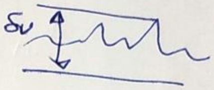
E8

shear velocity $\boxed{u_\tau = \sqrt{\dfrac{\tau_o}{\rho}}}$

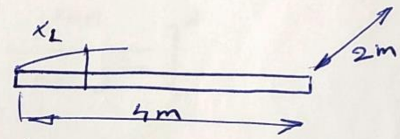
thickness of fluctuating (wall layer thickness) $\boxed{\delta_v = \dfrac{5\nu}{u_\tau}}$



Ex1

Air flows over a smooth, rigid 2 x 4 wide and long flat plate by 10 m/s. How far from the beginning of the plate will be laminar portion of the Boundary layer & calculate the drag force ? Check the B.L thickness ?

Sol.



Laminar flow portion at

$Re = 5*10^5$

$\nu_{air} = 1.51*10^{-5}\ m^2/s$
$\rho_{air} = 1.2\ kg/m^3$

$$\therefore Re = \frac{V x_L}{\nu} \Rightarrow 5*10^5 = \frac{10*x}{1.51*10^{-5}} \Rightarrow \boxed{x = 0.755\ m}$$

$$F_D = \frac{0.646*\rho U^2 x_L}{Re^{0.5}}$$

$$= \frac{0.646*1.2*10^2*0.755}{(5*10^5)^{0.5}} = 0.083\ N \text{ per width}$$

$$\therefore F_D = 0.083*2 = \boxed{0.166\ N}$$

$$\frac{\delta}{x} = \frac{4.64}{Re^{.5}} \Rightarrow \delta = \frac{4.64*0.755}{(5.10^5)^{0.5}} = 4.95*10^{-3}\ m$$

$$\therefore \boxed{\delta = 4.95\ mm}$$

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(1)

In general case $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$ let $\frac{1}{n} = m$

$$\therefore \frac{u}{U} = \left(\frac{y}{\delta}\right)^{m}$$

$$\tau_o = \rho U^2 \frac{d}{dx} \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{m} \quad \text{and} \quad 1 - \frac{u}{U} = 1 - \left(\frac{y}{\delta}\right)^{m}$$

$$\therefore \frac{u}{U}\left(1 - \frac{u}{U}\right) = \left(\frac{y}{\delta}\right)^{m} - \left(\frac{y}{\delta}\right)^{2m}$$

$$\therefore \tau_o = \rho U^2 \frac{d}{dx} \int_0^{\delta} \left(\frac{y}{\delta}\right)^{m} - \left(\frac{y}{\delta}\right)^{2m} dy$$

$$= \rho U^2 \frac{d}{dx} \left[\frac{y^{m+1}}{(m+1)\delta^m} - \frac{y^{2m+1}}{(2m+1)\delta^{2m}} \right]_0^{\delta}$$

$$= \rho U^2 \frac{d}{dx} \left[\frac{\cancel{\delta^{m}} \cdot \delta}{(m+1)\cancel{\delta^{m}}} - \frac{\cancel{\delta^{2m}}\,\delta}{(2m+1)\cancel{\delta^{2m}}} \right]$$

$$= \rho U^2 \frac{d\delta}{dx} \left[\frac{1}{m+1} - \frac{1}{2m+1} \right] = \rho U^2 \frac{d\delta}{dx} \underbrace{\left[\frac{(2m+1)-(m+1)}{(m+1)(2m+1)} \right]}$$

$$\frac{2m \cancel{+1} - m \cancel{-1}}{2m^2 + 3m + 1}$$

$$\therefore \tau_o = \rho U^2 \frac{d\delta}{dx} \frac{m}{2m^2 + 3m + 1}$$

or $$\boxed{\tau_o = \frac{m}{2m^2 + 3m + 1} \rho U^2 \frac{d\delta}{dx}}$$

$$\tau_o = 0.023 \rho U^2 \frac{1}{Re_{\delta}^{0.25}}$$ Blasius Equation

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(2)

$$\therefore\ 0.023\,\cancel{\rho}U^2\,\frac{1}{Re_\delta^{0.25}} = \frac{m}{2m^2+3m+1}\,\cancel{\rho}U^2\,\frac{d\delta}{dx}$$

$$\delta^{0.25}\,d\delta = \frac{0.023\,(2m^2+3m+1)}{m} * \left(\frac{U}{\nu}\right)^{-0.25} dx$$

