

2. The Greatest Integer Function

Positive values: $[1.9] = 1$; $[1.1] = 1$

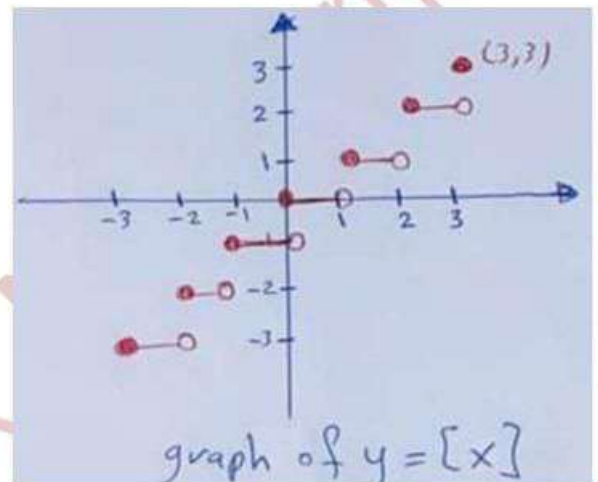
Zero values: $[0.3] = 0$; $[0.9] = 0$

Negative values: $[-0.5] = -1$; $[-1.1] = -2$

Example 1: Graph the following function: $y = [x]$; $-3 \leq x \leq 3$

Solution:

x	y = [x]
3	3
2.9	2
2	2
1.9	1
1	1
0.9	0
0	0
-0.1	-1
-1	-1
-1.1	-2
-2	-2
-2.1	-3
-3	-3

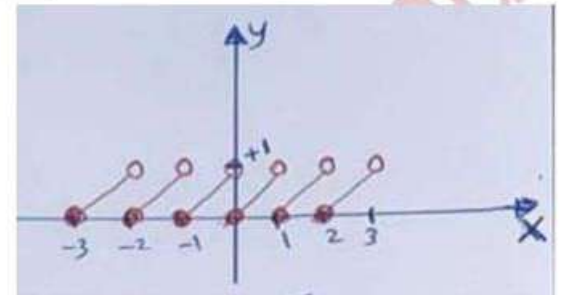


Example 2: Graph the following function: $y = x - [x]$; $-3 \leq x < 3$

x	$y = x - [x]$	y	Value of x
2	$x - 2$	$x - 2$	$2 \leq x < 3$
1	$x - 1$	$x - 1$	$1 \leq x < 2$
0	$x - 0$	x	$0 \leq x < 1$
-1	$x - (-1)$	$x + 1$	$-1 \leq x < 0$
-2	$x - (-2)$	$x + 2$	$-2 \leq x < -1$
-3	$x - (-3)$	$x + 3$	$-3 \leq x < -2$

x	y = x - 2	x	y = x - 1	x	y = x
2	0	1	0	0	0
3	1	2	1	1	1

x	y = x + 1	x	y = x + 2	x	y = x + 3
-1	0	-2	0	-3	0
0	1	-1	1	-2	1

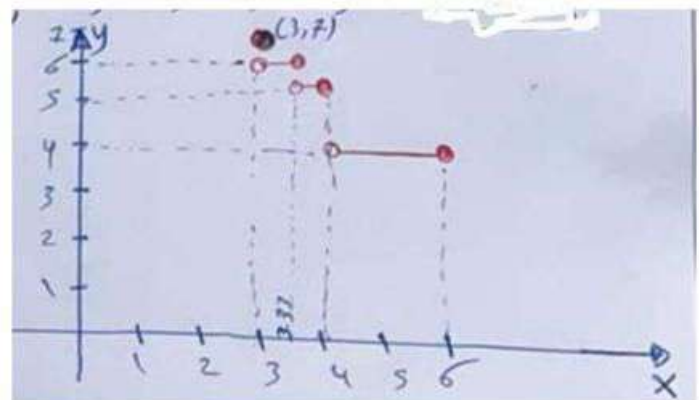


Example 3: Graph the following function: $y = \left[1 + \frac{2x}{x-2} \right]$; $3 \leq x \leq 6$

x	$y = \left[1 + \frac{2x}{x-2} \right]$	Value of x
3	7	$x = 3$
$x_2 = 3.33$	6	$3 < x \leq 3.33$
$x_3 = 4$	5	$3.33 < x \leq 4$
6	4	$4 < x \leq 6$

