مثال7/ جد قيمة $a \in R$ اذا علمت ان $\int_0^a (2x-1)dx = 42$

$$\int_0^a (2x-1)dx = 42$$

$$\to [x^2 - x]_0^a = 42$$

$$\to [a^2 - a] - [0] = 42$$

$$\to a^2 - a - 42 = 0$$

$$(a-7)(a+6) = 0 \Longrightarrow \quad a = 7 \quad or \quad a = -6$$

مثال8/ جد قيمة $a \in R$ اذا علمت ان $\int_a^2 (3+2x)dx = 6$

$$\int_a^2 (3+2x)dx = 6$$

$$\to [3x + x^2]_a^2 = 6$$

$$\to [3(2) + 2^2] - [3a + a^2] = 6$$

$$\to 10 - 3a - a^2 - 6 = 0$$

$$\to a^2 + 3a - 4 = 0$$

$$(a+4)(a-1) = 0$$

$$a = -4 \quad or \quad a = 1$$

مثال7/ جد قيمة $a \in R$ اذا علمت ان $\int_1^b (13-4x)dx = 9$

$$\int_1^b (13-4x)dx = 9$$

$$\to [13x - 2x^2]_1^b = 9$$

$$\to [13(b) - 2b^2] - [13 - 2] = 9$$

$$\to 13b - 2b^2 - 11 - 9 = 0 \quad \to \quad 2b^2 - 13b + 20 = 0$$

$$(2b-5)(b-4) = 0$$

$$2b - 5 = 0 \qquad 2b = 5 \qquad b = \frac{5}{2}$$

$$or \quad b - 4 = 0 \qquad b = 4$$

س٤/ $f(x)$ دالة مستمرة على الفترة $[-2,6]$ فاذا كان $\int_1^6 f(x)dx = 6$ وكان $\int_{-2}^{6}[f(x)+3]dx = 32$ فجد $\int_{-2}^{1} f(x)dx$

الحل /

$$\int_{-2}^{6}[f(x)+3]dx = 32$$

$$\int_{-2}^{6} f(x)dx + \int_{-2}^{6} 3dx = 32$$

$$\int_{-2}^{6} f(x)dx + [3x]_{-2}^{6} = 32$$

$$\int_{-2}^{6} f(x)dx + [18+6] = 32$$

$$\int_{-2}^{6} f(x)dx = 32 - 24$$

$$\int_{-2}^{6} f(x)dx = 8$$

$$\int_{-2}^{1} f(x)dx + \int_{1}^{6} f(x)dx = 8$$

$$\int_{-2}^{1} f(x)dx + 6 = 8$$

$$\int_{-2}^{1} f(x)dx = 8 - 6$$

$$\int_{-2}^{1} f(x)dx = 2$$

س٦/لتكن $f(x) = x^2 + 2x + k$ حيث $k \in R$ دالة نهايتها الصغرى $= -5$

جد $\int_1^3 f(x)dx$

الحل/ $f(x) = x^2 + 2x + k$

$f^{/}(x) = 2x + 2$

$f^{/}(x) = 0 \qquad 2x + 2 = 0 \qquad 2x = -2 \qquad x = -1$

$\therefore (-1, 5)$ نقطة نهاية صغرى محلية نعوضها في $f(x)$

$-5 = (-1)^2 + 2(-1) + k$

$-5 = 1 - 2 + k \qquad k = -5 + 1 \qquad k = -4$

$f(x) = x^2 + 2x - 4$

$\int_1^3 f(x)dx = \int_1^3 x^2 + 2x - 4\, dx$

$= \left[\frac{x^3}{3} + x^2 - 4x\right]_1^3$

$= \left[\frac{27}{3} + 9 - 12\right] - \left[\frac{1}{3} + 1 - 4\right]$

$= [9 + 9 - 12] - \left[\frac{1}{3} - 3\right] = 6 - \left(\frac{1-9}{3}\right) = 6 + \frac{8}{3} = \frac{26}{3}$

س٧/ اذا كان للمنحني $f(x) = (x-3)^3 + 1$ نقطة انقلاب (a , b) جد القيمة العددية

للمقدار: $\int_0^b f^{/}(x)dx - \int_0^a f^{//}(x)dx$

الحل /

بما ان للدالة نقطة انقلاب فاذن $f^{//}(x) = 0$

$f(x) = (x-3)^3 + 1$

$f^{/}(x) = 3(x-3)^2$

$f^{//}(x) = 6(x-3)$

$f^{//}(x) = 0 \qquad 6(x-3) = 0 \qquad x = 3$

نعوض قيمة x في الدالة $f(x)$

$y = f(3) = (3-3)^3 + 1 \qquad y = 1$

نقطة الانقلاب $(a, b) = (3, 1)$

$\int_0^1 f^{/}(x)dx - \int_0^3 f^{//}(x)dx = \int_0^1 3(x-3)^2 dx - \int_0^3 6(x-3)dx$

