1. Find ∇f for $f(x,y,z) = x^3y - y^2z + 3xz^2$. **1**

2. Find Jacobian matrix at (0,1,1) for $u = \sin 2x\,, v = 3y^2z,\ \ w = 2yz^2$.

3. Evaluate $\int_0^2 \int_0^1 (2x + 3y^2)dxdy$

4. Evaluate $\int_0^1 8t\left(1 - \sqrt{t}\right)^4 dt$

1. Find ∇f for $f(x,y,z) = 3x^2z + xy^2 - 2yz^3$. **2**

2. Find Jacobian matrix at (1,0,1) for $u = 2x^2y, v = \sin 3y\,,\ \ w = 3xz^2$.

3. Evaluate $\int_0^2 \int_0^1 (2y + 3x^2)dydx$

4. Evaluate $\int_0^1 8y\left(1 - \sqrt{y}\right)^4 dy$

1. Find ∇f for $f(x,y,z) = x^2y^2 + 3y^2z - 2xz^3$. **3**

2. Find Jacobian matrix at (1,0,1) for $u = 3x^2y, v = \sin 2y\,,\ \ w = 2xz^2$.

3. Evaluate $\int_0^2 \int_0^1 (3x^2 + 2y)dxdy$

4. Evaluate $\int_0^1 7y\left(1 - \sqrt{y}\right)^4 dy$

1. Find ∇f for $f(x,y,z) = 3x^2y - xy^2 + 2yz^3$. **4**

2. Find Jacobian matrix at (1,1,0) for $u = 2x^2y, v = 3xy^2,\ \ w = \sin 3z$.

3. Evaluate $\int_0^2 \int_0^1 (3y^2 + 2x)dydx$

4. Evaluate $\int_0^1 7t\left(1 - \sqrt{t}\right)^4 dt$

1. Find ∇f for $f(x,y,z) = x^3y - y^2z + 3xz^2$. **1**

$$\nabla f = f_x\, i + f_y\, j + f_z\, k = (3x^2y + 3z^2)\, i + (x^3 - 2yz)\, j + (-y^2 + 6xz)\, k$$

2. Find Jacobian matrix at $(0,1,1)$ for $u = \sin 2x\,, v = 3y^2z,\ \ w = 2yz^2$.

$$\mathrm{J} = \begin{bmatrix} 2\cos 2x & 0 & 0 \\ 0 & 6yz & 3y^2 \\ 0 & 2z^2 & 4yz \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 3 \\ 0 & 2 & 4 \end{bmatrix}$$

3. $$\int_0^2 \int_0^1 (2x + 3y^2)dxdy = \int_0^2 (x^2 + 3y^2x)\Big|_0^1 dy = \int_0^2 (1 + 3y^2)\, dy = y + y^3\Big|_0^2 = 10$$

4. Let $x = \sqrt{t}$ ⇨ $t = x^2$ ⇨ $dt = 2xdx$

$$\int_0^1 8t\left(1 - \sqrt{t}\right)^4 dt = \int_0^1 16x^3\,(1 - x)^4 dx = 16B(4,5)$$

$$= 16 \times \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)} = 16 \times \frac{3! \times 4!}{8!} = \frac{16 \times 3 \times 2 \times 4!}{8 \times 7 \times 6 \times 5 \times 4!} = \frac{2}{35}$$

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1. Find ∇f for $f(x,y,z) = 3x^2z + xy^2 - 2yz^3$. **2**

$$\nabla f = f_x\, i + f_y\, j + f_z\, k = (6xz + y^2)\, i + (2xy - 2z^3)\, j + (3x^2 - 6yz^2)\, k$$

2. Find Jacobian matrix at $(1,0,1)$ for $u = 2x^2y, v = \sin 3y\,,\ \ w = 3xz^2$.

$$\mathrm{J} = \begin{bmatrix} 4xy & 2x^2 & 0 \\ 0 & 3\cos 3y & 0 \\ 3z^2 & 0 & 6xz \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 3 & 0 \\ 3 & 0 & 6 \end{bmatrix}$$

