Measures of Central Tendency (مقاييس النزعة المركزية)

1. Arithmetic Mean (الوسط الحسابي)

It is one of the most important measures of central tendency and the most widely used in practical aspects. It is generally known as the sum of the values divided by their number and is symbolized by the symbol $\overline{x}$.

If any data set consisting of n values $x_1, x_2, \cdots, x_n$, then the arithmetic mean of these values $\overline{x}$ is defined as:

$\overline{x}$ =(Sum of all observations)/ (Total number of observation)

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Example 1: Determine the mean of the first six prime numbers.

Solution: The first six prime numbers are 2, 3, 5, 7,11 and 13.

$$\overline{x} = \sum_{i=1}^{6} x_i / 6$$

$$= \frac{2 + 3 + 5 + 7 + 11 + 13}{6} = 6.833$$

For calculating the mean when the frequency of the observations is given, such that $x_1, x_2, \cdots, x_n$ is the recorded observations, and $f_1, f_2, \cdots, f_n$ is the respective frequencies of the observations then:

$$\overline{x} = \frac{x_1.f_1 + x_2.f_2 + \cdots + x_n.f_n}{f_1 + f_2 + \cdots + f_n}$$

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i.f_i}{\sum_{i=1}^{n} f_i}$$

Example 2: Doctors tested the glomerular filtration rate (GFR) in children who received kidney transplants. Below are measurements from 15 children's GFR measured using diethylenetriaminepentaacetic acid.

21, 21, 21, 23, 27, 27, 30, 31, 37, 37, 42, 42, 42, 42, and 48. Compute the arithmetic mean.

Solution: Put the given values x_i and their corresponding frequency f_i as in the following table:

x_i	21	23	27	30	31	37	42	48	
f_i	3	1	2	1	1	2	4	1	$\sum_{i=1}^{8} f_i = 15$
$x_i.f_i$	63	23	54	30	31	74	168	48	$\sum_{i=1}^{8} x_i.f_i = 491$

$$\overline{x} = \frac{\sum_{i=1}^{8} x_i.f_i}{\sum_{i=1}^{8} f_i} = \frac{491}{15} = 32.733.$$

Example 3: The partial pressure of arterial carbon dioxide (PaCO2) was measured for 30 patients and the results were as shown in the following table:

Class	33-37	38-42	43-47	48-52	53-57	58-62
Frequency	4	5	6	2	9	4

Compute the arithmetic mean.

Solution: First, you need to find the midpoints x_i of each class, and then the following table is formed:

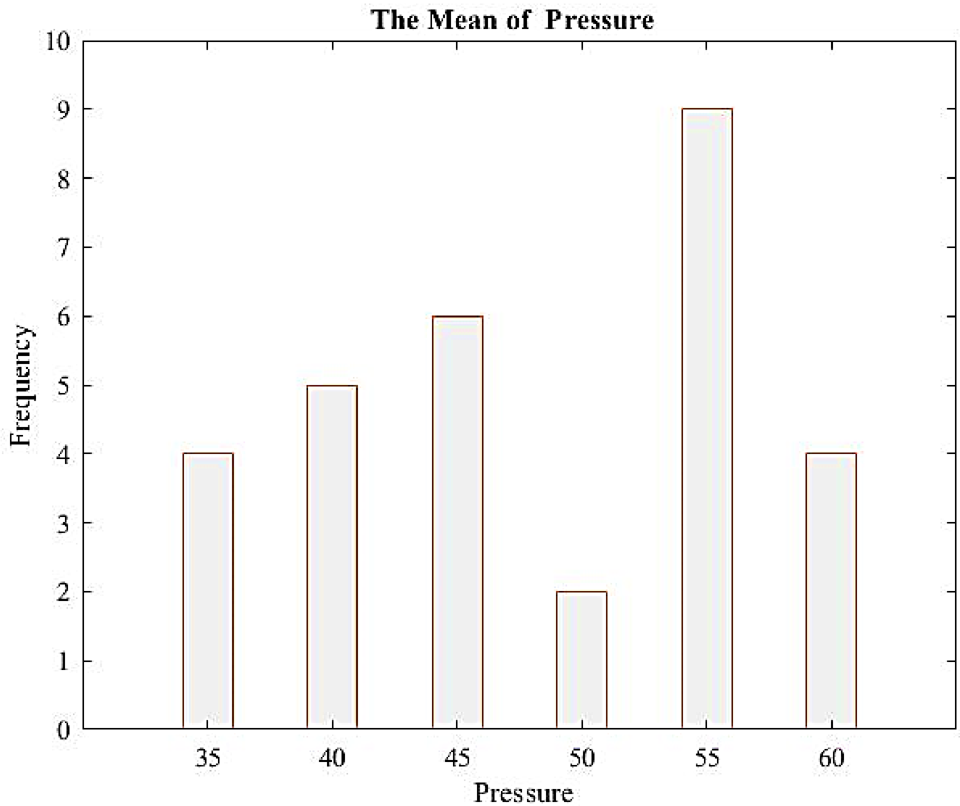
Class	33-37	38-42	43-47	48-52	53-57	58-62	
x_i	35	40	45	50	55	60	
f_i	4	5	6	2	9	4	$\sum f_i = 30$
$x_i.f_i$	140	200	270	100	495	240	$\sum x_i.f_i = 1445$

$$\overline{x} = \frac{\sum_{i=1}^{6} x_i.f_i}{\sum_{i=1}^{6} f_i} = \frac{1445}{30} = 48.167$$

