Prof. Dr. Eng. Ahmed S. Naji **Chem. Eng. Dept., Faculty of Eng.**
The lecturer of Physics **The University of Babylon**

Chapter Three: Motion along a Straight Line

Outline of this chapter

1. Position, Displacement, and Average Velocity
2. Instantaneous Velocity and Speed
3. Average and Instantaneous Acceleration
4. Motion with Constant Acceleration
5. Finding Velocity, Displacement and Acceleration

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1. Position, Displacement, and Average Velocity

Our universe is full of objects in motion. From **the stars, planets, and galaxies**; to the motion of people and animals; down to the microscopic scale of atoms and molecules—everything in our universe is in motion.

We can describe motion using the two disciplines of **kinematics and dynamics**. We study dynamics, which is concerned with the causes of motion, in **Newton's Laws of Motion**; but, **Kinematics** involves describing motion through properties such as position, time, velocity, and acceleration.

Position

To describe the motion of an object, you must first be able to describe its **position** (x): where it is at any particular time. More precisely, we need to specify its position relative to a convenient frame of reference. A frame of reference is an arbitrary set of axes from which the position and motion of an object are described.

Earth is often used as a frame of reference, and we often describe the position of an object as it relates to stationary objects on Earth.

Displacement

If an object moves relative to a frame of reference–for example, if a professor moves to the right relative to a whiteboard, then the objects position changes. This change in position is called **displacement**. The word displacement implies that an object has moved, or has been displaced. Although position is the numerical value of x along a straight line where an object might be located, displacement gives the change in position along this line. Since displacement indicates direction, it is a vector and can be either positive or negative, depending on the choice of positive direction.

Displacement

Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0, \quad (3.1)$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

$$\Delta x_{Total} = \sum \Delta x_i, \quad (3.2)$$

The total distance is given by the following equation

$$x_{\text{Total}} = |\Delta x_1| + |\Delta x_2|$$

Average Velocity

To calculate the other physical quantities in kinematics we must introduce the time variable. The time variable allows us not only to state where the object is (its position) during its motion, but also how fast it is moving. How fast an object is moving is given by the rate at which the position changes with time or rate. The rate is usually expressed as **the average velocity**. This vector quantity is simply the total displacement between two points divided by the time taken to travel between them.

Average Velocity

If x_1 and x_2 are the positions of an object at times t_1 and t_2, respectively, then

$$\text{Average velocity} = \bar{v} = \frac{\text{Displacement between two points}}{\text{Elapsed time between two points}} \tag{3.3}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

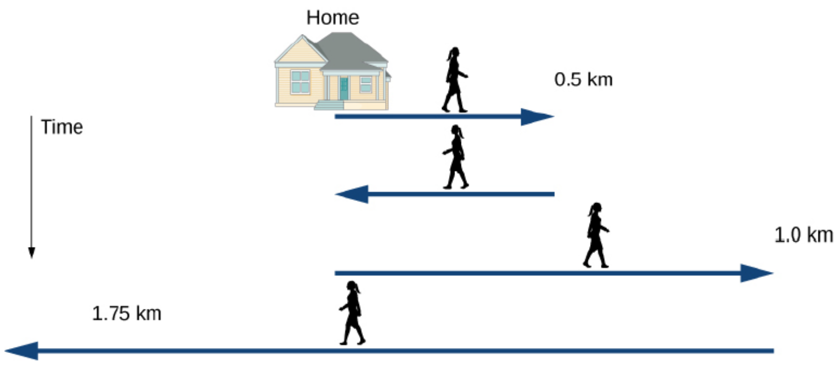
Example (1)

Jill sets out from her home to deliver flyers for her yard sale, traveling due east along her street lined with houses. At 0.5 km and 9 minutes later she runs out of flyers and has to retrace her steps back to her house to get more. This takes an additional 9 minutes. After picking up more flyers, she sets out again on the same path, continuing where she left off, and ends up 1.0 km from her house. This third leg of her trip takes 15 minutes. At this point she turns back toward her house, heading west. After 1.75 km and 25 minutes she stops to rest.

Now:

a. What is Jill's total displacement to the point where she stops to rest?
b. What is the magnitude of the final displacement?
c. What is the average velocity during her entire trip?
d. What is the total distance traveled?
e. Make a graph of position versus time?

A sketch of Jill's movements is shown in Figure 3.4.



