $E_1 - E_2 = 0.721\ m$

Or $\Delta E = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) = 2.17 - 1.449 = 0.721\ m$

Example:

A hydraulic jump occurs in a rectangular open channel. The water depths before and after the jump are 0.6 m and 1.5 m, respectively. Calculate the depth and the loss of head in the jump.

Solution:

$$\frac{y_2}{y_1} = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2q^2}{gy_1}} \quad \rightarrow \quad \frac{1.5}{0.6} = -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2q^2}{9.81x0.6}}$$

$$q = 3.045\ \frac{m^3}{s.m}$$

Or $\frac{q^2}{g} = \frac{1}{2}\, y_1 y_2 (y_1 + y_2) \rightarrow \frac{q^2}{9.81} = \frac{1}{2} x0.6x1.5x(0.6 + 1.5) \rightarrow$

$q = 3.045$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{3.045^2}{9.81}} = 0.981\ m$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 0.6 + \frac{(3.045)^2}{19.62x0.6^2} = 1.913\ m$$

$$E_2 = y_2 + \frac{q^2}{2gy_2^2} = 1.5 + \frac{(3.045)^2}{19.62(1.5)^2} = 1.71\ m$$

$$\Delta E = E_1 - E_2 = 1.913 - 1.71 = 0.203\ m$$

Or $\Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2} = \frac{(1.5-0.6)^3}{4x1.5x0.6} = 0.2025\ m$