University of Babylon
College of Information Technology
Department of Software

Operations Research

Lecture-6

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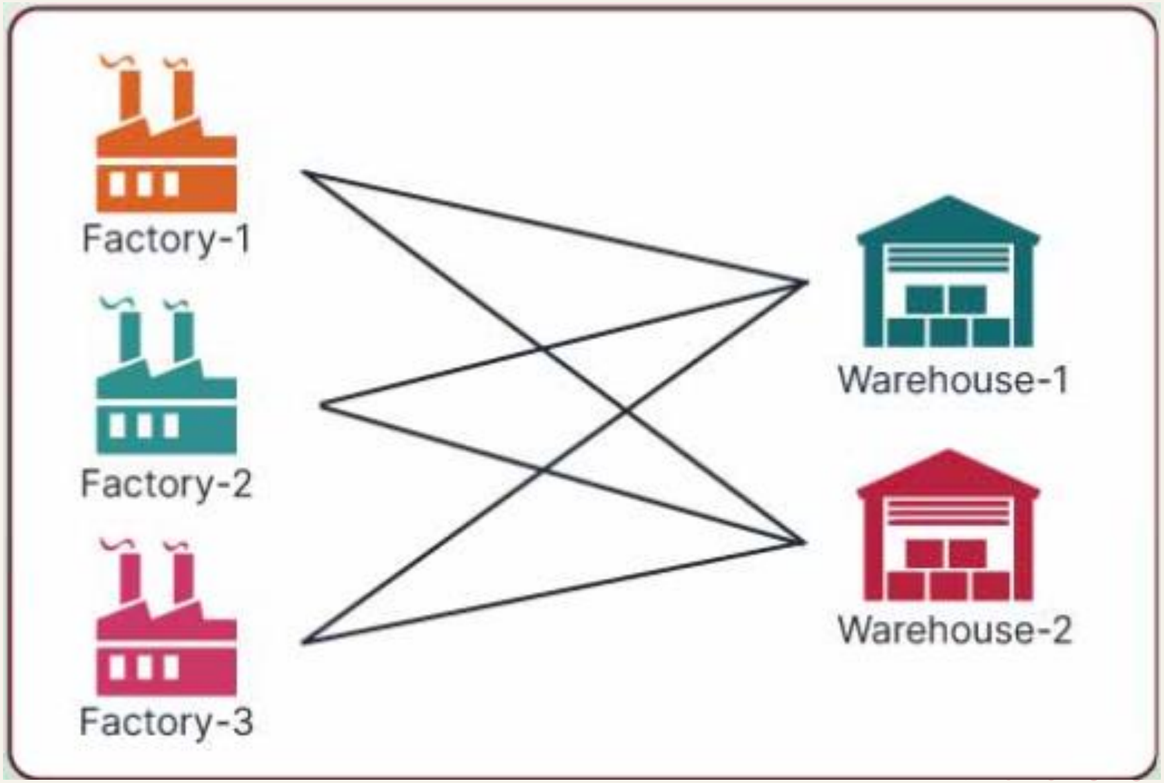
Transportation Problem

- A transportation problem in operation research is a special type of Linear Programming Problem used to optimize (minimize) the transportation cost and allocate resources from M source to N destination.



Transportation Problem

In simple words, the main objective of the Transportation problem is to deliver (from the source to the destination) the resources at the minimum cost.

- It involves transporting a single product from ‘m’ source (origin) to ‘n’ destinations.
- Assumptions: The supply level of each source and the demand at each destination are known.
- Objective: To minimize the total Transportation Cost.

