Probability (الاحتمالية)

The concept of probability: It is the possibility of a certain event occurring that we are not completely certain will occur. Probability plays a fundamental role in our daily lives by predicting the possibility of an event occurring. The value of probability is limited to zero and one correct, and zero for an impossible probability, while one correct for a certain probability. Let S be the sample space for an experiment and E be an event in this experiment. The probability of event E occurring is:

$$P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ represents the number of elements of event E and $n(S)$ represents the number of elements in the sample space.

Example 1: Here are statistics on the blood types in Iraq according to the 2024 population census.

O^+	A^+	B^+	AB^+	O^-	A^-	B^-	AB^-	
14,804,132	11,529,698	11,806,411	3,412,791	1,660,277	1,245,209	1,245,206	415,069	46,118,793

Suppose we choose a person at random. What is the probability that his blood type is B^+?

Solution:

$$P(B^+) = \frac{n(B^+)}{n(S)} = \frac{11{,}806{,}411}{46{,}118{,}793} = 0.256$$

Elementary Properties of Probabilities

Given a sample space S with n events, that is $S = \{E_1, E_2, \cdots, E_n\}$. The properties are as follows:

1. $P(S) = P(E_1 + E_2 + \cdots + E_n) = 1$, and $P(\emptyset) = 0$
2. $0 \leq P(E_i) \leq 1$, for all $i = 1, \cdots, n$.
3. $P(E_i + E_j) = P(E_i) + P(E_j)$.
4. $P(E) + P(\bar{E}) = 1$, where $\bar{E}$ is the complement of E, that is $\bar{E} = S - E$.

Conditional Probability (الاحتمال الشرطي)

The conditional probability of A given B is equal to the probability of $A \cap B$ divided by the probability of B, provided the probability of B is not zero.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}; \quad P(B) \neq 0$$

The Addition Rule

Given two events A and B, the probability that event A, or event B, or both occur is equal to the probability that event A occurs, plus the probability that event B occurs, minus the probability that the events occur simultaneously.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Independent Events (الاحداث المستقلة)

Suppose that the probability of event A is the same regardless of whether or not B occurs. In this situation, $P(A|B) = P(A)$. In such cases we say that A and B are independent events.

The multiplication rule for two independent events, then, may be written as:

$$P(A \cap B) = P(A)P(B);\ P(A) \neq 0,\ \ P(B) \neq 0.$$

Example 2: Let $S = \{1,2,3,4,5,6\}$. If an element is selected from S,

1. what is the probability that it is an odd number or divisible by 3?
2. what is the probability that it is an odd number and divisible by 3?

Solution:

The event the number is odd: $A = \{1,3,5\}$.

The event the number is divisible by 3: $B = \{3,6\}$.

1. The event odd number or divisible by 3: $A \cup B = \{1,3,5,6\}$

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3}$$

2. The event odd number and divisible by 3: $A \cap B = \{3\}$

$$P(A \cap B) = \frac{1}{6}$$

Probability Distributions (التوزيعات الاحتمالية)

Probability distributions for random variables play important roles in statistical analysis. Because they show all possible values of a random variable and the probabilities associated with those values, probability distributions can be summarized in ways that enable researchers to easily make objective decisions based on samples taken from the populations that these distributions represent.

The researcher usually wants to deal with numerical values associated with the sample points of the random experiment instead of dealing with the sample points themselves, as the sample points or possible results of the random experiment are sometimes qualities or names that are difficult to deal with mathematically. In this case, we convert these descriptive values into real numerical values called the values of the random variable. Random variables are used to express the results of the random experiment and events with numerical values instead of names or qualities. For example, when a coin is tossed three times in a row, the sample space is:

$$S = \{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{HTT}, \text{THT}, \text{TTH}, \text{TTT}\}$$

If we are interested in the number of times the image H appears only, regardless of other details, then the number of images in this case is a random variable x_i whose value changes with the change in the result of the random experiment. The values of x_i here are $x_i = 0,1,2,3$.

Random variables are of several types, two of which we mention: the first are discrete random variables and the second are continuous random variables.

Discrete Probability Distributions (التوزيعات الاحتمالية المنفصلة)

A discrete probability distribution is a table or rule that contains all the values of a discrete random variable with all the probabilities associated with each value of the discrete variable and is denoted by the symbol $p(x)$ such that: $p(x) \geq 0$ and $\sum p(x_i) = 1$. $p(x)$ is called the probability density function (pdf).

Example 3: Let x represent the number of heads in the experiment of tossing a coin four times. Find a law that represents the probability density function. Then calculate $p(x=2)$.

Solution:

The number of elements in the sample space is: $n(S) = 2^4 = 16$.

The number of elements of event E (the number of heads in the experiment of tossing a coin four times) is: $n(E) = \binom{4}{x}$

$$p(x) = \frac{n(E)}{n(S)} = \frac{\binom{4}{x}}{16}$$

$$p(x=2) = \frac{\binom{4}{2}}{16}$$

$$= \frac{\frac{4!}{2!\,(4-2)!}}{16} = \frac{\frac{4\times 3\times 2\times 1}{2\times 1\times 2\times 1}}{16} = \frac{6}{16} = \frac{3}{8}$$

Example 4: In 2014, 50 students from the College of Science at the University of Babylon donated blood to the wounded of the Ali al-Akbar Brigade who were wounded during the operations to liberate Talafar from Daeish gangs. The level of hemoglobin in their blood was summarized in the following table.

Class	12.8-13.8	13.9-14.9	15.0-16.0	16.1-17.1	17.2-18.2	18.3-19.3
f_i	3	5	15	16	10	1

Create a table containing all the values of a discrete random variable along with all the probabilities associated with each value of the discrete variable.

Solution:

x_i	13.3	14.4	15.5	16.6	17.7	18.8
f_i	3	5	15	16	10	1
$p(x=x_i)$	3/50	5/50	15/50	16/50	10/50	1/50
$p(x=x_i)$	0.06	0.1	0.3	0.32	0.2	0.02

Mean and Variance of Discrete Probability Distributions

The mean and variance of a discrete probability distribution can easily be found using the formulae below:

$$\mu = E(x) = \sum_{i=1}^{n} x_i p(x_i)$$

$$\sigma^2 = E(x_i - \mu)^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)$$

The standard deviation is simply the positive square root of the variance.

Example 5: What is $C.V$ of the distribution from Example 4?

Solution:

$$\mu = E(x) = \sum x_i p(x_i)$$

$$\mu = 13.3 \times 0.06 + 14.4 \times 0.1 + 15.5 \times 0.3 + 16.6 \times 0.32 + 17.7 \times 0.2 + 18.8 \times 0.02$$

$$\mu = 16.1$$

$$\sigma^2 = E(x_i - \mu)^2 = \sum (x_i - \mu)^2 p(x_i)$$

$$\sigma^2 = (13.3 - 16.1)^2 \times 0.06 + (14.4 - 16.1)^2 \times 0.1 + (15.5 - 16.1)^2 \times 0.3$$
$$+ (16.6 - 16.1)^2 \times 0.32 + (17.7 - 16.1)^2 \times 0.2 + (18.8 - 16.1)^2 \times 0.02$$

$$\sigma^2 = 1.6$$

$$C.V = \frac{\sigma}{\mu} \times 100\% = \frac{\sqrt{1.6}}{16.1} \times 100\% = 7.8\%$$

H.W. Calculate $C.V$ from the following discrete probability distribution table:

x_i	0	1	2	3	4
$p(x_i)$	0.15	0.15	0.35	0.25	0.10