Linear Programming

BY: LECTURER AHMED REYADH

5TH LECTURE

APRIL 2022

Introduction

- Linear programming is one of operation research techniques that developed at fourteenth of previous century. It used to solve various problems consist of linear objective function and linear constraints, which may include both equalities and inequalities.
- The linear programming model has three basic components:

1. Decision variables that we seek to determine.
2. Objective (goal) we need to be optimized.
3. Constraints that the solution must satisfy.

Types and scope of solving techniques

1. Graphical method
2. Simplex method
3. Big M method.
4. Two Phase method.

- LP used for solving inventories, resources allocation, optimum products mix, supplying, planning, problems and many more.

Formulation of LP Model

- **Example:** A company produces exterior and interior isolations. a market survey indicate that daily demand of interior cannot exceed exterior by 1 ton and the max interior daily market demand 2 ton.

Determine the optimum mix that maximize the profit. All the other details are mentioned in the following table.

	Exterior Isolation	Interior Isolation	Maximum Daily Storage
Raw Material R1	6	4	24
Raw Material R2	1	2	6
Profit per ton ($1,000)	5	4	

Solution:

- Let the daily production of exterior isolation is X_1.

The daily production of interior isolation is X_2.

- The objective is to maximize profit, so the objective function is

$$\mathbf{Max\ Z = 5\ X_1 + 4\ X_2}$$

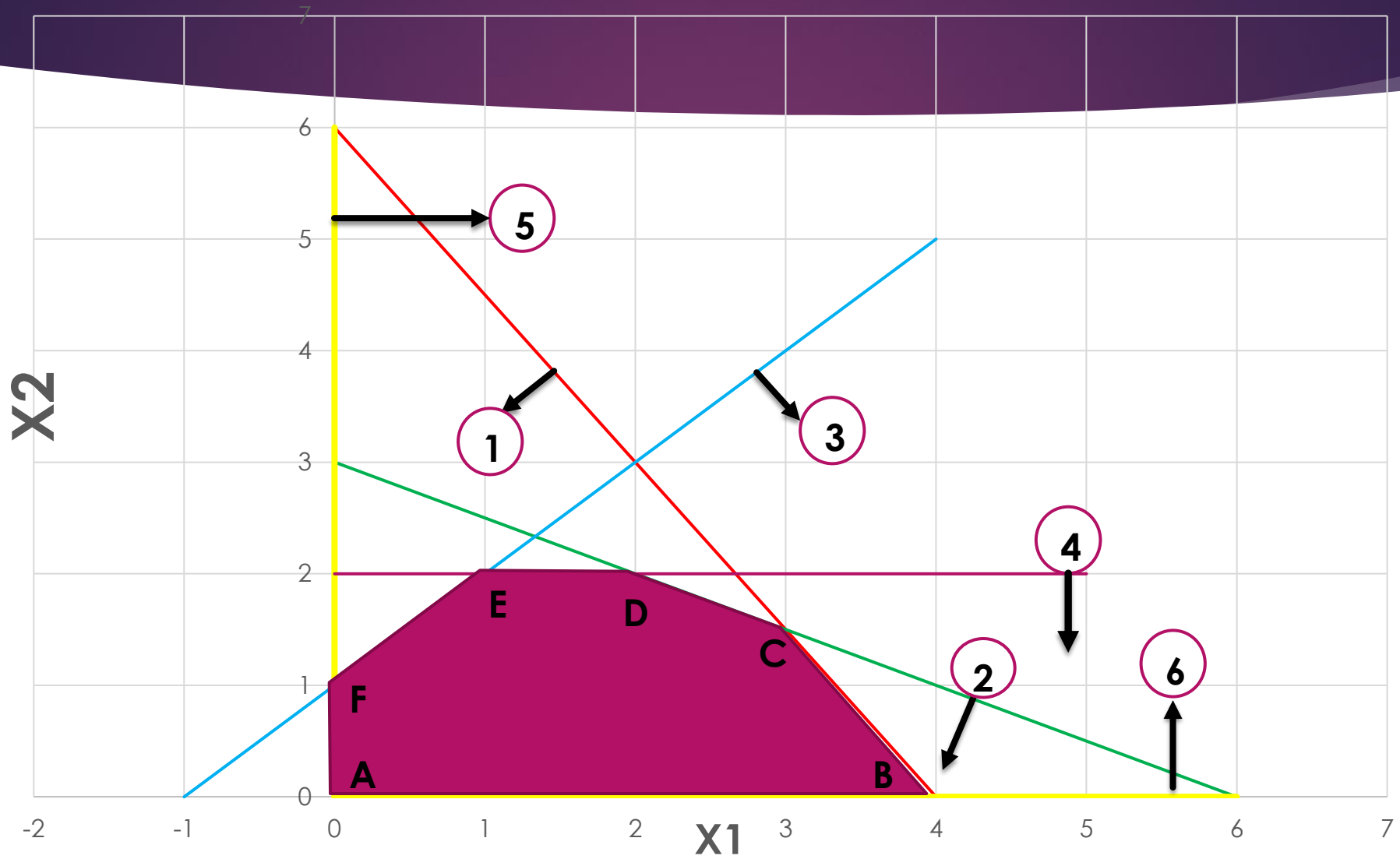
- The constraints will be formulated according to raw materials availability and their usage to exterior and interior isolation.
- Usage of first raw material (R1) and second raw material (R2).

