Yield Criteria

Commencement of plastic deformation in materials is predicted by yield criteria. Yield criteria are also called theories of yielding. A number of yield criteria have been developed for ductile and brittle materials.

1. Tresca yield criterion:

It states that when the maximum shear stress within an element is equal to or greater than a critical value, yielding will begin.

$\tau_{max} \geq K$

Where

K is shear yield stress (strength).

Or $\tau_{max} = (\sigma_1 - \sigma_3)/2 = k$ where σ_1 and σ_3 are principal stresses

Or $\sigma_1 - \sigma_3 = Y$

For uniaxial tension, we have $K = Y/2$

Therefore, $\sigma_1 - \sigma_3 = 2K = Y$

Here Y= yield stress. The intermediate stress σ_2 has no effect on yielding.

2. Von Mises criterion:

According to this criterion, yielding occurs when

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2 = 6k^2$$

For plane strain condition, we have: $\sigma_2 = (\sigma_1 + \sigma_3)/2$

$K = Y/\sqrt{3} = 0.577\ Y$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = constant$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6K^2 = 2Y^2$$

$$Y = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}}$$

Effective stress and effective strain

Effective stress is defined as that stress which when reaches critical value, yielding can commence.

For Tresca criterion, effective stress is

$$\bar{\sigma} = \sigma_1 - \sigma_3$$

For von Mises criterion, the effective stress is

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\}^{1/2}$$

Effective strain

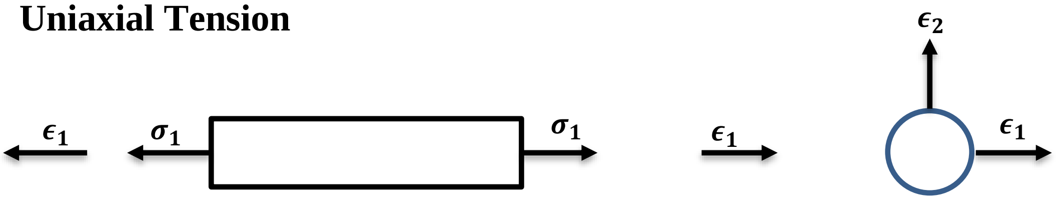
For Tresca criterion, effective strain is

$$\bar{\epsilon} = \frac{2}{3}(\epsilon_1 - \epsilon_3)$$

For von Mises criterion, the effective strain is

$$\bar{\epsilon} = \sqrt{\frac{2}{3}}[\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2]^{1/2}$$

Uniaxial Tension



$$\sigma_2 = \sigma_3 = 0$$

For von Mises criterion

$$\bar{\epsilon} = \sqrt{\frac{2}{3}}[\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2]^{1/2}$$

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0 \Rightarrow \epsilon_1 = -2\epsilon_2 = -2\epsilon_3$$

$$\bar{\epsilon} = \sqrt{\frac{2}{3}}\left[\epsilon_1^2 + \frac{\epsilon_1^2}{4} + \frac{\epsilon_1^2}{4}\right]^{1/2} = \sqrt{\frac{2}{3}}\left(\frac{3}{2}\right)^{1/2} \epsilon_1$$

$$\bar{\epsilon} = \epsilon_1$$

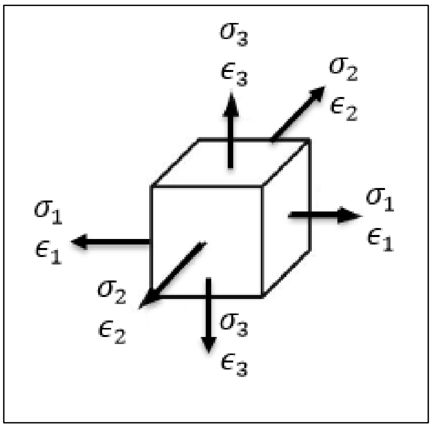
$$\bar{\sigma} = \frac{1}{\sqrt{2}}\{ [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\}^{1/2}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[(\sigma_1 - 0)^2 + (0 - 0)^2 + (0 - \sigma_1)^2]^{1/2}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}[2\sigma_1^2]^{1/2} = \frac{1}{\sqrt{2}}\sqrt{2}\sigma_1$$