Solution

a. From the above table, the total displacement is

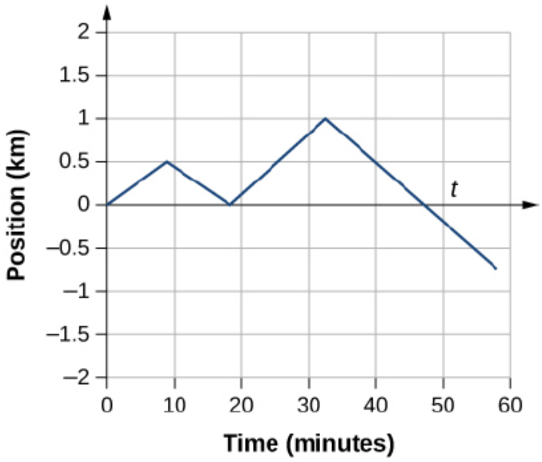
$$\sum \Delta x_i = 0.5 - 0.5 + 1.0 - 1.75 \text{ km} = -0.75 \text{ km}.$$

b. The magnitude of the total displacement is $|-0.75|$ km $= 0.75$ km.

c. Average velocity $= \frac{\text{Total displacement}}{\text{Elapsed time}} = \bar{v} = \frac{-0.75 \text{ km}}{58 \text{ min}} = -0.013$ km/min

d. The total distance traveled (sum of magnitudes of individual displacements) is $x_{\text{Total}} = \sum |\Delta x_i| = 0.5 + 0.5 + 1.0 + 1.75 \text{ km} = 3.75 \text{ km}.$

e. We can graph Jill's position versus time as a useful aid to see the motion; the graph is shown in Figure



2. Instantaneous Velocity and Speed

We have now seen how to calculate the average velocity between two positions. However, since objects in the real world move continuously through space and time, we would like to find the velocity of an object at any single point. We can find the velocity of the object anywhere along its path by using some fundamental principles of calculus. This section gives us better insight into the physics of motion and will be useful in later chapters.

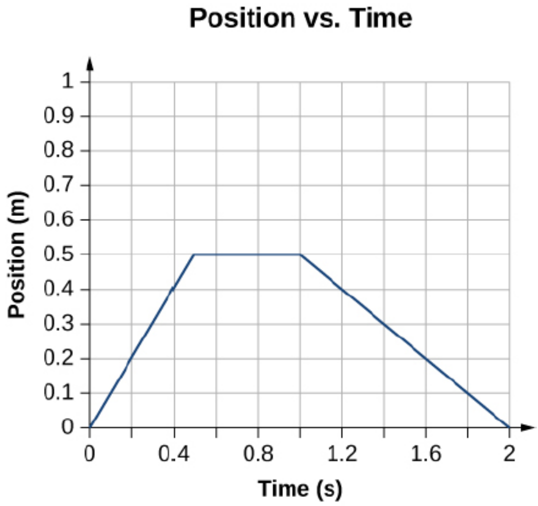
<u>Instantaneous Velocity</u>

The quantity that tells us how fast an object is moving anywhere along its path is the instantaneous velocity, usually called simply **velocity**. It is the average velocity between two points on the path in the limit that the time (and therefore the displacement) between the two points approaches zero. To illustrate this idea mathematically, we need to express position x as a continuous function of (t) denoted by x (t). The expression for the average velocity between two points using this notation is $\bar{v} = \frac{x(t_2)-x(t_1)}{t_2-t_1}$. To find the instantaneous velocity at any position, we let $t_1 = t$ and $t_2 = t + \Delta t$. After inserting these expressions into the equation for the average velocity and taking the limit as $\Delta t \to 0$, we find the expression for the instantaneous velocity:

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \frac{dx(t)}{dt}$$

Example (2)

For the position-time diagram which is shown in figure below. Draw the velocity-time diagram?

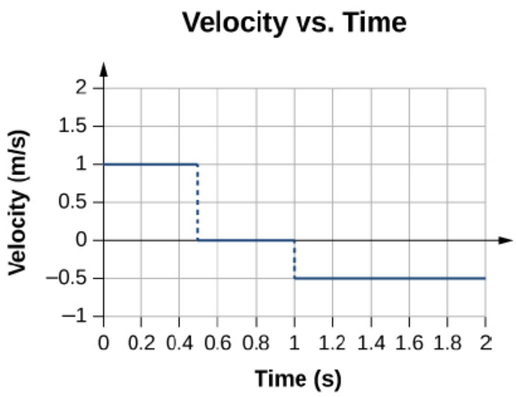


Solution

Time interval 0 s to 0.5 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5\text{ m}-0.0\text{ m}}{0.5\text{ s}-0.0\text{ s}} = 1.0\text{ m/s}$

Time interval 0.5 s to 1.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.5\text{ m}-0.5\text{ m}}{1.0\text{ s}-0.5\text{ s}} = 0.0\text{ m/s}$

Time interval 1.0 s to 2.0 s: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m}-0.5\text{ m}}{2.0\text{ s}-1.0\text{ s}} = -0.5\text{ m/s}$



Example (3)

The position of a particle is given by $x(t) = 3\,t + 0.5\,t^2$

a. Find the instantaneous velocity at t=2 sec.

b. Calculate the average velocity between 1 sec and 3 sec.

