Simplify the Boolean Function using Karnaugh Map (K-Map)

The second method that used to simplify the Boolean function is the Karnaugh map. K-map basically deals with the technique of inserting the values of the output variable in cells within a rectangle or square grid according to a definite pattern. The number of cells in the K-map is determined by the number of input variables and is mathematically expressed as two raised to the power of the number of input variables, i.e., 2^n, where the number of input variables is n.

Thus, to simplify a logical expression with **two inputs**, we require a K-map with ($2^2 = 4$) cells. A **four-input** logical expression would lead to a ($2^4 = 16$) celled-K-map, and so on.

Advantages of K-Maps

1- The K-map simplification technique is simpler and less error-prone compared to the method of solving the logical expressions using Boolean laws.
2- It prevents the need to remember each and every Boolean algebraic theorem.
3- It involves fewer steps than the algebraic minimization technique to arrive at a simplified expression.
4- K-map simplification technique always results in minimum expression if carried out properly.

Disadvantages of K-Maps

1- As the number of variables in the logical expression increases, the K-map simplification process becomes complicated.
2- The minimum logical expression arrived by using the K-map simplification procedure may or may not be unique depending on the choices made while forming the groups

K-mapping & Minimization Steps

Step 1: generate K-map based on the number of input variables n

- Put a 1 in all specified minterms
- Put a 0 in all other boxes (optional)

Step 2: group all adjacent 1s without including any 0s. All groups must be rectangular and contain a "power-of-2" number of 1s 1, 2, 4, 8, 16, 32, …

Step 3: define product terms using variables common to all minterms in group

Step 4: sum all essential groups plus a minimal set of remaining groups to obtain a minimum SOP.

1- Two variables K-Map

Number of input variables are 2
Hence the number of squares = $2^n = 2^2 = 4$

Inputs A B	Decimal equivalent	Minterms		Output F
0 0	0	m_0	$\bar{A}\bar{B}$	
0 1	1	m_1	$\bar{A}B$	
1 0	2	m_2	$A\bar{B}$	
1 1	3	m_3	AB	

And K-Map of two variables is:

	$\bar{B}$ 0	B 1
$\bar{A}$ 0	0	1
A 1	2	3

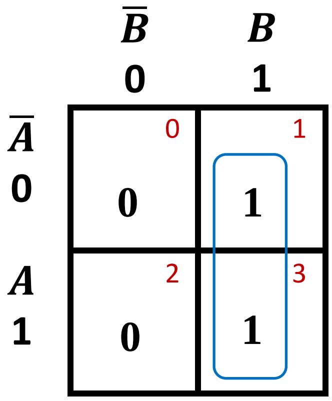
Example: simplify the Boolean expression by using K-Map

$$F = \bar{A}B + AB$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F = B$$

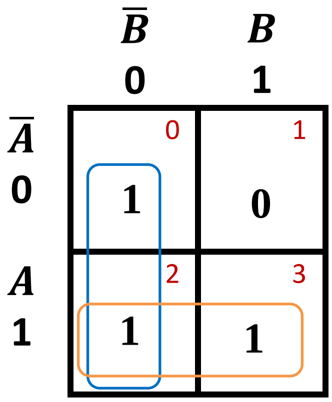
Example: simplify the Boolean expression by using K-Map

$$F(A,B) = \sum m(2,0,3)$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F(A,B) = \bar{B} + A$$

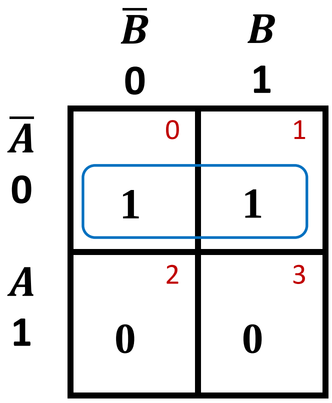
Example: simplify the Boolean expression by using K-Map

$$F = \overline{A}B + \overline{A}\overline{B}$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F = \overline{A}$$