3. $$\int_0^2 \int_0^1 (2y + 3x^2)dydx = \int_0^2 (y^2 + 3x^2y)\Big|_0^1 dx = \int_0^2 (1 + 3x^2)\, dx = x + x^3\Big|_0^2 = 10$$

4. Let $x = \sqrt{y}$ ⇨ $y = x^2$ ⇨ $dy = 2xdx$

$$\int_0^1 8y\left(1 - \sqrt{y}\right)^4 dy = \int_0^1 16x^3\,(1 - x)^4 dx = 16B(4,5)$$

$$= 16 \times \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)} = 16 \times \frac{3! \times 4!}{8!} = \frac{16 \times 3 \times 2 \times 4!}{8 \times 7 \times 6 \times 5 \times 4!} = \frac{2}{35}$$

1. Find ∇f for $f(x,y,z) = x^2y^2 + 3y^2z - 2xz^3$. 3

$$\nabla f = f_x\, i + f_y\, j + f_z\, k = (2xy^2 - 2z^3)\, i + (2x^2y + 6yz)\, j + (3y^2 - 6xz^2)\, k$$

2. Find Jacobian matrix at (1,0,1) for $u = 3x^2y, v = \sin 2y, \ w = 2xz^2$.

$$\mathrm{J} = \begin{bmatrix} 6xy^2 & 3x^2 & 0 \\ 0 & 2\cos 2y & 0 \\ 2z^2 & 0 & 4xz \end{bmatrix} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

3. $$\int_0^2 \int_0^1 (3x^2 + 2y)dxdy = \int_0^2 (x^3 + 2yx)\Big|_0^1 dy = \int_0^2 (1 + 2y)\, dy = y + y^2\Big|_0^2 = 6$$

4. Let $x = \sqrt{y} \quad \Rightarrow \quad y = x^2 \quad \Rightarrow \quad dy = 2xdx$

$$\int_0^1 7y\left(1 - \sqrt{y}\right)^4 dy = \int_0^1 14x^3\,(1-x)^4 dx = 14B(4,5)$$

$$= 14 \times \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)} = 14 \times \frac{3! \times 4!}{8!} = \frac{14 \times 3 \times 2 \times 4!}{8 \times 7 \times 6 \times 5 \times 4!} = \frac{1}{20}$$

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1. Find ∇f for $f(x,y,z) = 3x^2y - xy^2 + 2yz^3$. 4

$$\nabla f = f_x\, i + f_y\, j + f_z\, k = (6xy - y^2)\, i + (3x^2 - 2xy + 2z^3)\, j + 6yz^2\, k$$

2. Find Jacobian matrix at (1,1,0) for $u = 2x^2y, v = 3xy^2, \ w = \sin 3z$.

$$\mathrm{J} = \begin{bmatrix} 4xy & 2x^2 & 0 \\ 3y^2 & 6xy & 0 \\ 0 & 0 & 3\cos 3z \end{bmatrix} = \begin{bmatrix} 4 & 2 & 0 \\ 3 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3. $$\int_0^2 \int_0^1 (3y^2 + 2x)dydx = \int_0^2 (y^3 + 2xy)\Big|_0^1 dx = \int_0^2 (1 + 2x)\, dx = x + x^2\Big|_0^2 = 6$$

4. Let $x = \sqrt{t} \quad \Rightarrow \quad t = x^2 \quad \Rightarrow \quad dt = 2xdx$

$$\int_0^1 7t\left(1 - \sqrt{t}\right)^4 dt = \int_0^1 14x^3\,(1-x)^4 dx = 14B(4,5)$$

$$= 14 \times \frac{\Gamma(4)\Gamma(5)}{\Gamma(9)} = 14 \times \frac{3! \times 4!}{8!} = \frac{14 \times 3 \times 2 \times 4!}{8 \times 7 \times 6 \times 5 \times 4!} = \frac{1}{20}$$