$$\frac{\delta^{1.25}}{1.25} = \frac{0.023\,(2m^2+3m+1)}{m}\left(\frac{U}{\nu}\right)^{-0.25} x \quad * \frac{x^{1/4}}{x^{1/4}}$$

$$\frac{\delta^{1.25}}{x^{1.25}} = \frac{0.023 * 1.25\,(2m^2+3m+1)}{m} * \frac{1}{\left(\frac{U}{\nu}\right)^{0.25} \cdot x^{0.25}}$$

$$\boxed{\frac{\delta}{x} = 0.0585\left(\frac{2m^2+3m+1}{m}\right)^{0.8} * \frac{1}{Re_x^{0.2}}}$$

To check let $n=7 \Rightarrow m = 1/7$

$$\frac{2m^2+3m+1}{m} \Rightarrow \therefore \frac{2(\frac{1}{7})^2+3(\frac{1}{7})+1}{\frac{1}{7}} = \frac{\frac{2}{49}+\frac{3}{7}+\frac{7}{7}}{\frac{1}{7}} = \frac{\frac{72}{49}}{\frac{1}{7}} = \frac{72}{7}$$

$$\frac{\delta}{x} = 0.0585 * \left(\frac{72}{7}\right)^{0.8}\frac{1}{Re_x^{0.2}}$$

$$\boxed{\frac{\delta}{x} = \frac{0.38}{Re_x^{0.2}}}$$

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(3)

$$\therefore \tau_o = \frac{m \ \rho U^2}{2m^2+3m+1} \frac{d}{dx}\left[0.0585\left(\frac{2m^2+3m+1}{m}\right)^{0.8} \frac{x}{Re_x^{0.2}}\right]$$

$$= 0.0585 \, \rho U^2 \left(\frac{m}{2m^2+3m+1}\right)^{0.2} * 0.8 \frac{1}{Re_x^{0.2}} \qquad \left(\frac{U}{\nu}\right)^{0.2} \cdot x^{0.2}$$

$$\boxed{\tau_o = 0.047 \, \rho U^2 \left(\frac{m}{2m^2+3m+1}\right)^{0.2} \frac{1}{Re_x^{0.2}}} = \tau_w \quad \text{wall shear stress}$$

To check $m = \frac{1}{7}$

$$\frac{m}{2m^2+3m+1} = \frac{7}{72} \qquad \therefore 0.047 * \left(\frac{7}{72}\right)^{0.2} = 0.03$$

$$\boxed{\therefore \tau_o = 0.03 \, \rho U^2 \frac{1}{Re_x^{0.2}}} \quad \text{same.}$$

$$c_{fl} = \frac{\tau_o}{\frac{1}{2}\rho U^2} \Rightarrow \boxed{c_{fs} = 0.094 \left(\frac{m}{2m^2+3m+1}\right)^{0.2} \frac{1}{Re_x^{0.2}}}$$

check again $0.094 * \left(\frac{7}{72}\right)^{0.2} = 0.059$

$\therefore$

$$c_{fs} = \frac{1}{L}\int_0^L c_{fl}\,dx = \frac{1}{L} 0.094 \left(\frac{m}{2m^2+3m+1}\right)^{0.2} \frac{1}{\left(\frac{U}{\nu}\right)^{0.2}} \int_0^L x^{-0.2}\,dx$$

$$\Downarrow \quad \frac{L^{0.8}}{0.8} * \frac{L^{0.2}}{L^{0.2}}$$

$=$

$$\boxed{c_{fs} = 0.118 \left(\frac{m}{2m^2+3m+1}\right)^{0.2} \frac{1}{Re_L^{0.2}}} \quad \simeq 0.074$$

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L13 (4)

Displacement thickness

$$\delta d = \int_0^\delta 1-\frac{u}{U}\,dy = \int_0^\delta 1-\left(\frac{y}{\delta}\right)^m dy = y-\frac{y^{m+1}}{(m+1)\,\delta^m}\Big]_0^\delta$$

$$= \delta - \frac{\cancel{\delta^m}\cdot\delta}{(m+1)\cancel{\delta^m}} = \delta\left[1-\frac{1}{m+1}\right] = \delta\left[\frac{m+1-1}{m+1}\right] =$$

$$\therefore \boxed{\delta d = \left(\frac{m}{m+1}\right)\delta} \quad \text{or} \quad \boxed{\frac{\delta d}{\delta} = \frac{m}{m+1}}$$