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 + 1.5t^2$ m/s.

Substituting $t = 2.0$ s into this equation gives $v(2.0\text{ s}) = [3.0 + 1.5(2.0)^2]$ m/s $= 9.0$ m/s.

b. To determine the average velocity of the particle between 1.0 s and 3.0 s, we calculate the values of $x(1.0\text{ s})$ and $x(3.0\text{ s})$:

$$x(1.0\text{ s}) = \left[(3.0)(1.0) + 0.5(1.0)^3\right]\text{ m} = 3.5\text{ m}$$

$$x(3.0\text{ s}) = \left[(3.0)(3.0) + 0.5(3.0)^3\right]\text{ m} = 22.5\text{ m.}$$

Then the average velocity is

$$\bar{v} = \frac{x(3.0\text{ s}) - x(1.0\text{ s})}{t(3.0\text{ s}) - t(1.0\text{ s})} = \frac{22.5 - 3.5\text{ m}}{3.0 - 1.0\text{ s}} = 9.5\text{ m/s.}$$

Example (4)
Consider the motion of a particle in which the position is $x(t) = 3\,t + 3\,t^2$
a. What is the instantaneous velocity at t= 0.25 sec, t= 0.5 sec, and t= 1 sec?
b. What is the speed of the particle at these times?

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 - 6.0t$ m/s $v(0.25\text{ s}) = 1.50$ m/s, $v(0.5\text{ s}) = 0$ m/s, $v(1.0\text{ s}) = -3.0$ m/s
b. Speed $= |v(t)| = 1.50$ m/s, 0.0 m/s, and 3.0 m/s

3. Average and Instantaneous Acceleration

Average acceleration is the rate at which velocity changes:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$$

Example (5)
A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Solution

First, identify the knowns: $v_0 = 0$, $v_f = -15.0$ m/s (the negative sign indicates direction toward the west), $\Delta t = 1.80$ s.

Second, find the change in velocity. Since the horse is going from zero to −15.0 m/s, its change in velocity equals its final velocity:

$$\Delta v = v_f - v_0 = v_f = -15.0 \text{ m/s}.$$

Last, substitute the known values (Δv and Δt) and solve for the unknown $\bar{a}$:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0 \text{ m/s}}{1.80 \text{ s}} = -8.33 \text{m/s}^2.$$

Instantaneous Acceleration
Instantaneous acceleration a, or acceleration at a specific instant in time, is obtained using the same process discussed for instantaneous velocity

$$a(t) = \frac{d}{dt}v(t).$$

Example (6)
A particle is in motion and is accelerating. The functional form of the velocity is $v(t) = 20\,t - 5\,t^2$.
a. Find the functional form of the acceleration.
b. Find the instantaneous velocity at t= 1, 2, 3, and 5 s.
c. Find the instantaneous acceleration at t= 1, 2, 3, and 5 s.
d. Interpret the results of (c) in terms of the directions of the acceleration and velocity vectors.

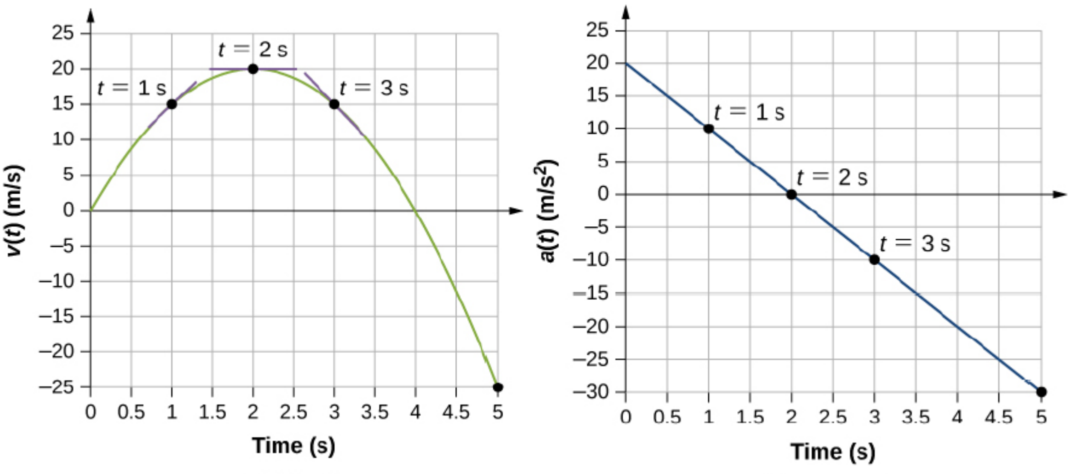
Solution

a. $a(t) = \frac{dv(t)}{dt} = 20 - 10t \text{ m/s}^2$
b. $v(1\text{ s}) = 15\text{ m/s}, v(2\text{ s}) = 20\text{ m/s}, v(3\text{ s}) = 15\text{ m/s}, v(5\text{ s}) = -25\text{ m/s}$
c. $a(1\text{ s}) = 10\text{ m/s}^2, a(2\text{ s}) = 0\text{ m/s}^2, a(3\text{ s}) = -10\text{ m/s}^2, a(5\text{ s}) = -30\text{ m/s}^2$
d. At $t = 1$ s, velocity $v(1\text{ s}) = 15\text{ m/s}$ is positive and acceleration is positive, so both velocity and acceleration are in the same direction. The particle is moving faster.

