

University of Babylon
College of Information Technology
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Operations Research

Lecture-8

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Least Cost Method

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Step 1: Balance the problem

Step 2: Select the lowest cost from the entire matrix and allocate the minimum of supply or demand.

Step 3: Remove the row or column whose supply or demand is satisfied and prepare a new matrix.

Step 4: Repeat the procedure until all the allocations are over.

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Least Cost Method-Example 1

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	D 1	D 2	D 3	D 4	Supply
S 1	3	1	7	4	300
S 2	2	6	5	9	400
S 3	8	3	3	2	500
Demand	250	350	400	200	1200

Least Cost Method-Example 1

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- **Step 1:** The least cost is 1 in cell(S1,D2)
- Allocate the smaller value to the cell (S1,D2) which is 300
- Remaining supply for S1=300-300=0
- Remaining demand for D2= 350-300=50
- Cancel this row

	D 1	D 2	D 3	D 4	Supply
S 1	3	300	7	4	300-300=0
S 2	2	6	5	9	400
S 3	8	3	3	2	500
Demand	250	350-300=50	400	200	1200

Least Cost Method-Example 1

- **Step 2:** The least cost is 2 in cell (S2,D1) and cell (S3,D4) ,lets select cell (S2,D1) .
- Allocate the smaller value to the cell (S2,D1) which is 250
- Remaining supply for S2=400-250=150
- Remaining demand for D1= 250-250=0
- Cancel this column

	D 1	D 2	D 3	D 4	Supply	
S 1	3	300	1	7	4	300-300=0
S 2	250	2	6	5	9	400-250=150
S 3	8	3	3	2	500	
Demand	250-250=0	350-300=50	400	200	1200	

Least Cost Method-Example 1

- **Step3:** The least cost is 2 in cell (S3,D4).
- Allocate the smaller value to the cell (S3,D4) which is 200
- Remaining supply for S3=500-200=300
- Remaining demand for D1= 200-200=0
- Cancel this column

	D 1	D 2	D 3	D 4	Supply
S 1	3	300 1	7	4	$300-300=0$
S 2	250 2	6	5	9	$400-250=150$
S 3	8	3	3	200 2	$500-200=300$
Demand	$250-250=0$	$350-300=50$	400	$200-200=0$	1200

Least Cost Method-Example 1

- **Step4:** The least cost is 3 in cell (S3,D2) and cell (S3,D3) ,lets select cell (S3,D2) .
- Allocate the smaller value to the cell (S3,D4) which is 50
- Remaining supply for S3=300-50=250
- Remaining demand for D1= 50-50=0
- Cancel this column

	D 1	D 2	D 3	D 4	Supply
S 1	3	300 1	7	4	$300-300=0$
S 2	250 2	6	5	9	$400-250=150$
S 3	8	50 3	3	200 2	$300-50=250$
Demand	$250-250=0$	$50-50=0$	400	$200-200=0$	1200

Least Cost Method-Example 1

- **Step 5:** The least cost is 3 in cell (S3,D3) .
- Allocate the smaller value to the cell (S3,D4) which is 250
- Remaining supply for S3=250-250=0
- Remaining demand for D3= 400-250=150
- Cancel this row

	D 1	D 2	D 3	D 4	Supply		
S 1	3	300	1	7	4	300-300=0	
S 2	250	2	6	5	9	400-250=150	
S 3	8	50	3	250	3	200	250-250=0
Demand	250-250=0	50-50=0	400-250=150	200-200=0	2	1200	

Least Cost Method-Example 1

- **Step 6:** The only remaining cost is 5 in cell (S2,D3) .
- Supply and demand both are equal \rightarrow 150
- Remaining supply for S2=150-150=0
- Remaining demand for D3= 150-150=0

	D 1	D 2	D 3	D 4	Supply			
S 1	3	300	1	7	4	300-300=0		
S 2	250	2	6	150	5	9	150-150=0	
S 3	8	50	3	250	3	200	2	250-250=0
Demand	250-250=0	50-50=0	150-150=0	200-200=0	1200			

Least Cost Method-Example 1

Step 7:,

the total cost =

$$(300 \times 1) + (250 \times 2) + (150 \times 5) + (50 \times 3) + (250 \times 3) + (200 \times 2) = 2850$$

	D 1	D 2	D 3	D 4	Supply
S 1	3	300 1	7	4	300-300=0
S 2	250 2	6	150 5	9	150-150=0
S 3	8	50 3	250 3	200 2	250-250=0
Demand	250-250=0	50-50=0	150-150=0	200-200=0	

Vogel's Approximation Method (VAM)

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Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's $\min(\text{supply}, \text{demand})$. If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Vogel's Approximation Method (VAM)

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Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Vogel's Approximation Method (VAM) -Example

- **Total supply=Total demand=135, the problem is balanced.**

Factories	Destination centers				Supply
	D ₁	D ₂	D ₃	D ₄	
F ₁	3	2	7	6	50
F ₂	7	5	2	3	60
F ₃	2	5	4	5	25
Demand	60	40	20	15	

Vogel's Approximation Method (VAM) -Example

- **Step1:** Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	3	2	7	6	50	1
F_2	7	5	2	3	60	1
F_3	2	5	4	5	25	2
Demand	60	40	20	15		
Column difference	1	3	2	2		

Vogel's Approximation Method (VAM) -Example

- **Step2:** Identify maximum penalty $\rightarrow 3$
- Select the least cost in corresponding column $\rightarrow 2$
- $\text{Min}(\text{supply, demand}) = \text{min}(50, 40) = 40$.
- Allocate 40 in that cell and strike the column .

