B.Sc. Fourth Year – Laser Physics Department - Laser Design I-LECTURE ONE

The word 'laser is an acronym for 'Light Amplification by Stimulated Emission of Radiation'. Albert Einstein in 1917 showed that the process of stimulated emission must exist, but it was not until 1960 that T. H. Maiman first achieved laser action at optical frequencies in ruby.

The development of lasers since 1960 has been extremely rapid and although applications for lasers had a very slow start during their first decade, new applications for laser radiation are being found now almost every day.

1. Emission and absorption of radiation

when an electron in an atom undergoes transitions between two energy states or levels it either emits or absorbs a photon, which can be described in terms of a wave of frequency (ν) where $\boldsymbol{\nu = \Delta E/h}$, ΔE being the energy difference between the two levels concerned.

Let us consider the electron transitions which may occur between the two energy levels of the hypothetical atomic system shown in Fig. (1) below.

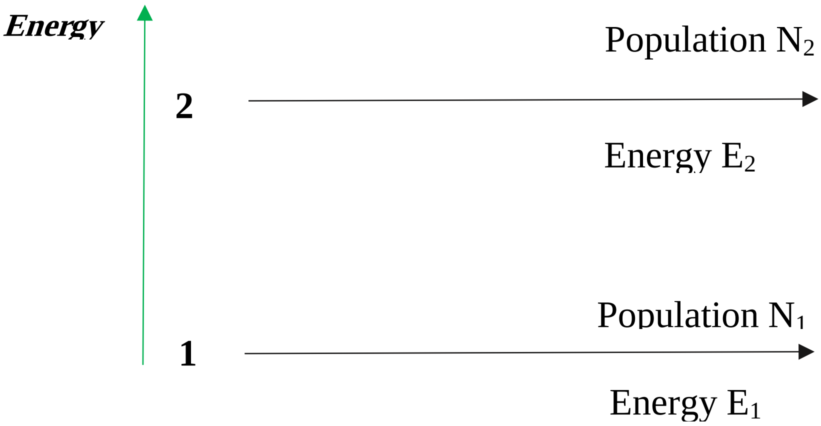

Figure (1). Two- energy level system

If the electron is in the lower level E_1 then in the presence of photons of energy ($E_2 - E_1$) it may be excited to the upper-level E_2 by absorbing a photon. Alternatively, if the electron is in the level E_2 it may return to the ground state with the emission of a photon. The emission process may occur in two distinct ways. These are:

(a) The *spontaneous emission* process in which the electron drops to the lower level in an entirely random way.

(b) The *stimulated emission* process in which the electron is 'triggered' to undergo the transition by the presence of photons of energy $(E_2 - E_1)$.

There is nothing mystical in this, as the electron would undergo this process sooner or later spontaneously; the transition is simply initiated by the presence of the stimulating photon. The absorption and emission processes are illustrated in Figs (2) (a), (b) and (c).

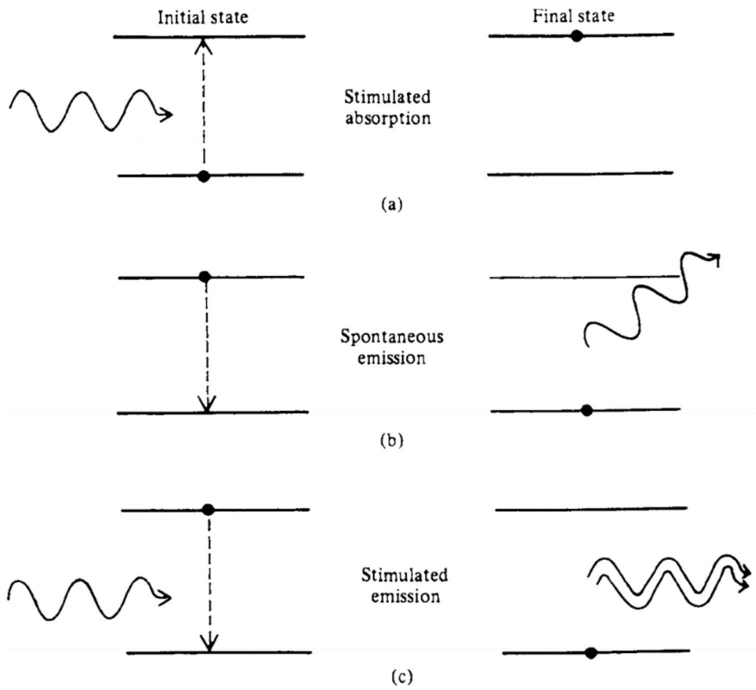

Figure (2). Energy level diagram illustrating (a) absorption, (b) spontaneous emission and (c) stimulated emission. The black dot indicates the state of the atom before and after the transition.

Under normal circumstances, we do not observe the stimulated process because the probability of the spontaneous process occurring is much higher. The average time the electron exists in the excited state before making a spontaneous transition is called the lifetime (τ_{21}) of the excited state. The '21' here indicates the energy levels involved. The probability that a particular atom will undergo spontaneous emission within a time interval (dt) is given by:

$$A_{21}\, dt = \frac{dt}{\tau_{21}} \quad \text{..................}\mathbf{(1)}$$

where (A_{21}) is the spontaneous transition rate.

Because the spontaneous radiation from any atom is emitted at random, the radiation emitted by a large number of atoms will clearly be incoherent. In contrast to this, the stimulated emission process results in coherent radiation since the waves associated with the stimulating and stimulated photons to have identical frequencies, are in phase, have the same state of polarization and travel in the same direction.