$= [(x-3)^3]_0^1 - [3(x-3)^2]_0^3$

$= [(1-3)^3 - (0-3)^3] - [3(3-3)^2 - 3(0-3)^2]$

$= [-8 + 27] - 3[0 - 9] = 19 + 27 = 46$

مثال/

اذا كان $\int_1^3 g(x)dx = 2$, $\int_1^3 f(x)dx = 6$ جد $\int_1^3 [f(x) - g(x) + 4x]dx$

$$\int_1^3 [f(x) - g(x) + 4x]dx = \int_1^3 f(x)dx - \int_1^3 g(x) + \int_1^3 4x\, dx$$
$$= 6 - 2 + [2x^2]_1^3$$
$$= 4 + [2(9 - 1)]$$
$$= 4 + 16 = 20$$

مثال/اوجد $\int_0^{\frac{\pi}{4}} sec^2 x\, dx$

$$\int_0^{\frac{\pi}{4}} sec^2 x\, dx = [tan\, x]_0^{\frac{\pi}{4}} = tan\, \frac{\pi}{4} - tan\, 0 = 1 - 0 = 1$$

مثال/اوجد $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} csc^2 x\, dx$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} csc^2 x\, dx = [-cot\, x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = -cot\, \frac{\pi}{2} + cot\, \frac{\pi}{4} = 0 + 1 = 1$$

مثال/اوجد $\int_0^{\frac{\pi}{3}} sec\, x\, tan\, x\, dx$

الحل /

$$\int_0^{\frac{\pi}{3}} sec\, x\, tan\, x\, dx = [sec\, x]_0^{\frac{\pi}{3}} = sec\, \frac{\pi}{3} - sec\, 0 = 2 - 1 = 1$$

مثال/اوجد $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosx}{\sqrt{sin\, x}} dx$

الحل/

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{cosx}{\sqrt{sin\, x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (sinx)^{-\frac{1}{2}} cos\, x\, dx$$
$$= 2[\sqrt{\sin x}]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$
$$= 2\left[\sqrt{\sin\frac{\pi}{2}} - \sqrt{\sin\frac{\pi}{6}}\right]$$
$$= 2\left[1 - \frac{1}{\sqrt{2}}\right] = 2 - \sqrt{2}$$

س/ جد قيمة $a \in R$ اذا علمت ان $\int_1^a \left(x+\frac{1}{2}\right)dx = 2\int_0^{\frac{\pi}{4}} sec^2x dx$

الحل /

$$\int_1^a \left(x+\frac{1}{2}\right)dx = 2\int_0^{\frac{\pi}{4}} sec^2x dx$$

$$\left[\frac{x^2}{2}+\frac{1}{2}x\right]_1^a = 2[\tan x]_0^{\frac{\pi}{4}}$$

$$\left[\frac{a^2}{2}+\frac{1}{2}a\right]-\left[\frac{1}{2}+\frac{1}{2}\right] = 2\left[tan\frac{\pi}{4}-\tan 0\right]$$

$$\frac{a^2}{2}+\frac{a}{2}-1 = 2(1-0)$$

$$a^2+a-2=4$$

$$a^2+a-6=0$$

$$(a+3)(a-2)=0$$

$$a=-3 \quad or \quad a=2$$

مشتقة دالة اللوغاريتم الطبيعي

اذا كانت

$$y = \ln x \implies y' = \frac{1}{x}$$

$$y = \ln u(x) \implies y' = \frac{1}{u(x)} \frac{du}{dx}$$

س٩/ جد $\frac{dy}{dx}$ لكل مما يأتي :-

1) $y = \ln(2x^2 - 3)$

$$\frac{dy}{dx} = \frac{4x}{2x^2-3}$$

---

2) $y = \ln(5x^3 - x + 1)$

الحل/

$$\frac{dy}{dx} = \frac{15x^2-1}{5x^3-x+1}$$

---

3) $y = \ln\sqrt{x}$ $\qquad y = \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x \qquad y' = \frac{1}{2} \cdot \frac{1}{x}$