$$\bar{\sigma} = \sigma_1$$

Plane Stress vs Plane Strain



$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$$

$$\epsilon_1 = -\epsilon_3$$

$$d\epsilon_1 = -d\epsilon_3$$

$$\frac{d\epsilon_1 - d\epsilon_2}{\sigma_1 - \sigma_2} = \frac{d\epsilon_2 - d\epsilon_3}{\sigma_2 - \sigma_3}$$

$$\frac{\cancel{d\epsilon_1} - 0}{\sigma_1 - \sigma_2} = \frac{0 + \cancel{d\epsilon_1}}{\sigma_2 - \sigma_3}$$

$$\sigma_1 - \sigma_2 = \sigma_2 - \sigma_3$$

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$$

$\sigma_2 = P$ Where P = hydrostatic pressure

$$P = \frac{\sigma_1 + \sigma_3}{2}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\{ [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\}^{1/2}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \frac{(\sigma_1 + \sigma_3)}{2}\right)^2 + \left(\frac{(\sigma_1 + \sigma_3)}{2} - \sigma_3\right)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\bar{\sigma} = \frac{\sqrt{3}}{2}(\sigma_1 - \sigma_3)$$

$\bar{\epsilon} = \sqrt{\frac{2}{3}}[\epsilon_1^2 + 0 + \epsilon_3^2]^{1/2}$ $\bar{\epsilon} = \sqrt{\frac{2}{3}}[\epsilon_1^2 + \epsilon_3^2]^{1/2} = \sqrt{\frac{2}{3}} \times \sqrt{2}\epsilon_1 = \frac{2}{\sqrt{3}}\epsilon_1$

$$\bar{\epsilon} = \frac{2}{\sqrt{3}}\epsilon_1$$

Example In the plane strain forward extrusion the thickness is reducing from 20 mm to 15 mm, the plastic stress- strain curve for the material is given by:

$\bar{\sigma} = 400(\bar{\epsilon})^{0.4}\ MPa$. Determine:

1. The effective strain.

2. The principal stress if the hydrostatic pressure 100 MPa.

3. Work done.

(a) $\bar{\epsilon} = \sqrt{\frac{2}{3}}[\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2]^{1/2}$

$\epsilon_2 = 0$

$\epsilon = ln\frac{t_1}{t_\circ} \Rightarrow \epsilon_3 = \ln\frac{15}{20} = -0.287$

$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0 \Rightarrow \epsilon_1 = -\epsilon_3 \Rightarrow \epsilon_1 = -(-0.287) = 0.287$

$\bar{\epsilon} = \sqrt{\frac{2}{3}}[(0.287)^2 + 0^2 + (0.287)^2]^{1/2} \approx 0.331$

Or $\bar{\epsilon} = \frac{2}{\sqrt{3}}\epsilon_1 = \frac{2}{\sqrt{3}}(0.287) \approx 0.331$

(b) $P = \frac{\sigma_1+\sigma_3}{2}$

$\sigma_2 = \frac{\sigma_1+\sigma_3}{2}$ $100 = \frac{\sigma_1+\sigma_3}{2}$ $\sigma_1 + \sigma_3 = 200 \ldots\ldots\ldots(1)$

$\bar{\sigma} = 400(\bar{\epsilon})^{0.4} = 400(0.331)^{0.4} \approx 258\ MPa.$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]\}^{1/2}$$

$$258 = \frac{1}{\sqrt{2}}[(\sigma_1 - 100)^2 + (100 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \ldots\ldots\ldots(2)$$

$\sigma_1 = 248.15\ MPa$

$\sigma_3 = -48.52\ MPa$

(c) $w = \int \bar{\sigma} d\bar{\epsilon} = \int_0^{0.331} 400\,(\bar{\epsilon})^{0.4} d\epsilon = 400\int_0^{0.331}\frac{\bar{\epsilon}^{1.4}}{1.4} = 400\left[\frac{0.331^{1.4}}{1.4} - 0\right]$

$w = 61.3\ MPa$