Momentum Thickness

$$\theta d = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy$$

$$\theta d$$

remember $\tau_o = \rho U^2 \frac{d}{dx}\boxed{\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy}$

$$= \rho U^2 \frac{d\theta d}{dx}$$

But $\tau_o = \frac{m}{2m^2+3m+1}\,\rho U^2 \frac{d\delta}{dx}$

$$\therefore \theta d = \frac{m}{2m^2+3m+1}\,\delta \quad \text{or} \quad \boxed{\frac{\theta d}{\delta} = \frac{m}{2m^2+3m+1}}$$

Energy Thickness

$$\delta e = \int_0^\delta \frac{u}{U}\left(1-\left(\frac{u}{U}\right)^2\right) dy$$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^m \Rightarrow \left(\frac{u}{U}\right)^2 = \left(\frac{y}{\delta}\right)^{2m}$$

$$\therefore \frac{u}{U}\left[1-\left(\frac{u}{U}\right)^2\right] = \left(\frac{y}{\delta}\right)^m\left[1-\left(\frac{y}{\delta}\right)^{2m}\right] = \left(\frac{y}{\delta}\right)^m - \left(\frac{y}{\delta}\right)^{3m}$$

$$\therefore \delta e = \frac{y^{m+1}}{(m+1)\,\delta^m} - \frac{y^{3m+1}}{(3m+1)\,\delta^{3m}}\Big]_0^\delta = \delta\left[\frac{1}{m+1}-\frac{1}{3m+1}\right]$$

$$= \frac{3m+\cancel{1}-m-\cancel{1}}{3m^2+4m+1}\,\delta \Rightarrow \boxed{\frac{\delta e}{\delta} = \frac{2m}{3m^2+4m+1}}$$

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T1

Find the displacement , momentum ~~thickn~~ and energy thicknesses of the boundary layer if its velocity profile — is.. $\frac{u}{U} = \sin\left(\frac{\pi y}{2\delta}\right)$

Sol.

① displacement thickness

$$\delta_d = \int_0^\delta 1-\frac{u}{U}\,dy = \int_0^\delta 1-\sin\frac{\pi y}{2\delta}\,dy$$

$$\cos\frac{\pi y}{2\delta}$$

$$= y + \frac{2\delta}{\pi}\int_0^\delta \left(-\sin\frac{\pi y}{2\delta}\right)\cdot\frac{\pi}{2\delta}$$

$$= y + \frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta}\Big]_0^\delta = (\delta-0)+\Big(\underbrace{\cos\frac{\pi}{2}}_{=0}-\underbrace{\cos 0}_{1=}\Big)\frac{2\delta}{\pi}$$

$$= \delta - \frac{2\delta}{\pi} = \boxed{0.363\ \delta}$$

② momentum thick

$$\theta_d = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^\delta \sin\frac{\pi y}{2\delta}\left(1-\sin\frac{\pi y}{2\delta}\right)dy$$

$$= \int_0^\delta \sin\frac{\pi y}{2\delta} - \sin^2\frac{\pi y}{2\delta}\,dy \rightarrow \frac{1}{2}\left[1-\cos 2\left(\frac{\pi y}{2\delta}\right)\right]$$

$$= -\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \frac{1}{2}\left[y-\frac{\delta}{\pi}\sin\frac{\pi y}{\delta}\right]\Big\}_0^\delta$$

$$= -\frac{2\delta}{\pi}\Big[\underbrace{\cos\frac{\pi}{2}}_{=0}-\cos 0\Big] - \frac{1}{2}\Big[(\delta-0)-\frac{\delta}{\pi}\big(\underbrace{\sin\pi}_{=0}-\underbrace{\sin 0}_{=0}\big)\Big]$$

$$= \frac{2\delta}{\pi} - \frac{1}{2}\left[\delta + 0\right] = \delta\left[\frac{2}{\pi}-\frac{1}{2}\right] = \boxed{0.137\delta}$$