4) $y = \sqrt{\ln 2x}$

5) $y = (\ln x^2)^3$

الحل

$$\frac{dy}{dx} = 3(\ln x^2)^2 \left(\frac{2x}{x^2}\right) = \frac{6(\ln x^2)^2}{x}$$

---

6) $y = \ln \sin 5x$

$$\frac{dy}{dx} = \frac{\cos 5x(5)}{\sin 5x} = 5\cot 5x$$

---

7) $y = \ln \tan 3x$

$$\frac{dy}{dx} = \frac{\sec^2 3x\,(3)}{\tan 3x} = \frac{3\sec^2 3x}{\tan 3x}$$

---

8) $y = \ln 3x$

الحل /

$$\frac{dy}{dx} = \frac{3}{3x} = \frac{1}{x}$$

$9)\ y = \ln\left(\frac{x}{2}\right)$

الحل /

$$y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$$

$$\frac{dy}{dx} = \frac{1}{x} - 0 = \frac{1}{x}$$

$10)\ y = \ln(x^2)$

الحل /

$$\frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x}$$

$11)\ y = \ln\left(\frac{1}{x}\right)^3$

الحل /

$$y = \ln\left(\frac{1}{x}\right)^3 = \ln\left(\frac{1^3}{x^3}\right) \Longrightarrow \quad y = \ln x^{-3}$$

$$y = \ln x^{-3}$$

$$\frac{dy}{dx} = \frac{-3x^{-4}}{x^{-3}} = -3x^{-1} = \frac{-3}{x}$$

$12)\ y = (\ln x)^2$

$$\frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x} = \frac{2\ln x}{x}$$

$13)\ y = \ln(2 - \cos x)$

$$\frac{dy}{dx} = \frac{\sin x}{2 - \cos x}$$

---

1) $\int_0^3 \frac{1}{x+1} dx = [ln|x+1|]_0^3$

$= ln(3+1) - ln(0+1)$

$= ln4 - ln1$

$= ln4 - 0 = ln2^2 = 2ln2$

---

2) $\int_0^4 \frac{2x}{x^2+9} dx = [ln|x^2+9|]_0^4$

$= ln(16+9) - ln(0+9)$

$= ln25 - ln9$

$= ln\frac{25}{9} = ln\left(\frac{5}{3}\right)^2 = 2ln\frac{5}{3}$

---

3) $\int_0^1 \frac{3x^2+4}{x^3+4x+1} dx = [ln|x^3+4x+1|]_0^1$

$= ln(1+4+1) - ln(0++0+1)$

$= ln6 - ln1$

$= ln6 + 0 = ln6$

---

4) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{sec^2 x}{2+tan\,x} dx = [ln|2+tan\,x|]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$= ln\left|2+tan\frac{\pi}{4}\right| - ln\left|2+tan(-\frac{\pi}{4})\right|$

$= ln|2+1| - ln|2-1|$

$= ln3 - ln\,1 = ln3 - 0 = ln3$

---

5) $\int \frac{ln\,x}{x} dx = \int hn\,x \cdot \frac{1}{x} dx = \frac{(ln\,x)^2}{2} + c$

---

6) $\int \frac{dx}{xln\,x} = ln|ln\,x| + c$

---

7) $\int \frac{dx}{x(1ln\,x)^2}$

$let \;\; u = 1 + ln\,x \qquad du = \frac{1}{x}$

$\int \frac{dx}{x(1ln\,x)^2} = \int \frac{du}{u^2} = \int u^{-2} du = -u^{-1} + c = -\frac{1}{1+ln\,x} + c$

8) $\int_0^{\frac{\pi}{3}} \sec x \sin x dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\, dx$

$$= [-\ln|\cos x|]_0^{\frac{\pi}{3}}$$

$$= -\ln\left|\cos\frac{\pi}{3}\right| + \ln|\cos 0|$$

$$= -\ln\frac{1}{2} + \ln 1 = -\ln\frac{1}{2} + 0$$

9) $\int \frac{x}{2x^2+5} dx = \frac{1}{6}\int \frac{6x}{2x^2+5} dx = \frac{1}{6}\ln|2x^2+5| + c$

==

الدالة الآسية : e^x (أساس e) هي عكس دالة اللوغاريتم الطبيعي ونستنتج جميع خواصها من هذه الحقيقة

مشتقة الدالة الآسية e^u هي $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$