Transportation Problem Formulation

a_i the quantity available at the source S_i

b_j the quantity required at the destination D_j

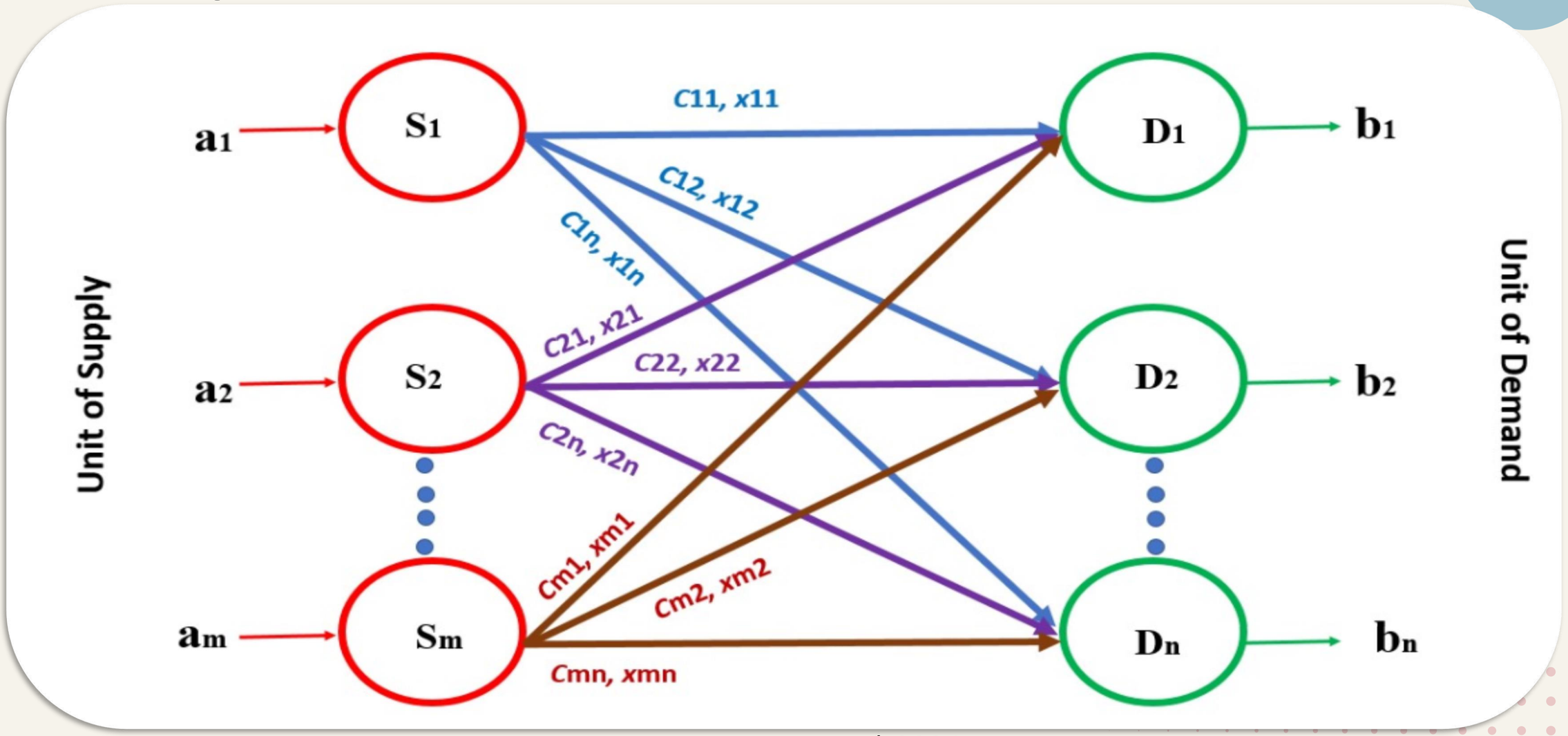
c_{ij} cost of transportation of one unit resource from S_i to D_j

x_{ij} unit of resources transported from from S_i to D_j

$$1 \leq i \leq m$$

$$1 \leq j \leq n$$

Transportation Problem Formulation



Transportation Problem Formulation

Objective:

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{c_{ij}\ x_{ij}}$$

Subject to:

- Supply constraints: (for each source i) $\boldsymbol{x_{i1} + x_{i2} + x_{i3} \leq Supply_i}$
- demand constraints: (for each destination j) $\boldsymbol{x_{j1} + x_{j2} + x_{j3} = Demand_j}$