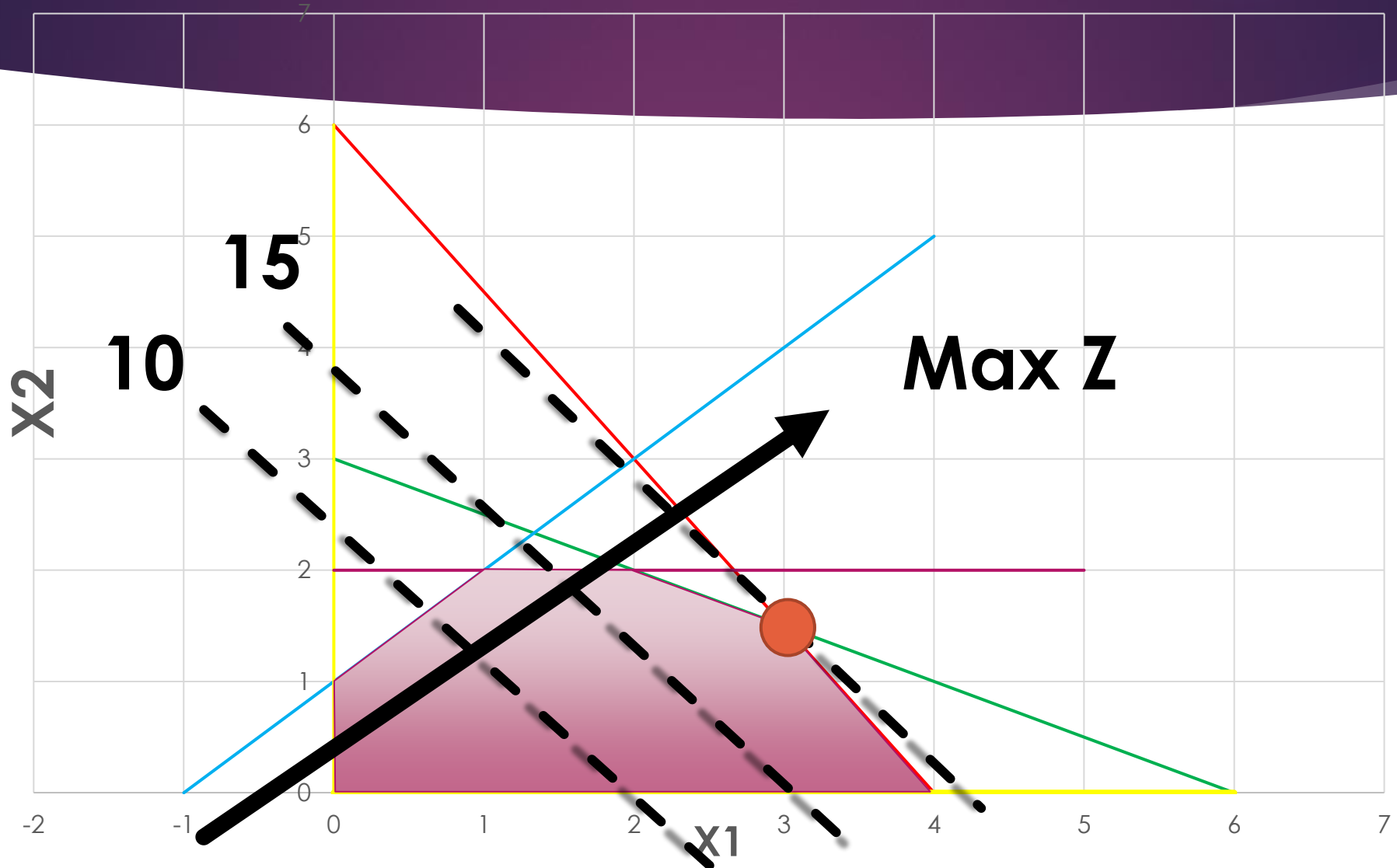
1) $6\ X_1 + 4\ X_2 \leq 24$ (0,6) (4,0)
2) $X_1 + 2\ X_2 \leq 6$ (0,3) (6, 0)
3) $X_2 - X_1 \leq 1$
4) $X_2 \leq 2$