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Energy thickness T2

$$\delta_e = \int_0^\delta \frac{u}{U}\left[1-\left(\frac{u}{U}\right)^2\right]dy \qquad \frac{u}{U} = \sin\frac{\pi y}{2\delta}$$

$$= \int_0^\delta \sin\frac{\pi y}{2\delta}\left[1-\sin^2\left(\frac{\pi y}{2\delta}\right)\right]dy$$

$$= -\int_0^\delta -\sin\frac{\pi y}{2\delta}\cos^2\frac{\pi y}{2\delta} = -\left[\frac{1}{3}\cos^3\frac{\pi y}{2\delta}\cdot\frac{2\delta}{\pi}\right]_0^\delta$$

$$= -\frac{2\delta}{3\pi}\left[\cos^3\frac{\pi}{2} - \cos^3 0\right] = -\frac{2\delta}{3\pi}\left[0-1\right] = \frac{2\delta}{3\pi}$$

$$= \boxed{0.212\,\delta}$$

Ex:- Find the B.L. thickness of B.L of a velocity profile $\sin\frac{\pi y}{2\delta}$

$$\tau_o = \rho U^2\frac{d\theta}{dx} = 0.137\,\rho U^2\frac{d\delta}{dx}$$

$$\tau_o = \mu\left.\frac{du}{dy}\right|_{y=0} = \mu U\cos\frac{\pi y}{2\delta}\cdot\left.\frac{\pi}{2\delta}\right|_{y=0}$$

$$\tau_o = \boxed{\frac{\mu\pi U}{2\delta}}$$

$$\therefore\ 0.137\,\rho U^{\cancel{2}}\frac{d\delta}{dx} = \frac{\mu\pi}{2\delta}\cancel{U} \Rightarrow \delta\,d\delta = \frac{\mu\pi}{2\rho U\times 0.137}dx$$

$$\frac{\delta^2}{\cancel{2}} = \frac{\pi}{\cancel{2}\times 0.137}\,\frac{x}{\frac{\rho U}{\mu}} \times \frac{x}{x} \Rightarrow \left(\frac{\delta}{x}\right)^2 = \left(\frac{\pi}{0.137}\right)^{0.5}\frac{1}{Re_x^{0.5}}$$

$$\frac{\delta}{x} = 4.79\,\frac{1}{Re_x^{0.5}}$$

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T3

Ex prove that the momentum thickness of the B.L. that has a velocity profile of $\frac{u}{U} = \frac{3}{2}\eta - \frac{1}{2}\eta^3$ is given by $\boxed{\theta = \frac{39}{280}\delta}$ when $\eta = \frac{y}{\delta}$

Sol.

$$\theta d = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy \qquad \text{let } \eta = \frac{y}{\delta} \therefore y = \delta\eta$$

$$dy = \delta\, d\eta$$

at $y=0 \rightarrow \eta = 0$

$y = \delta \rightarrow \eta = 1$

$$\therefore \theta d = \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left[1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right] \delta\, d\eta$$

$$= \delta\int_0^1 \frac{3}{2}\eta - \frac{9}{4}\eta^2 + \frac{3}{4}\eta^4 - \frac{1}{2}\eta^3 + \frac{3}{4}\eta^4 - \frac{1}{4}\eta^6 \, d\eta$$

$$= \delta\left[\frac{3}{4}\eta^2 - \frac{9}{12}\eta^3 + \frac{3}{20}\eta^5 - \frac{1}{8}\eta^4 + \frac{3}{20}\eta^5 - \frac{1}{28}\eta^7\right]_0^1$$

$$= \delta\left[\cancel{\frac{3}{4}} - \cancel{\frac{9}{12}} + \frac{3}{20} - \frac{1}{8} + \frac{3}{20} - \frac{1}{28}\right]$$

$$= \left[\frac{42 - 35 + 42 - 10}{280}\right]\delta = \frac{39}{280}\delta$$

OR

$$\theta d = \int_0^\delta \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 dy$$

$$= \int_0^\delta \frac{3}{2}\frac{y}{\delta} - \frac{9y^2}{4\delta^2} + \frac{3y^4}{4\delta^4} - \frac{y^3}{2\delta^3} + \frac{3y^4}{4\delta^4} - \frac{y^6}{4\delta^6} \Big] dy$$

$$= \frac{3y^2}{4\delta} - \frac{9y^3}{12\delta^2} + \frac{3y^5}{20\delta^4} - \frac{y^4}{8\delta^3} + \frac{3y^5}{20\delta^4} - \frac{y^7}{28\delta^6}\Big]_0^\delta = \frac{39}{280}\delta$$

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Ex 14: if the linear velocity profile applied to B.L. (Tu) in laminar and turbulent, compare between the hydrodynamic thickness? Q For same* linear V.P of L. & T. prove that the L is thinner than T.

