

Integral

If $F(x)$ is an antiderivative of $f(x)$, then the expression $F(x) + C$ where C is an arbitrary constant, is called the indefinite integral of $f(x)$.

Integral Calculus Formula Sheet

No.	Derivation Formulas	Integration Formulas
1.	$\frac{d}{dx}(C) = 0$	$\int 0 \cdot dx = C$
2.	$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
3.	$\frac{d}{dx}(\sin ax) = a \cos ax$	$\int \cos ax dx = (1/a) \sin ax + C$
4.	$\frac{d}{dx}(\cos ax) = -a \sin ax$	$\int \sin ax dx = -(1/a) \cos ax + C$
5.	$\frac{d}{dx}(\tan ax) = a \sec^2 ax$	$\int \sec^2 ax dx = (1/a) \tan ax + C$
6.	$\frac{d}{dx}(\cot ax) = -a \csc^2 ax$	$\int \csc^2 ax dx = -(1/a) \cot ax + C$
7.	$\frac{d}{dx}(\sec ax) = a \sec ax \tan ax$	$\int \sec ax \tan ax dx = (1/a) \sec ax + C$
8.	$\frac{d}{dx}(\csc ax) = -a \csc ax \cot ax$	$\int \csc ax \cot ax dx = -(1/a) \csc ax + C$
9.	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
10.	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
11.	$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$	$\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$
12.	$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln x + C$
13.	$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = (1/a)e^{ax} + C$

The Integrals Concerning the Inverse Trigonometric Functions

14.	$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + c_1 = -\cos^{-1} \frac{u}{a} + c_2$
15.	$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c_1 = -\frac{1}{a} \cot^{-1} \frac{u}{a} + c_2$
16.	$\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + c_1 = -\frac{1}{a} \csc^{-1} \frac{u}{a} + c_2$

Examples - I

- $$\int x\sqrt{2x} dx = \sqrt{2} \int (x)^{\frac{3}{2}} dx = \sqrt{2} (x)^{\frac{5}{2}} \times \frac{2}{5} + c = \frac{2\sqrt{2}}{5} (x)^{\frac{5}{2}} + c$$
- $$\int \sqrt{3x+1} dx = \frac{1}{3} \int 3(3x+1)^{\frac{1}{2}} dx = \frac{1}{3} (3x+1)^{\frac{3}{2}} \times \frac{2}{3} + c = \frac{2}{9} (3x+1)^{\frac{3}{2}} + c$$
- $$\int \frac{dx}{(2x-3)^2} = \frac{1}{2} \int 2(2x-3)^{-2} dx = -\frac{1}{2} (2x-3)^{-1} + c = -\frac{1}{2(2x-3)} + c$$
- $$\int \frac{xdx}{\sqrt{x^2-3}} = \frac{1}{2} \int 2x(x^2-3)^{-1/2} dx = \frac{1}{2} (x^2-3)^{1/2} \times 2 + c = \sqrt{x^2-3} + c$$
- $$\int \frac{xdx}{x^2-3} = \frac{1}{2} \ln|x^2-3| + c$$
- $$\int \sqrt{1+\cos 2x} dx = \int \sqrt{2\cos^2 x} dx = \int \sqrt{2} \cos x dx = \sqrt{2} \sin x + c$$
- $$\int \frac{\sin 2x}{\sqrt{1+\cos 2x}} dx = \frac{-1}{2} \int (1+\cos 2x)^{-1/2} (-2\sin 2x) dx = -\sqrt{1+\cos 2x} + c$$
- $$\int \frac{\sin 2x}{1+\cos 2x} dx = \frac{-1}{2} \int \frac{-2\sin 2x}{1+\cos 2x} dx = \frac{-1}{2} \ln|1+\cos 2x| + c$$
- $$\int (2e^{-2x} + 3x^2) dx = -e^{-2x} + x^3 + c$$
- $$\int \tan 2x dx = \int \frac{\sin 2x}{\cos 2x} dx = -\frac{1}{2} \ln|\cos 2x| + c$$
- $$\int \frac{dx}{\sqrt{2-x^2}} = \sin^{-1} \frac{x}{\sqrt{2}} + c$$