This means that with stimulated emission the amplitude of an incident wave can grow as it passes through a collection of excited atoms in what is clearly an amplification process.

As the absorption transition, in common with stimulated emission, can only occur in the presence of photons of appropriate energy, it is often referred to as stimulated absorption. These two processes may be regarded as the inverse of one another.

The above discussion ignores the fact that the emission and absorption processes do not simply involve photons of a precisely defined energy. Consequently, there is a range of frequencies associated with these processes.

2. Einstein relations

Einstein showed that the parameters describing the above three processes are related through the requirement that for a system in thermal equilibrium the rate of upward transitions (E_1 to E_2) must equal the rate of the downward transition processes (E_2 to E_1).

Let us suppose that our simple two-level atomic energy level system is in equilibrium inside a blackbody cavity. Although we indicated at the end of the previous section that the upward and downward transitions involve a spread of frequencies, we may assume that this will be very small compared with that of a blackbody.

Nevertheless, in further considering the transitions we must take into account the behaviour of photons within these frequency distributions.

If there are (N_1) atoms per unit volume in the collection with energy (E_1)

then the upward transition or absorption rate will be proportional to both N_1 and to the number of photons available at the correct frequency. Now (ρ_ν), the energy density at a frequency (ν) , is given by:

$$\rho_\nu = N\ h\nu \quad \ldots\ldots\ldots\ldots\ldots\ldots\textbf{(2)}$$

where (N) is the number of photons per unit volume having frequency ν. Therefore, we may write the upward transition rate as

Stimulated absorption rate = $N_1\ P_\nu\ B_{12}$, where (B_{12}) is a constant for a given pair of energy levels (the '12' indicates the energy levels involved)(transition from 2 to 1 level)

Similarly, if there are (N_2) atoms per unit volume in the collection with energy (E_2) then the induced transition rate from level 2 to level 1 is

Stimulated emission rate = $N_2\ P_\nu\ B_{12}$ **,** where again (B_{21}) is a constant.

The spontaneous transition rate depends on the average lifetime, τ_{21} , of the atoms in the excited state. The probability that a particular atom will undergo a spontaneous transition in a time dt is dt/τ_{21} , which equals A_{21} dt , where A_{21} is a constant. Thus as there are N_2 atoms in the upper level, then

Spontaneous transition rate = $N_2\ A_{21}$**.**

The total downward transition rate is the sum of the induced and spontaneous contributions, that is:

$$N_1\ \rho_\nu\ B_{12} = N_2\ \rho_\nu\ B_{21} + N_2\ A_{21}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(3)$$

(A_{21}), (B_{21}) and (B_{12}) are called the *Einstein coefficients*; the relationships between them can be established as follows.

For a system in equilibrium, the upward and downward transition rates must be equal and hence we have

Thus:

$$\rho_\nu = \frac{N_2\ A_{21}}{N_1\ B_{12} - N_2\ B_{21}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

or

$$\rho_\nu = \frac{\frac{A_{21}}{B_{21}}}{\left(\frac{B_{12}}{B_{21}}\right)\left(\frac{N_1}{N_2}\right) - 1} \quad \text{.................. (5)}$$

Now, the populations of the various energy levels of a system in thermal equilibrium are given by Boltzmann statistics to be

$$\frac{N_1}{N_2} = \exp\left[\frac{(E_2 - E_1)}{kT}\right] = \exp\left(\frac{h\nu}{kT}\right) \quad \text{.................. (6)}$$

Therefore, substituting eq.(1.4) into Eq. (1.2) gives

$$\rho_\nu = \frac{\frac{A_{21}}{B_{21}}}{\left[\left(\frac{B_{12}}{B_{21}}\right)\exp\left(\frac{h\nu}{kT}\right)\right] - 1} \quad \text{....................... (7)}$$

Since the collection of atoms in the system we are considering is in thermal equilibrium it must give rise to radiation which is identical with blackbody radiation, the radiation density of which can be described by

$$\rho_\nu = \frac{8\ \pi\ h\ \nu^3}{c^3}\left(\frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right) \quad \text{............................. (8)}$$

Comparing Eqs. (5) and (6) for ($\rho\nu$), we see that

$B_{12} = B_{21}$ =B .... (9)

and

$$\frac{A_{21}}{B_{21}} = \frac{8\ \pi\ h\ \nu^3}{c^3} \quad \text{.. (10)}$$

Equations (7) and (8) are referred to as the *Einstein relations*. The second relation enables us to evaluate the ratio of the rate of spontaneous emission to the rate of stimulated emission for a given pair of energy levels. We see that this ratio is given by

$$R = \frac{N_2\ A_{21}}{N_2\ \rho_\nu\ B_{21}} = \frac{8\pi\ h\ \nu^3}{\rho_\nu\ c^3} \quad \text{.................................... (11)}$$

or

$$R = \exp\left(\frac{h\,\nu}{kT}\right) - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)}$$

The above discussion indicates that the process of stimulated emission competes with the processes of spontaneous emission and absorption. Clearly, if we wish to amplify a beam of light by stimulated emission then we must increase the rate of this process in relation to the other two processes.

for a given pair of energy levels we must increase both the radiation density and the population density (N_2) of the upper level in relation to the population density (N_1) of the lower level. Indeed, we shall show that to produce laser action we must create a condition in which $\mathbf{N_2 > (g_2/g_1)N_1}$ even though $\mathbf{E_2 > E_1}$, that is we must create a so-called population inversion. Before describing this situation in detail, it will be instructive to look more closely at the process of absorption.