Sol.

$$\frac{u}{U} = \frac{y}{\delta} \quad \text{laminar}$$

$$\tau_o = \rho U^2 \frac{d}{dx}\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$= \rho U^2 \frac{d}{dx}\int_0^\delta \frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 dy$$

$$= \rho U^2 \frac{d}{dx}\left(\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right)\Big]_0^\delta$$

$$= \rho U^2 \frac{d\delta}{dx}\left(\frac{1}{2}-\frac{1}{3}\right) \Rightarrow \tau_o = 0.167\,\rho U^2 \frac{d\delta}{dx}$$

$$\tau_o = \mu \frac{du}{dy} = \mu U \frac{1}{\delta} = \boxed{\frac{\mu U}{\delta}}$$

$$\therefore\ 0.167\,\rho U^{\cancel{2}} \frac{d\delta}{dx} = \mu \cancel{U} \frac{1}{\delta}$$

$$\delta\, d\delta = \frac{\mu}{\rho U}\cdot\frac{1}{0.167}\, dx$$

$$\frac{\delta^2}{2} = \frac{1}{\frac{\rho U}{\mu}}\,\frac{1}{0.167}\, x \cdot \frac{x}{x}$$

$$\left(\frac{\delta}{x}\right)^2 = \frac{2/0.167}{Re_x} \Rightarrow \boxed{\frac{\delta}{x} = \frac{3.46}{Re_x^{0.5}}}$$

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}} \quad n=1 \qquad \tau_o = 0.023\,\rho U^2 \frac{1}{Re_\delta^{0.25}}$$

$$0.023\,\rho \cancel{U}^2 \frac{1}{\left(\frac{U}{\nu}\right)^{0.25}\delta^{0.25}} = 0.167\,\rho \cancel{U}^2 \frac{d\delta}{dx}$$

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TS

$$\delta^{0.25}\, d\delta = \frac{0.023}{0.167}\,\frac{1}{\left(\frac{U}{\nu}\right)^{0.25}}\, dx$$

$$\frac{\delta^{1.25}}{1.25} = 0.138\,\frac{x}{\left(\frac{U}{\nu}\right)^{0.25}} \Rightarrow \delta^{1.25} = 0.172\,\frac{1}{\left(\frac{U}{\nu}\right)^{0.25}}\, x \cdot \frac{x^{0.25}}{x^{0.25}}$$

$$\left(\frac{\delta}{x}\right)^{1.25} = \frac{0.172}{Re_x^{0.25}} \Rightarrow \frac{\delta}{x} = \frac{(0.172)^{\frac{1}{1.25}}}{Re_x^{0.25/1.25}}$$

$$\boxed{\frac{\delta}{x} = \frac{0.248}{Re_x^{0.2}}}$$ Turbulent

$$\frac{\frac{\delta_L}{x}}{\frac{\delta_T}{x}} = \frac{\frac{3.46}{Re_x^{0.5}}}{\frac{0.248}{Re_x^{0.2}}} \Rightarrow \frac{\delta_L}{\delta_T} = \frac{3.46}{0.248} * \frac{Re_x^{0.2}}{Re_x^{0.5}}$$

$$\simeq 13.95\, Re_x^{-0.3} \simeq \frac{13.95}{Re_x^{0.3}}$$

$$Re_x = 5*10^5 \Rightarrow \frac{\delta_L}{\delta_T} = \frac{13.95}{(5*10^5)^{0.3}} = 0.272$$

∴ Laminar BL. is thinner than the Turbulent B.L.

Ex water moves in 3 m/s over a plate. At what distance from the beginning of the plate at which the boundary layer thickness will be 1.2 mm? [use $\frac{\delta}{x} = 4.96\, Re_x^{-0.5}$]

Sol.

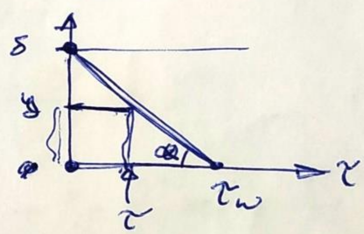
$$\frac{\delta}{x} = \frac{4.96}{\left(\frac{U}{\nu}\right)^{0.5} x^{0.5}} \Rightarrow x^{0.5} = \frac{\left(\frac{U}{\nu}\right)^{0.5} \cdot \delta}{4.96} = \frac{\left(\frac{3}{10^{-6}}\right)^{0.5} * \frac{1.2}{1000}}{4.96}$$

$$\therefore x = 0.1157\ m$$

Ex If the shear stress of the laminar boundary layer varies linearly from $\tau = 0$ at $y = \delta$ to $\tau = \tau_w$ at $y = 0$, find the momentum thickness?