Transportation Problem Formulation

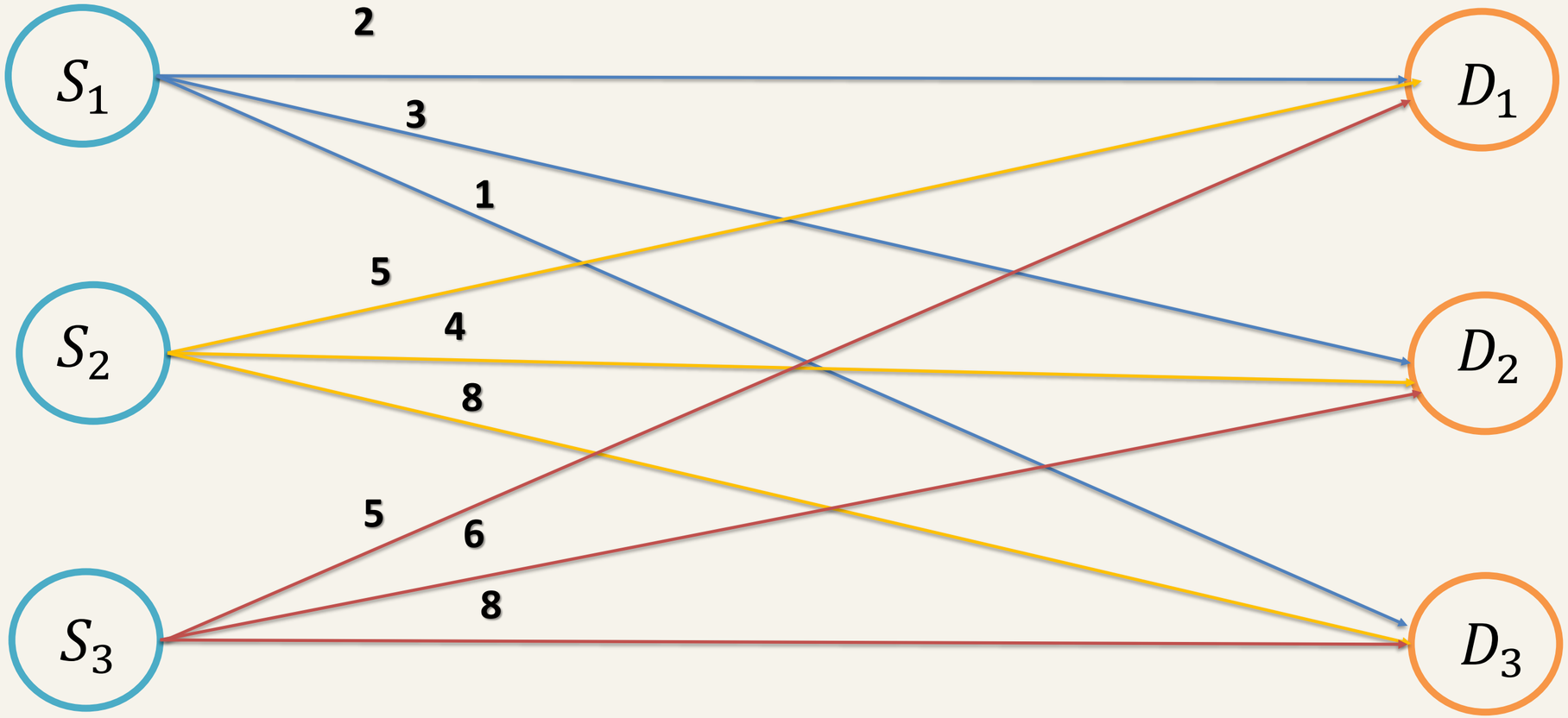
Source \ Destination	D_1	D_2	D_3	D_4	.....	D_n	Supply
S_1	$c11$	$c12$	$c13$	$c14$	.....	$c1n$	a_1
S_2	$c21$	$c22$	$c23$	$c24$	.....	$c2n$	a_2
S_3	$c31$	$c32$	$c33$	$c34$	.....	$c3n$	a_3
S_4	$c41$	$c42$	$c43$	$c44$	.....	$c4n$	a_4
	.....	.....	.....	.....	.....	.....	
S_m	$cm1$	$cm2$	$cm3$	$cm4$	.....	cmn	a_m
Demand	b_1	b_2	b_3	b_4		b_n	

Transportation Problem Formulation

Example-1: Let's consider a transportation problem with three sources and three destinations, where the costs per unit transported are given as follows: from Source 1 to Destination 1, 2, and 3 are $2, $3, and $1 respectively, from Source 2 to Destination 1, 2, and 3 are $5, $4, and $8 respectively, and from Source 3 to Destination 1, 2, and 3 are $5, $6, and $8 respectively. The supplies are 50 units for Source 1, 60 units for Source 2, and 50 units for Source 3, while the demands are 30 units for Destination 1, 70 units for Destination 2, and 60 units for Destination 3. Draw a graph that represent the problem and Formulate it. Finally draw a table *Draw a graph that represent the problem and Formulate it. Finally draw a table.*