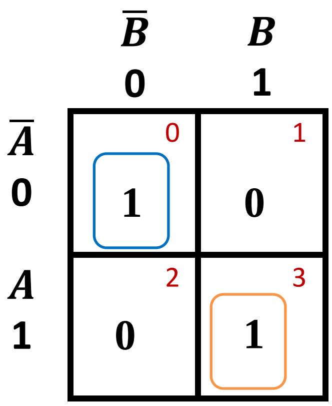
Example: simplify the Boolean expression by using K-Map

$$F(A,B) = \sum m(0,3)$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F(A,B) = \sum m(0,3) = \overline{A}\overline{B} + AB$$

2- Three Variables K-Map

Number of input variables are 3
Hence the number of squares = $2^n = 2^3 = 8$

The truth table is

Inputs A B C	Decimal equivalent	Minterms		Output F
0 0 0	0	m_0	$\bar{A}\bar{B}\bar{C}$	
0 0 1	1	m_1	$\bar{A}\bar{B}C$	
0 1 0	2	m_2	$\bar{A}B\bar{C}$	
0 1 1	3	m_3	$\bar{A}BC$	
1 0 0	4	m_4	$A\bar{B}\bar{C}$	
1 0 1	5	m_5	$A\bar{B}C$	
1 1 0	6	m_6	$AB\bar{C}$	
1 1 1	7	m_7	ABC	

And the K-Map of three variables is:

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	3	2
A 1	4	5	7	6

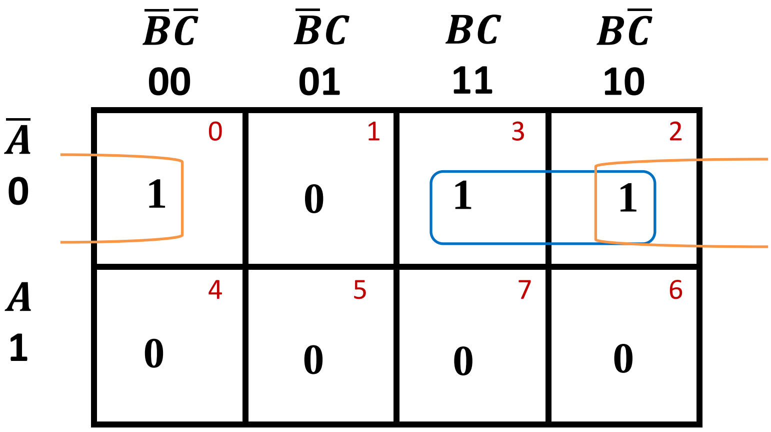
Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

Solution:

Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$



$$F(A,B,C) = \bar{A}\bar{C} + \bar{A}B$$

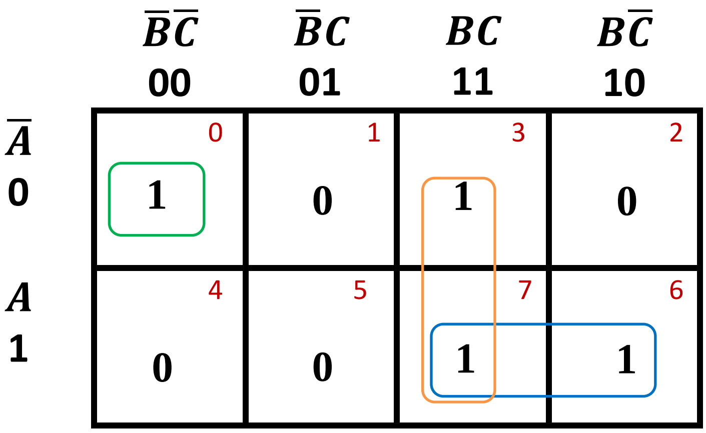
Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \sum m(0,3,7,6)$$

Solution:

Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$



$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + BC + AB$$

3- Four Variables K-map

Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

The truth table is

Inputs A B C D	Decimal equivalent	Minterms		Output F
0 0 0 0	0	m_0	$\bar{A}\bar{B}\bar{C}\bar{D}$	
0 0 0 1	1	m_1	$\bar{A}\bar{B}\bar{C}D$	
0 0 1 0	2	m_2	$\bar{A}\bar{B}C\bar{D}$	
0 0 1 1	3	m_3	$\bar{A}\bar{B}CD$	
0 1 0 0	4	m_4	$\bar{A}B\bar{C}\bar{D}$	
0 1 0 1	5	m_5	$\bar{A}B\bar{C}D$	
0 1 1 0	6	m_6	$\bar{A}BC\bar{D}$	
0 1 1 1	7	m_7	$\bar{A}BCD$	
1 0 0 0	8	m_8	$A\bar{B}\bar{C}\bar{D}$	
1 0 0 1	9	m_9	$A\bar{B}\bar{C}D$	
1 0 1 0	10	m_{10}	$A\bar{B}C\bar{D}$	
1 0 1 1	11	m_{11}	$A\bar{B}CD$	
1 1 0 0	12	m_{12}	$AB\bar{C}\bar{D}$	
1 1 0 1	13	m_{13}	$AB\bar{C}D$	
1 1 1 0	14	m_{14}	$ABC\bar{D}$	
1 1 1 1	15	m_{15}	$ABCD$	

And the K-Map of four variables is:

	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\bar{B}$ 10	8	9	11	10

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: 1	1: 0	3: 0	2: 1
$\bar{A}B$ 01	4: 0	5: 0	7: 1	6: 0
AB 11	12: 0	13: 1	15: 0	14: 0
$A\bar{B}$ 10	8: 1	9: 0	11: 0	10: 1

$$F(A,B,C,D) = \bar{B}\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \sum m(0,2,4,6,12,14,15,8,10)$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0: 1	1: 0	3: 0	2: 1
$\bar{A}B$ 01	4: 1	5: 0	7: 0	6: 1
AB 11	12: 1	13: 0	15: 1	14: 1
$A\bar{B}$ 10	8: 1	9: 0	11: 0	10: 1

$$F(A,B,C,D) = \bar{D} + ABC$$

K-Map with Don't Care Conditions

In certain cases some of the minterms may never occur or it may not matter what happens if they do

– In such cases we fill in the Karnaugh map with an X that meaning don't care

– When minimizing an X is like a "joker"

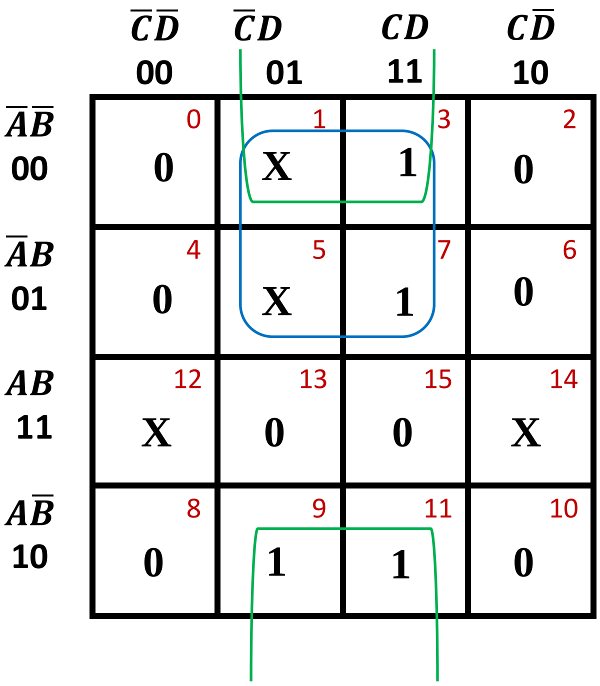
• X can be 0 or 1 - whatever helps best with the minimization

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \sum m(3,7,9,11) + \sum d(1,5,12,14)$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$



$$F(A,B,C,D) = \bar{A}D + \bar{B}D$$