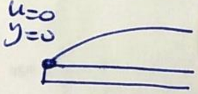
Sol.

$$\frac{\delta}{\tau_w} = \frac{y}{\tau_w - \tau}$$



or

$$\cot\alpha = \frac{\tau}{\delta - y} = \frac{\tau_w}{\delta} \Rightarrow \tau = \tau_w\left[\frac{\delta - y}{\delta}\right]$$



$$\therefore \tau = \tau_w\left(1 - \frac{y}{\delta}\right) = \mu \frac{du}{dy}$$

$$\therefore \frac{du}{dy} = \frac{\tau_w}{\mu}\left(1 - \frac{y}{\delta}\right)$$

$$u = \frac{\tau_w}{\mu}\left(y - \frac{y^2}{2\delta}\right) + C_1$$

$y = 0 \quad u = 0 \Rightarrow C_1 = 0$

$$\therefore u = \frac{\tau_w}{\mu}\left(y - \frac{y^2}{2\delta}\right)$$

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12-b

T7

$y = \delta \longrightarrow u = U$

$$\therefore U = \frac{\tau_w}{\mu}\left[\delta - \frac{\delta^2}{2\delta}\right] = \frac{\tau_w}{\mu}\left(\frac{\delta}{2}\right)$$

$$\therefore U = \frac{\tau_w \delta}{2\mu}$$

$$\therefore \frac{u}{U} = \frac{\frac{\tau_w}{\cancel{\mu}}\left[y - \frac{y^2}{2\delta}\right]}{\frac{\tau_w \delta}{2\cancel{\mu}}} = \frac{2}{\delta}\left[y - \frac{y^2}{2\delta}\right]$$

$$\frac{u}{U} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}$$

$$\therefore \theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(2\frac{y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - 2\frac{y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta 2\frac{y}{\delta} - 4\frac{y^2}{\delta^2} + 2\frac{y^3}{\delta^3} - \frac{y^2}{\delta^2} + 2\frac{y^3}{\delta^3} - \frac{y^4}{\delta^4}$$

$$= \frac{y^2}{\delta} - \frac{4}{3}\frac{y^3}{\delta^2} + \frac{2}{4}\frac{y^4}{\delta^3} - \frac{1}{3}\frac{y^3}{\delta^2} + \frac{2}{4}\frac{y^4}{\delta^3} - \frac{1}{5}\frac{y^5}{\delta^4}\Big]_0^\delta$$

$$= \delta\left[1 - \frac{4}{3} + \frac{1}{2} - \frac{1}{3} + \frac{1}{2} - \frac{1}{5}\right] = \delta\left[2 - \frac{5}{3} - \frac{1}{5}\right]$$

$$= \delta\left[\frac{30 - 25 - 3}{15}\right] = \boxed{\frac{2}{15}\delta}$$

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T_3-b

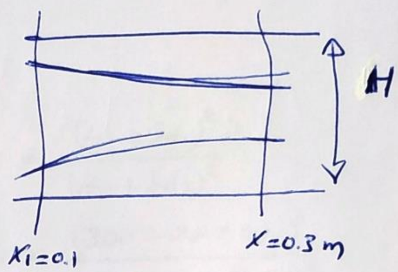
Q Evaluate $\frac{\delta d}{\delta}$ and $\frac{\theta d}{\delta}$ of the B.L. if its velocity profile is $\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$

T_8

Sinusoidal

Ex Find $P_1 - P_2$ or ΔP

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7}$$



$\delta = x Re^{-0.2}$

$$\delta d = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta 1 - \left(\frac{y}{\delta}\right)^{1/7} dy = y - \frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}}\Big]_0^\delta = \delta - \frac{7}{8}\delta = \boxed{\frac{\delta}{8}}$$

$$\tau = \rho U^2 \frac{d}{dx}\int_0^\delta \left(\frac{y}{\delta}\right)^{1/7}\left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$$

$$= \rho U^2 \frac{d}{dx}\int_0^\delta \left(\frac{y}{\delta}\right)^{1/7} - \left(\frac{y}{\delta}\right)^{2/7} dy = \rho U^2\left[\frac{7}{8} - \frac{7}{9}\right]\frac{d\delta}{dx}$$