- **The non negativity conditions**

5) $X_1 \geq 0$
6) $X_2 \geq 0$

Graphical Method




15
10
Max Z
X2
X1
7
6
5
4
3
2
1
0
-2
-1
0
1
2
3
4
5
6
7

Point	(X1, X2)	Z
A	(0, 0)	0
B	(4, 0)	20
C	(3, 1.5)	21
D	(2, 2)	18
E	(1, 2)	13
F	(0, 1)	4

Simplex Method

- This method is developed at 1947 and provides an algorithm which consists in moving from one vertex of the region of feasible solution to another in a manner that the value of the objective function at the succeeding vertex is less or more at minimum or maximum problems.
- This procedure of jumping from one vertex to another in a finite number of steps

Important notes

- We will added a slack variable (S) to each constraint to form an equality equations (if equality is available there is no need to add "S" to constraint) .
- If the constraints sign ($\leq$) the slack variables (S) sign will positive.
- If the constraints sign ($\geq$) the slack variables sign (S) will negative.
- If right side is negative multiply the constraints by (-1) and reverse the constraints sign.

Solution Procedure:

1. Transform to formal equations by adding slack Variables

maximize $Z = 5X_1 + 4X_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

$$Z - 5X_1 - 4X_2 - 0S_1 - 0S_2 - 0S_3 - 0S_4 = 0$$

Subject to:

$$6X_1 + 4X_2 + S_1 = 24$$

$$X_1 + 2X_2 + S_2 = 6$$

$$-X_1 + X_2 + S_3 = 1$$

$$X_2 + S_4 = 2$$

$$X_1, X_2, S_1, S_2, S_3, S_4 \geq 0$$

Basic	X1	X2	S1	S2	S3	S4	SOLUTION	RATIO
Z	- 5	- 4	0	0	0	0	0	
S1	6	4	1	0	0	0	24	
S2	1	2	0	1	0	0	6	
S3	- 1	1	0	0	1	0	1	
S4	0	1	0	0	0	1	2	

- The Solution will be explained in the attached PDF file

Assignment

- Solve the following problem by graphical and simplex methods:

$$\text{Maximize } Z = 2X_1 + 3X_2$$

subject to:

$$2X_1 + X_2 \leq 4$$

$$X_1 + 2X_2 \leq 5$$

$$X_1, X_2 \geq 0$$

Assignment 2

- A wiring company manufacturing three types of wires using two raw materials cooper, and isolations. The inventories of each raw material is detailed below. The production process need two types of skilled labour for wire production and packaging processes, The following table gives the availability of the resources, their usage by the three products, and the profits per unit.

	Resources required			
Resources	**Wire1**	**Wire 2**	**Wire 3**	**Daily availability**
Cooper (kg)	2	1	3	**42 kg**
Wire production skill (hr.)	2	1	2	**40 hr.**
Packaging skill (hr.)	1	0.5	1	**45 hr.**
Profit ($)	**24**	**22**	**45**	

THANK YOU