$$12. \int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{x^2 + 2x + 1 + 1}$$

$$= \int \frac{dx}{(x+1)^2 + 1} = \tan^{-1}(x+1) + c$$

$$13. \int \frac{dx}{x\sqrt{x^2-3}} = \frac{1}{\sqrt{3}} \sec^{-1} \frac{x}{\sqrt{3}} + c$$

$$14. \int \frac{2x}{x^2 - 6x + 10} dx = \int \frac{2x - 6 + 6}{x^2 - 6x + 10} dx$$

$$= \int \frac{2x - 6}{x^2 - 6x + 10} dx + \int \frac{6}{x^2 - 6x + 9 + 1} dx$$

$$= \ln|x^2 - 6x + 10| + \int \frac{6}{(x-3)^2 + 1} dx$$

$$= \ln|x^2 - 6x + 10| + 6 \tan^{-1}(x-3) + c$$

Definite Integration

If $\int f(x) dx = F(x) + c$, then the definite integral of $f(x)$ on an interval $[a, b]$ is

expressed as: $\int_a^b f(x) dx = F(b) - F(a)$.

Examples - II

$$1. \int_0^3 x\sqrt{9-x^2} dx = -\frac{1}{2} \int_0^3 -2x(9-x^2)^{1/2} dx = -\frac{1}{2} (9-x^2)^{3/2} \times \frac{2}{3} \Big|_0^3$$

$$= -\frac{1}{3} ((9-3^2)^{3/2} - (9-0^2)^{3/2}) = 9$$

$$2. \int_0^{\pi/2} \sin 2x dx = -\frac{1}{2} \cos 2x \Big|_0^{\pi/2} = -\frac{1}{2} (\cos \pi - \cos 0) = -\frac{1}{2} (-1 - 1) = 1$$

$$3. \int_0^{2/3} \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3} \sin^{-1} \frac{3x}{2} \Big|_0^{2/3} = \frac{1}{3} (\sin^{-1} 1 - \sin^{-1} 0) = \frac{\pi}{6}$$

$$4. \int_1^4 \frac{1}{x^2 - 2x + 10} dx = \int_1^4 \frac{1}{x^2 - 2x + 1 + 9} dx = \int_1^4 \frac{1}{(x-1)^2 + 9} dx = \frac{1}{3} \tan^{-1} \frac{x-1}{3} \Big|_1^4$$

$$= \frac{1}{3} \left(\tan^{-1} \frac{4-1}{3} - \tan^{-1} 0 \right) = \frac{1}{3} \left(\frac{\pi}{4} - 0 \right) = \frac{\pi}{12}$$

Exercises

Evaluate the following integrals

$$1. \int (x + \sqrt{x}) dx \quad 2. \int \sqrt[3]{3x-2} dx \quad 3. \int \frac{xdx}{\sqrt{1-4x^2}}$$

$$4. \int \frac{xdx}{x^2+3} \quad 5. \int \frac{dx}{\sqrt{1-4x^2}} \quad 6. \int_0^{\sqrt{3}} \frac{dx}{x^2+3}$$

$$7. \int \sqrt{1-\cos 2x} dx \quad 8. \int \frac{(x-1)^2}{x\sqrt{x}} dx \quad 9. \int \frac{dx}{x^2-2x+5}$$

Ans:

$$1. \frac{x^2}{2} + \frac{2x^{3/2}}{3} + c \quad 2. \frac{(3x-2)^{4/3}}{4} + c \quad 3. -\frac{\sqrt{1-4x^2}}{4} + c$$

$$4. \frac{\ln(x^2+3)}{2} + c \quad 5. \frac{\sin^{-1} 2x}{2} + c \quad 6. \frac{\pi}{4\sqrt{3}}$$

$$7. -\sqrt{2} \cos x + c \quad 8. \frac{2x^{3/2}}{3} - 4\sqrt{x} - \frac{2}{\sqrt{x}} + c \quad 9. \frac{1}{2} \tan^{-1} \left(\frac{x-1}{2} \right) + c$$

Work in the college

Let $f(x) = e^{\sqrt{x^2-x-20}}$. Then find $\int \frac{(2x-1)f(x)dx}{\ln(f(x))^2}$.