$$\tau = \frac{7}{72}\rho U^2 \frac{d\delta}{dx} \qquad \text{and } \tau = \mu \frac{du}{dy} = 0$$

$$\tau = 0.023\,\rho U^2 \frac{1}{Re_\delta^{0.25}}$$

$$\frac{7}{72}\cancel{\rho}\cancel{U^2}\frac{d\delta}{dx} = 0.023\,\cancel{\rho}\cancel{U^2}\frac{1}{\left(\frac{U}{\nu}\right)^{0.25}\delta^{0.25}}$$

$$\delta^{0.25}d\delta = \frac{0.023 * 72}{7} * \frac{1}{\left(\frac{U}{\nu}\right)^{0.25}} dx \Rightarrow x$$

$$\frac{\delta^{1.25}}{1.25} = 0.237\,\frac{x^{1.25}}{Re_x^{0.25}} \Rightarrow \delta = (1.25 * 0.237)^{1/1.25} \cdot x \cdot \frac{1}{Re_x^{0.2}}$$

$$\boxed{\therefore \delta = 0.378\,\frac{x}{Re_x^{0.2}}}$$

at $x = 0.1 \longrightarrow \delta = 0.378 * \frac{0.1}{\left(\frac{10 * 0.1}{10^{-6}}\right)^{0.2}} = 2.385$ mm

$x = 0.3 \longrightarrow \delta = 0.378\,\frac{0.3}{\left(\frac{10 * 0.3}{10^{-6}}\right)^{0.2}} = 5.743$ mm

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Tu-b

T9

$\therefore \delta d_1 = \frac{3}{8} * 2.385 = 0.894$ mm

$\delta d_2 = \frac{3}{8} * 5.743 = 2.15$ mm

at ① $A_1 = (H - 2\delta d_1)^2$

② $A_2 = (H - 2\delta d_2)^2$

$U_1 = 10$ m/s

$$\therefore A_2 U_2 = A_1 U_1 \Rightarrow U_2 = 10 * \frac{(H - 2\delta d_1)^2}{(H - 2\delta d_2)^2}$$

$$= 10 * \frac{(300 - 2 * 0.894)^2}{(300 - 2 * 2.15)^2}$$

$$= 10.171 \text{ m/s}$$

Bernolli Eqt

$$\frac{P_1}{\gamma_1} + \frac{U_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{U_2^2}{2g} + \cancel{z_2} + \cancel{h_L}^{=0}$$

$$P_1 - P_2 = \underbrace{\frac{\rho}{2}}_{10^3/2}\left[(10.171)^2 - (10)^2\right] = 1724 \text{ Pa}$$

$$\frac{kg}{m^3} \cdot \frac{m^2}{s^2}$$

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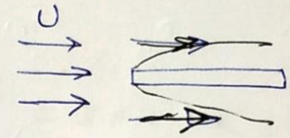
Ex: Assume linear profile of the boundary layer velocity. (1) prove that the drag force can be calculated from $F_D = 1.15\, \mu U b \sqrt{Re_L}$ (2) F_D where $U = 1$ m/s, $L = 0.3$ m, $b = 0.3$ m, fluid is water

where b is the width of the plate

Sol.

$$\frac{u}{U} = \frac{y}{\delta}$$



$$F_D = 2\int_0^L \tau_o\, dx$$

$$\tau_o = \rho U^2 \frac{d}{dx}\int_0^{\delta} \frac{u}{U}\left(1-\frac{u}{U}\right) dy = \frac{1}{6}\rho U^2 \frac{d\delta}{dx} \quad \text{--- (1)}$$

$$\tau_o = \mu \left.\frac{du}{dy}\right|_{y=0} = \mu U \frac{1}{\delta} = \frac{\mu U}{\delta} \quad \text{--- (2)}$$

$$\frac{1}{6}\rho U^2 \frac{d\delta}{dx} = \frac{\mu U}{\delta} \Rightarrow \delta\, d\delta = \frac{6\mu}{\rho U}\, dx$$

$$\frac{\delta^2}{2} = \frac{6\mu}{\rho U} x + C \qquad x=0 \quad \delta = 0 \quad C = 0$$

$$\delta^2 = \frac{12 x^2}{Re_x} \Rightarrow \frac{\delta}{x} = \frac{3.46}{Re_x^{0.5}} \quad \text{in (1) or (2)}$$