At $t = 2$ s, velocity has increased to$v(2\text{ s}) = 20\text{ m/s}$, where it is maximum, which corresponds to the time when the acceleration is zero. We see that the maximum velocity occurs when the slope of the velocity function is zero, which is just the zero of the acceleration function.

At $t = 3$ s, velocity is $v(3\text{ s}) = 15\text{ m/s}$ and acceleration is negative. The particle has reduced its velocity and the acceleration vector is negative. The particle is slowing down.

At $t = 5$ s, velocity is $v(5\text{ s}) = -25\text{ m/s}$ and acceleration is increasingly negative. Between the times $t = 3$ s and $t = 5$ s the particle has decreased its velocity to zero and then become negative, thus reversing its direction. The particle is now speeding up again, but in the opposite direction.



(a) Velocity (b) Acceleration

4. Motion with Constant Acceleration

To get our first two equations, we start with the definition of average velocity $\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta \mathbf{t}}$ **,** Substituting the simplified notation for Δx and Δt yields, $\bar{\mathbf{v}} = \frac{\mathbf{x} - \mathbf{x_o}}{\mathbf{t}}$. Now, Solving for x gives us: $\mathbf{x} = \mathbf{x_o} + \bar{\mathbf{v}}\mathbf{t}$**,** where the average velocity is $\bar{\mathbf{v}} = \frac{v_o + v}{2}$. On the other hand, acceleration could be found from $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta \mathbf{t}} = \frac{v - v_o}{t}$ and lastly, $\mathbf{v} = \mathbf{v_o} + \mathbf{a}t$. We can write down the equation of kinematic motion as:

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

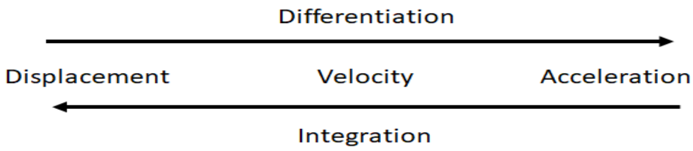
$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

5. Finding Velocity and Displacement and Acceleration

In the above equation, differentiation is used to get velocity and acceleration from the displacement and velocity respectively. If needed to get displacement and velocity from the velocity and acceleration respectively, then the integration will be used.



Summary of This chapter's equations

Displacement	$\Delta x = x_f - x_i$
Total displacement	$\Delta x_{Total} = \sum \Delta x_i$
Average velocity (for constant acceleration)	$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$
Instantaneous velocity	$v(t) = \frac{dx(t)}{dt}$
Average speed	$\text{Average speed} = \overline{s} = \frac{\text{Total distance}}{\text{Elapsed time}}$
Instantaneous speed	$\text{Instantaneous speed} = \lvert v(t) \rvert$
Average acceleration	$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$
Instantaneous acceleration	$a(t) = \frac{dv(t)}{dt}$
Position from average velocity	$x = x_0 + \overline{v}t$
Average velocity	$\overline{v} = \frac{v_0 + v}{2}$
Velocity from acceleration	$v = v_0 + at$ (constant a)
Position from velocity and acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$ (constant a)
Velocity from distance	$v^2 = v_0^2 + 2a(x - x_0)$ (constant a)
Velocity of free fall	$v = v_0 - gt$ (positive upward)
Height of free fall	$y = y_0 + v_0 t - \frac{1}{2}gt^2$
Velocity of free fall from height	$v^2 = v_0^2 - 2g(y - y_0)$
Velocity from acceleration	$v(t) = \int a(t)dt + C_1$
Position from velocity	$x(t) = \int v(t)dt + C_2$

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Sheet No. 3

Q1) *A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is* $2.10 \times 10^4\ \mathrm{m/s^2}$*, and 1.85 ms* $(1\,\mathrm{ms} = 10^{-3}\ \mathrm{s})$ *elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?*

Q2) ***Professional Application*** *Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?*

Q3) ***Professional Application*** *A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in* $\mathrm{m/s^2}$ *and in multiples of* $g\ (g = 9.80\ \mathrm{m/s^2})$*. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of* g *?*

Q4) *Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be* $y_0 = 0$*.*

Q5) *Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?*