Transportation Problem Formulation

graph that represent the problem in Example 1



Transportation Problem Formulation

The problem formulation in Example 1

Objective:

$$\boldsymbol{Minimize\ Z = 2x_{11} + 3x_{12} + 1x_{13} + 5x_{21} + 4x_{22} + 8x_{23} + 5x_{31} + 6x_{32} + 8x_{33}}$$

Subject to:

Supply:

$$x_{11} + x_{12} + x_{13} \leq 50$$
$$x_{21} + x_{22} + x_{23} \leq 60$$
$$x_{31} + x_{32} + x_{33} \leq 50$$

Demand:

$$x_{11} + x_{21} + x_{31} = 30$$
$$x_{12} + x_{22} + x_{32} = 70$$
$$x_{13} + x_{23} + x_{33} = 60$$

$$x_{ij} \geq 0$$

Transportation Problem Formulation

The table in Example 1

	Destination 1	Destination 2	Destination 3	Supply
Source 1	2	3	1	50
Source 2	5	4	8	60
Source 3	5	6	8	50
Demand	30	70	60	

Transportation Problem Types

1- Balanced Transportation Problem

- **Definition**: A transportation problem is balanced when the total supply from all sources equals the total demand at all destinations.

  Condition: ∑Supply=∑Demand

2-Unbalanced Transportation Problem

- **Definition:** A transportation problem is unbalanced when the total supply does not equal the total demand.

  Condition: ∑Supply≠∑Demand

Solution:

- ❑ If supply exceeds demand, dummy demand nodes are added to balance the problem. The transportation cost to the dummy demand nodes is set to zero.
- ❑ If demand exceeds supply, dummy supply nodes are added.

Transportation Problem Types

Example-2: check which types of transportation problem it is?

	Destination 1	Destination 2	Destination 3	Supply
Source 1	2	7	4	5
Source 2	3	3	1	8
Source 3	5	4	7	7
Source 4	1	6	2	14
Demand	7	9	18	

Transportation Problem Types

Answer:

From the above, we have:

Total supply=5+8+7+14=34

Total demand=7+9+18=34

Hence, Total supply=total demand

Therefore, it is Balanced Transportation problem

Transportation Problem Types

Example-3: check which types of transportation problem it is?

	Destination 1	Destination 2	Destination 3	Supply
Source 1	4	3	2	10
Source 2	2	5	0	13
Source 3	3	8	6	12
Demand	8	5	4	

Transportation Problem Types

Answer:

From the above, we have:

Total supply=10+13+12=35

Total demand=8+5+4=17

Hence, Total supply≠total demand

Therefore, it is unbalanced Transportation problem

- Since the total supply (35) is greater than the total demand (17), we need to introduce a **dummy destination** to balance the problem.
- The dummy destination will take the surplus supply, which is:

Dummy Demand=Total Supply−Total Demand=35−17=18 units

Transportation Problem Types

So, we introduce a dummy destination with a demand of 18 units, and the transportation cost to this dummy destination is set to 0.

	Destination 1	Destination 2	Destination 3	Dummy	Supply
Source 1	4	3	2	0	10
Source 2	2	5	0	0	13
Source 3	3	8	6	0	12
Demand	8	5	4	18	balance

Transportation Problem Formulation

Example 4: We have three factories (sources) and three warehouses (destinations), with transportation costs, supplies, and demands as follows:

Transportation Costs:

From Source 1 to Destination 1, 2, and 3: $4, $3, and $7 per unit respectively.
From Source 2 to Destination 1, 2, and 3: $6, $2, and $5 per unit respectively.
From Source 3 to Destination 1, 2, and 3: $5, $4, and $3 per unit respectively.

Supply at Sources:

Source 1: 40 units
Source 2: 50 units
Source 3: 40 units

Demand at Destinations:

Destination 1: 50 units
Destination 2: 70 units
Destination 3: 60 units

- *Home Work:*
- *Draw a graph that represent the problem*
- *Formulate it.*
- *draw a table.*
- *Check the type of this problem.*

THANK YOU