$$\tau_o = \frac{1}{6}\rho U^2 \frac{d}{dx}\left[\frac{3.46 x}{\left(\frac{U}{\nu}\right)^{0.5} x^{0.5}}\right] = \frac{1}{6}\rho U^2 * 3.46\, \frac{d}{dx}\left[\frac{x^{0.5}}{\left(\frac{U}{\nu}\right)^{0.5}}\right]$$

$$\tau_o = \frac{3.46}{6}\rho U^2 * 0.5 \frac{1}{Re_x^{0.5}} = 0.288\, \rho U^2 \frac{1}{Re_x^{0.5}}$$

$$F_D = 2\int_0^L \tau_o\, dx = 2\int_0^L 0.288\, \rho U^2 \frac{1}{\left(\frac{U}{\nu}\right)^{0.5}} x^{-0.5}\, dx$$

$$= \frac{2 * 0.288}{0.5}\rho U^2 \frac{1}{\left(\frac{U}{\nu}\right)^{0.5}} L^{1/2} \cdot \frac{L^{1/2}}{L^{1/2}} = 1.15\, \rho U^2 L \frac{1}{Re_L^{0.5}}$$

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$$F_D = 1.15\,\rho U^2 L\,\frac{1}{Re_L^{0.5}}\cdot\frac{Re_L^{0.5}}{Re_L^{0.5}}$$

$$F_D = 1.15\,\cancel{\rho} U^2 \cancel{L}\,\frac{Re_L^{0.5}}{\frac{\cancel{\rho} U \cancel{L}}{\mu}} = \boxed{1.15\,\mu U\,Re_L^{0.5}}$$

$$Re_L = \frac{UL}{\nu} = \frac{1*0.3}{10^{-6}} = 0.3*10^6 = 3*10^5 < 5*10^5$$

Laminar

$$F_D = 1.15 * \overset{\rho}{1.26} * \overset{\mu}{10^{-3}} * 1 * (3*10^5)^{0.5} = \boxed{0.794\ N}$$

$$\frac{\delta}{x} = \frac{3.46}{Re_x^{0.5}}$$

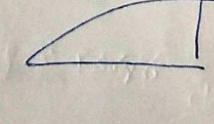
$$x = 0.1 \longrightarrow Re_{0.1} = \frac{\rho U L}{\mu} = \frac{1.26*0.3*0.1}{10^{-3}}$$

$$= 37.8$$

$$\delta = 0.1 * \frac{3.46}{37.8} = 9.15\ mm$$

$$\delta_d = \int_0^{\delta} 1-\frac{u}{U}\,dy = \int_0^{\delta} 1-\frac{y}{\delta}\,dy = y - \frac{y^2}{2\delta}\Big]_0^{\delta} = \delta - \frac{\delta^2}{2\delta}$$

$$= \delta - \frac{\delta}{2} = \frac{\delta}{2} \qquad \therefore\ \delta_d = \frac{9.15}{2} = 4.58\ mm$$



Ex2 Water flows over a flat plate (2m-long and 3m width) by 12 m/s. Find:

$\nu_w = 10^{-6}\ m^2/s$

1. shear velocity
2. wall layer thickness
3. Boundary layer thickness at a) begining b) mid-span c) ending of the plate
4. Drag force

Sol.

$$Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu} = \frac{12 * 2}{10^{-6}} = 24 * 10^{6} \text{ or } 2.4 * 10^{7}$$

$\therefore n = 7$

$$\tau_o = \frac{0.03 \rho U^2}{Re^{0.2}} = \frac{0.03 * 10^3 * (12)^2}{(24 * 10^6)^{0.2}} = 144.4\ N/m^2$$

1. shear velocity $u_\tau = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{144.4}{10^3}} = 0.38\ m/s$

2. wall layer thickness $\delta_v = \frac{5\nu}{u_\tau} = \frac{5 * 10^{-6}}{0.38}$

$= 0.013\ mm$

3. B.L at x location $\delta = \frac{0.38 x}{Re_x^{0.2}}$

$$Re_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu} = \frac{12}{10^{-6}} x = 12 * 10^6 x$$

$$\therefore \delta = \frac{0.38 x}{(12 * 10^6)^{0.2} x^{0.2}} \Rightarrow \delta = 0.0146\ x^{0.8}$$

4. Drag force (H.W).