Binomial Distribution (توزيع ذي الحدين)

The binomial distribution is one of the most common probability distributions in applied statistics. It is used when a random process or experiment yields only one of two mutually exclusive outcomes. This distribution is used for n −repeated experiments, where the outcomes are classified into two categories: success (p), which is the probability of the event occurring, and failure (q), which is the probability of it not occurring, such that:

$p + q = 1.$

The probability of getting exactly x successes in n trials is as follows:

$$P(x) = \binom{n}{x} p^x q^{n-x} \quad ; \quad x = 0,1,2,\cdots,n$$

Example 1: Suppose 10% of a given population is colorblind. If a random sample of 25 people is taken from this population, find the probability that:

(a) Three people are colorblind.

(b) Two or fewer will be colorblind.

(c) Two or more will be colorblind.

(d) Two, three, or four will be colorblind.

Solution: $p = 10\% = 0.1,\ q = 1 - p = 0.9,\ n = 25$

(a) $P(x = 3) = \binom{25}{3} (0.1)^3 (0.9)^{22}$

$$= \frac{25!}{22! \times 3!} \times (0.1)^3 (0.9)^{22}$$

$$= \frac{25 \times 24 \times 23 \times 22!}{22! \times 3 \times 2} \times 0.001 \times 0.0985 = 0.2265$$

(b) $P(x \le 2) = P(x = 2) + P(x = 1) + P(x = 0)$

$$= 0.2658 + 0.1995 + 0.0718 = 0.5371$$

(c) $P(x \ge 2) = 1 - P(x \le 1) = 1 - (0.1995 + 0.0718) = 0.7287$

(d) $P(2 \le x \le 4) = P(x = 2) + P(x = 3) + P(x = 4)$

$$= 0.2658 + 0.2265 + 0.1384 = 0.6307$$

Mean and Standard Deviation of Binomial Distribution

For a Binomial distribution, μ, the expected number of successes, σ^2, the variance, and σ, the standard deviation for the number of success are given by the formulas:

$$\mu = np, \quad \sigma^2 = npq, \quad \sigma = \sqrt{npq}$$

Example 2: When looking at a person's eye color, it turns out that 1.5% of people in the world has green eyes. Find $C.V$ for a group of 20 people.

Solution: $p = 0.015, \ \ q = 1 - 0.015 = 0.985, \ \ n = 20$

$$\mu = np = 20 \times 0.015 = 0.3$$

$$\sigma^2 = npq = 0.3 \times 0.985 = 0.2955$$

$$C.V = \frac{\sigma}{\mu} \times 100\% = \frac{\sqrt{0.2955}}{0.3} \times 100\% = 181.2\%$$

Example 3: Approximately 10% of all people are left-handed. Consider a grouping of fifteen people. Find the mean, and the standard deviation.

Solution: $p = 0.1, \ \ q = 1 - 0.1 = 0.9, \ \ n = 15$

$$\mu = np = 15 \times 0.1 = 1.5$$

$$\sigma = \sqrt{npq} = \sqrt{15 \times 0.1 \times 0.9} = 1.16$$

H.W

1. 24% of patients hospitalized with acute myocardial infarction (MI) had not completed their cardiac medication regimen by day 7 after discharge. Find $C.V$ for 12 people were hospitalized with acute myocardial infarction.
2. A company manufactures eyeglasses. It tested the number of defective lenses it manufactures. The percentage of defective lenses due to scratches was 16.9%. Suppose 25 pairs of eyeglasses were tested. Calculate the coefficient of variation.

Poisson Distribution

The next discrete distribution we will study is the Poisson distribution. This distribution has been widely used as a probability model in biology and medicine.

If x is the number of times a random event occurs in a given time or space (or volume of matter), the probability of x occurring is given by the formula

$$P(x) = \frac{e^{-\lambda}\lambda^{x}}{x!},$$

where $\lambda > 0$ is called the parameter of the distribution.

An interesting feature of the Poisson distribution is the fact that the mean and variance are equal to λ.

Example 4: Births in a hospital occur randomly with a mean 1.8 births per hour.

1. What is the probability of observing 4 births in a given hour at the hospital?
2. What about the probability of observing more than or equal to 2 births in a given hour at the hospital?

Solution: mean $= \lambda = 1.8$

1. $P(x = 4) = \dfrac{e^{-1.8} \times 1.8^{4}}{4!} = \dfrac{0.1653 \times 10.4976}{4 \times 3 \times 2} = 0.0723$
2. $P(x \geq 2) = 1 - P(x < 2) = 1 - \big(P(x = 1) + P(x = 0)\big)$

$$= 1 - \left(\frac{e^{-1.8} \times 1.8^{1}}{1!} + \frac{e^{-1.8} \times 1.8^{0}}{0!}\right)$$

$$= 1 - (0.1653 \times 1.8 + 0.1653) = 0.5372$$

Example 5: A random variable has a Poisson distribution such that

$P(x = 1) = 0.2P(x = 2)$. Find $P(x = 3)$.

Solution:

$$\frac{e^{-\lambda} \times \lambda^{1}}{1!} = 0.2 \times \frac{e^{-\lambda} \times \lambda^{2}}{2!} \quad \Rightarrow \quad \lambda = 10$$

$$P(x = 3) = \frac{e^{-10} \times 10^{3}}{3!} = 0.0076$$

H.W.

In a certain population an average of 13 new cases of esophageal cancer are diagnosed each year. If the annual incidence of esophageal cancer follows a Poisson distribution, find the probability that in a given year the number of newly diagnosed cases of esophageal cancer will be:

(a) Exactly 10

(b) No more than 2

(c) At least 2

(d) Between 1 and 3

(e) Fewer than 3