مثال / لتكن $y = e^{2x}$ جد $\frac{dy}{dx}$

$$y = e^{2x} \Longrightarrow \frac{dy}{dx} = e^{2x}(2) = 2e^{2x}$$

مثال / لتكن $y = e^{\tan x}$ جد $\frac{dy}{dx}$

$$y = e^{\tan x} \Longrightarrow \frac{dy}{dx} = e^{\tan x} \sec^2 x$$

مثال / لتكن $y = e^{-5x^2+3x+5}$ جد $\frac{dy}{dx}$

$$y = e^{-5x^2+3x+5} \Longrightarrow \frac{dy}{dx} = e^{-5x^2+3x+5}(-10x+3)$$

مثال / لتكن $y = x^2 e^x$ جد $\frac{dy}{dx}$

الحل /

$$y = x^2 e^x \Longrightarrow \frac{dy}{dx} = x^2 e^x + e^x 2x$$

مثال / لتكن $y = e^{x^2} \ln|2x|$ جد $\frac{dy}{dx}$

الحل /

$$y = e^{x^2} \ln|2x| \Longrightarrow \frac{dy}{dx} = e^{x^2} \cdot \frac{2}{2x} + \ln|2x| e^{2x} \cdot 2x$$

مثال / لتكن $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$ جد $\frac{dy}{dx}$

$$y = \frac{e^x+e^{-x}}{e^x-e^{-x}} \Longrightarrow \frac{dy}{dx} = \frac{(e^x-e^{-x})(e^x-e^{-x})-(e^x+e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})^2}$$

$$= \frac{(e^{2x}-2e^0+e^{-2x})-(e^{2x}+2e^0+e^{-2x})}{(e^x-e^{-x})^2}$$

$$= \frac{-4}{(e^x-e^{-x})^2}$$

مثال / لتكن $y = \cos(e^{\pi x})$ جد $\frac{dy}{dx}$

$$y = \cos(e^{\pi x}) \Longrightarrow \frac{dy}{dx} = -\sin(e^{\pi x}) \cdot e^{\pi x} \cdot \pi = -\pi e^{\pi x} \sin(e^{\pi x})$$

تكامل الدالة الاسية e^u

$\int e^u du = e^u + c$ تقودنا الى صيغة التكامل $d(e^u) = e^u \frac{du}{dx}$

1) $\int xe^{x^2} dx = \frac{1}{2}\int e^{x^2}\, 2x\, dx = \frac{1}{2}\, e^{x^2} + c$

2) $\int_{ln3}^{ln5} e^{2x} dx = \left[\frac{1}{2}\, e^{2x}\right]_{ln3}^{ln5} = \frac{1}{2}[e^{2ln5} - e^{2ln3}]$

$= \frac{1}{2}[e^{ln\,25} - e^{ln9}]$

$= \frac{1}{2}[25 - 9] = \frac{1}{2}[16] = 8$

3) $\int_0^{ln2} e^{-x} dx = [-\, e^{-x}]_0^{ln2}$

$= -\, e^{-ln2} +\, e^0$

$= -\, e^{ln2^{-1}} + 1 = -\frac{1}{2} + 1 = \frac{1}{2}$

4) $\int_0^1 (1 + e^x)^2\, e^x dx = \left[\frac{1}{3}(1 +\, e^x)^3\right]_0^1 =$

$= \frac{1}{3}[(1 +\, e^1)^3 - (1 +\, e^0)^3]$

$= \frac{1}{3}[(1 +\, e^x)^3 - (1 + 1)^3]$

$= \frac{1}{3}[(1 +\, e^x)^3 - 8]$

5) $\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = \left[\, e^{\sqrt{x}}\right]_1^4 = e^{\sqrt{4}} - e^{\sqrt{1}}$

$= e^2 - e^1 = e^2 - e$

6) $\int_0^{\frac{\pi}{2}} e^{\cos x} sinx\, dx = [-e^{\cos x}]_0^{\frac{\pi}{2}}$

$= -e^{\cos\frac{\pi}{2}} + e^{\cos 0} = -e^0 + e^1 = -1 + e$

7) $\int sec^2 3x\, e^{tan3\theta} dx = \frac{1}{3}\int e^{tan3\theta}\, 3sec\, 3x\, dx = \frac{1}{3}\, e^{tan3\theta} + c$

8) $\int_1^2 xe^{-lnx} dx = \int_1^2 xe^{(lnx)^{-1}} dx$

$= \int_1^2 x \cdot x^{-1} dx = \int_1^2 dx$

$= \int_1^2 dx = [x]_1^2 = 2 - 1 = 1$

9) $\int \sqrt{e^{2x-4}} dx = \int \sqrt{e^{2(x-2)}}\, dx = \int e^{x-2} dx = e^{x-2} + c$