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	3	40 2	7	6	$50 - 40 = 10$	1
F_2	7	5	2	3	60	1
F_3	2	5	4	5	25	2
Demand	60	$40 - 40 = 0$	20	15		
Column difference	1	3	2	2		

Vogel's Approximation Method (VAM) -Example

- **Step3: find the absolute row and column differences for the remaining . Then repeat step 2.**
- **Identify maximum penalty $\rightarrow 3$**
- **Select the least cost in corresponding row $\rightarrow 3$**
- **Min(supply, demand)=min(10,60)=10**
- **Allocate 10 in that cell and strike the row .**

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	10 3	40 2	7	6	10-10=0	1 3
F_2	7	5	2	3	60	1 1
F_3	2	5	4	5	25	2 2
Demand	60-10=50	0	20	15		
Column difference	1	3	2	2		
	1	-	2	2		

Vogel's Approximation Method (VAM) -Example

- **Step4: repeat the above steps.**
- Identify maximum penalty $\rightarrow 5$
- Select the least cost in corresponding column $\rightarrow 2$
- $\text{Min}(\text{supply, demand}) = \min(25, 50) = 25$
- Allocate 25 in that cell and strike the row.

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	10 3	40 2	7	6	0	1 3 -
F_2	7	5	2	3	60	1 1 1
F_3	25 2	5	4	5	25-25=0	2 2 2
Demand	50-25=25	0	20	15		
Column difference	1	3	2	2		
	1	-	2	2		
	5	-	2	2		

Vogel's Approximation Method (VAM) -Example

- **Step5: repeat the above steps.**
- Identify maximum penalty $\rightarrow 7$
- Select the least cost in corresponding column $\rightarrow 7$
- $\text{Min}(\text{supply, demand}) = \min(60, 25) = 25$
- Allocate 25 in that cell and strike the column.

	D_1	D_2	D_3	D_4	Supply	Row difference
F_1	10 3	40 2	7	6	0	1 3 - -
F_2	25 7	5	2	3	60-25=35	1 1 1 1
F_3	25 2	5	4	5	0	2 2 - -
Demand	25-25=0	0	20	15		
Column difference	1	3	2	2		
	1	-	2	2		
	5	-	2	2		
	7	-	2	3		

Vogel's Approximation Method (VAM) -Example

- **Step 6: repeat the above steps.**
- **Identify maximum penalty $\rightarrow 3$**
- **Select the least cost in corresponding column $\rightarrow 3$**
- **Min(supply, demand) = $\min(35, 15) = 15$**
- **Allocate 15 in that cell and strike the column.**

	D ₁	D ₂	D ₃	D ₄	Supply	Row difference
F ₁	10 3	40 2	7	6	0	1 3 - - -
F ₂	25 7	5	2	15 3	35-15=20	1 1 1 1 1
F ₃	25 2	5	4	5	0	2 2 - - -
Demand	25-25=0	0	20	15-15=0		
Column difference	1	3	2	2		
	1	-	2	2		
	5	-	2	2		
	7	-	2	3		
	-	-	2	3		

Vogel's Approximation Method (VAM) -Example

- **Step 7: every row and column vanishes.**
- **Now allocate the remaining supply or demand value for that cell.**
- **Total cost= $(10 \times 3) + (40 \times 2) + (25 \times 7) + (20 \times 2) + (15 \times 3) + (25 \times 2) = 420$**

	D ₁	D ₂	D ₃	D ₄	Supply	Row difference
F ₁	10 3	40 2	7	6	0	1 3 - - -
F ₂	25 7	5	20 2	15 3	20-20=0	1 1 1 1 1
F ₃	25 2	5	4	5	0	2 2 - - -
Demand	25-25=0	0	20-20=0	0		
Column difference	1	3	2	2		
	1	-	2	2		
	5	-	2	2		
	7	-	2	3		
	-	-	2	3		

Vogel's Approximation Method (VAM) –Home Work

- **Solve this transportation problem by using VAM and find the total cost.**

	D 1	D 2	D 3	Supply
S 1	2	5	9	20
S 2	3	6	11	15
S 3	4	8	13	25
Demand	10	25	25	

The background features three vertical bars on the left: a wide light pink bar, a narrower teal bar, and a narrow light beige bar. The right side of the image is white with two rectangular areas of a light pink dot grid pattern, one in the top right and one in the bottom right.

THANK YOU