$$1 + \frac{2x_2}{x_2-2} = 6 \quad \text{;;;} \quad \frac{2x_2}{x_2-2} = 5 \quad \text{;;;} \quad 2x_2 = 5x_2 - 10 \quad \text{;;} \quad 3x_2 = 10 \quad \text{;;;} \quad x_2 = 3.33$$

$$1 + \frac{2x_3}{x_3-2} = 5 \quad \text{;;;} \quad \frac{2x_3}{x_3-2} = 4 \quad \text{;;;} \quad 2x_3 = 4x_3 - 8 \quad \text{;;} \quad 2x_3 = 8 \quad \text{;;;} \quad x_3 = 4$$



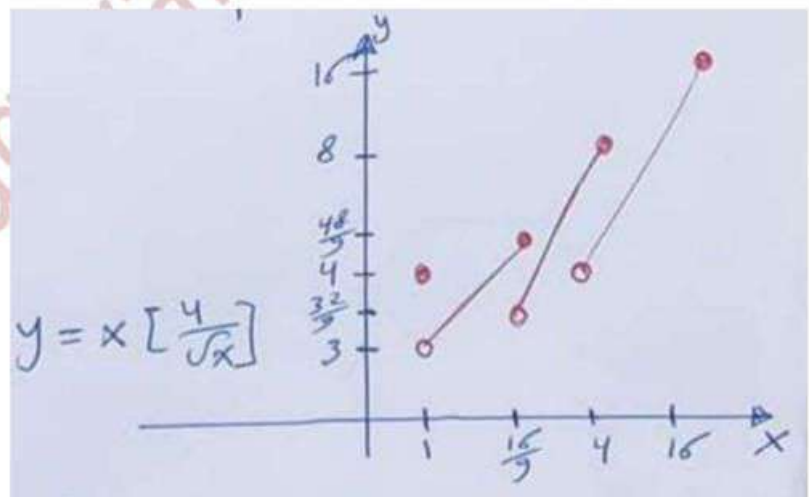
Example 4: Graph the following function: $y = x \left[\frac{4}{\sqrt{x}} \right]$; $1 \leq x \leq 16$

x	$\left[\frac{4}{\sqrt{x}} \right]$	$y = x \left[\frac{4}{\sqrt{x}} \right]$	Value of x
16	1	x	$4 < x \leq 16$
4	2	2x	$\frac{16}{9} < x \leq 4$
$\frac{16}{9}$	3	3x	$1 < x \leq \frac{16}{9}$
1	4	4x = 4	x = 1

$$\frac{4}{\sqrt{x}} = 2 \quad \therefore \quad \frac{4}{2} = \sqrt{x} \quad \therefore \quad x = 4$$

$$\frac{4}{\sqrt{x}} = 3 \quad \therefore \quad \frac{4}{3} = \sqrt{x} \quad \therefore \quad x = \frac{16}{9}$$

x	y = x	x	y = 2x	x	y = 3x
16	16	4	8	$\frac{16}{9}$	$\frac{48}{9}$
4	4	$\frac{16}{9}$	$\frac{32}{9}$	1	3



Example 5: Graph the following function: $y = x \left[\frac{x+5}{x-2} \right]$; $3 \leq x \leq 3.75$

Solution:

x	$\left[\frac{x+5}{x-2} \right]$	$y = x \left[\frac{x+5}{x-2} \right]$	Value of x
3	8	$8x = 24$	$x = 3$
3.16	7	$7x$	$3 < x \leq 3.16$
3.4	6	$6x$	$3.16 < x \leq 3.4$
3.75	5	$5x$	$3.4 < x \leq 3.75$

$$\frac{x+5}{x-2} = 7 \quad ;;; \quad x + 5 = 7x - 14 \quad ;;; \quad x = 3.16$$

$$\frac{x+5}{x-2} = 6 \quad ;;; \quad x + 5 = 6x - 12 \quad ;;; \quad x = 3.4$$

x	$y = 7x$	x	$y = 6x$	x	$y = 5x$
3	21	3.16	18.9	3.4	17
3.16	22.12	3.4	20.4	3.75	18.75