In Matlab

```
X=[35:5:60];F=[4,5,6,2,9,4];
Mean=sum(X.*F)/sum(F)
bar(X,F);
xlabel('Pressure'); ylabel('Frequency');
title('The Mean of Pressure')
```

Ans:Mean = 48.1667



Properties of the Mean:

1) For a given set of data there is one and only one arithmetic mean.
2) The arithmetic mean is easily understood and easy to compute.
3) Extreme values have an influence on the mean and, in some cases, can so distort it that it becomes undesirable as a measure of central tendency. As an example of how extreme values affect the mean, consider the following situation. Suppose the wages of five doctors for a particular operation were 850, 750, 700, 800 and 50 thousand Iraqi dinars. The mean wages of the five

doctors were found to be 630 thousand dinars, a value that is not representative of the data set as a whole. The single outlier had the effect of deflating the mean.

4) When a fixed value is added to all the data, the new arithmetic mean is the sum of the first arithmetic mean and that value.

2. Median (الوسيط)

The median $\overline{Me}$ of a finite set of values is that value which divides the set into two equal parts such that the number of values equal to or greater than the median is equal to the number of values equal to or less than the median. If the number of values is odd, the median will be the middle value when all values have been arranged in order of magnitude. When the number of values is even, there is no single middle value. Instead there are two middle values. In this case the median is taken to be the mean of these two middle values, when all values have been arranged in the order of their magnitudes.

In other world, the median is the value that lies in the middle of the data when the data set is ordered.

Median for ungrouped data: Is divided into two parts.

If the data set has an

1. Odd number of entries: median is the middle data entry.
2. Even number of entries: median is the mean of the two middle data entries.

Example 3: Porcellini studied 13 HIV-positive patients who were treated with highly active antiretroviral therapy (HAART) for at least 6 months. The CD4 T cell counts $(\times 10^{-6}/L)$ at baseline for the 13 subjects are listed below:

230, 205, 313, 207, 227, 245, 173, 58, 103, 181, 105, 301, 169. Compute the mean and median.

Solution: First, we arrange the values in ascending order:

58 103 105 169 173 181 **205** 207 227 230 245 301 313

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{2517 \times 10^{-6}/L}{13} = 193.615 \times 10^{-6}/L$$

The number of values is 13 (odd).

So, the value that is ranked seventh represents the median.

$$\overline{Me} = 205 \times 10^{-6}/L$$

In Matlab

```
x=[58 103   105 169 173 181 205 207 227 230 245 301 313];
Mean=mean(x)
Me=x(7)
```

Ans: Mean = 193.6154
Me = 205

Example 4: A statistic was conducted on the number of children born in ten hospitals in Babylon during April 2020, and it was as follows:

884, 340, 408, 531, 618, 438, 625, 377, 723, 562. Compute the mean and median.

Solution: Arraying the ten numbers in order of magnitude from smallest to largest gives: 340, 377, 408, 438, 538, 562, 618, 625,723, 884.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5513}{10} = 551.3$$

The number of values is 10 (even). The two middle values are 538 and 562. The median, then, is $\overline{Me} = (538 + 562)/2 = 550$

Properties of the Median

1) As is true with the mean, there is only one median for a given set of data.
2) The median is easy to calculate.
3) It is not as drastically affected by extreme values as is the mean.
4) When a fixed value is added to all the data, the new median is the sum of the first median and that value.

3. Mode (المنوال)

The mode $\overline{Mo}$ of a set of values is that value which occurs most frequently. If all the values are different there is no mode; on the other hand, a set of values may have more than one mode.

Example 5: The following values represent the heart rates of seven young mice: 500, 570, 560, 450, 570, 560 and 570. Compute the mean, median and mode.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{3780}{7} = 540$$

To find the median: Arraying the seven values in order of magnitude from smallest to largest gives: 450, 500, 560, 560, 570, 570, 570.

The middle value is: 560 which is the median.

$\overline{Mo} = 570.$

Example 6: The sample consisting of the values 10, 21, 33, 53, and 54 has no mode since all the values are different.

H.W.

1. The data below represent the duration of follow-up care in months after surgery for twenty people.

   103, 68, 62, 60, 60, 54, 49, 44, 42, 41, 38, 36, 34, 30, 19, 19, 19, 19, 17 and16. Compute the mean, median and mode.

2. Calculate the arithmetic mean from the following frequency distribution table

Class	20-24	25-29	30-34	35-39	40-44	45-49
f_i	8	11	21	28	17	15