***B.Sc. Fourth Year – Laser Physics Department - Laser Design I-LECTURE THREE***

## 7. Threshold conditions - laser losses

It was explained above that a steady state level of oscillation is reached when the rate of amplification is balanced by the rate of loss. This is the situation in continuous output (CW) lasers; it is a little different in pulse lasers. Thus, while a population inversion is a necessary condition for laser action, it is not a sufficient one because the minimum (i.e. threshold value) of the gain coefficient must be large enough to overcome the losses and sustain oscillations. The threshold gain, in turn, through Eq. (1.15) specifies the minimum population inversion required. The total loss of the system is due to a number of different processes; the most important ones include:

1. Transmission at the mirrors — the transmission from one of the mirrors usually provides the useful output, the other mirror is made as reflective as possible to minimize losses.
2. Absorption and scattering at the mirrors.
3. Absorption in the laser medium due to transitions other than the desired transitions (as mentioned earlier most laser media have many energy levels, not all of which will be involved in the laser action).
4. Scattering at optical inhomogeneities in the laser medium - this applies particularly to solid-state lasers.
5. Diffraction losses at the mirrors.

To simplify matters, let us include all the losses except those due to transmission at the mirrors in a single effective loss coefficient ($\gamma$) which reduces the effective gain coefficient to $(k - \gamma)$. We can determine the threshold gain by considering the change in irradiance of a beam of light undergoing a round trip within the laser cavity. We assume that the laser medium fills the space between the mirrors ($M_1$) and ($M_2$) which have reflectances ($R_1$) and ($R_2$) and a separation ($L$). Then in traveling from ($M_1$) to ($M_2$), the beam irradiance increases from ($I_0$) to ($I$) where from Eq. (16), $I = I_0 \exp\left[\left(k - \gamma\right)L\right]$.After reflection at ($M_2$), the beam irradiance will be $R_2\, I_0 \exp\left[(k-\gamma)L\right]$ and after a complete round trip the final irradiance will be such that the round trip gain *(G)* is

$$G = \frac{final\ irradiance}{initial\ irradiance} = R_1\ R_2 \exp\left[2(k-\gamma)L\right] \quad \textbf{(ACTIVE MEDIUM GAIN)}$$

If *(G)* is greater than unity a disturbance at the laser resonant frequency will undergo a net amplification and the oscillations will grow; if *(G)* is less than unity the oscillations will die out. Therefore, we can write the ***threshold condition*** **as**

$$G = R_1\ R_2 \exp\left[2(k_{th}-\gamma)L\right] = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

where $(k_{th})$ is the threshold gain. It is important to realize that the threshold gain is equal to the steady-state gain in continuous output lasers

The small signal gain required to support steady-state operation depends on the laser medium through ($k$) and ($\gamma$), and on the laser construction through $R_1$, $R_2$ and $L$. From Eq. (16) we can see that

$$k_{th} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

where the first term represents the volume losses and the second the loss in the form of the useful output. If (k) is high then it is relatively easy to achieve laser action

## 8-The rate equations of 4-levels:

The rate equations describe the rate of change of the populations of the laser medium energy levels in terms of the emission and absorption processes and pump rate. We shall consider the ideal four-level system shown in Fig. (8).

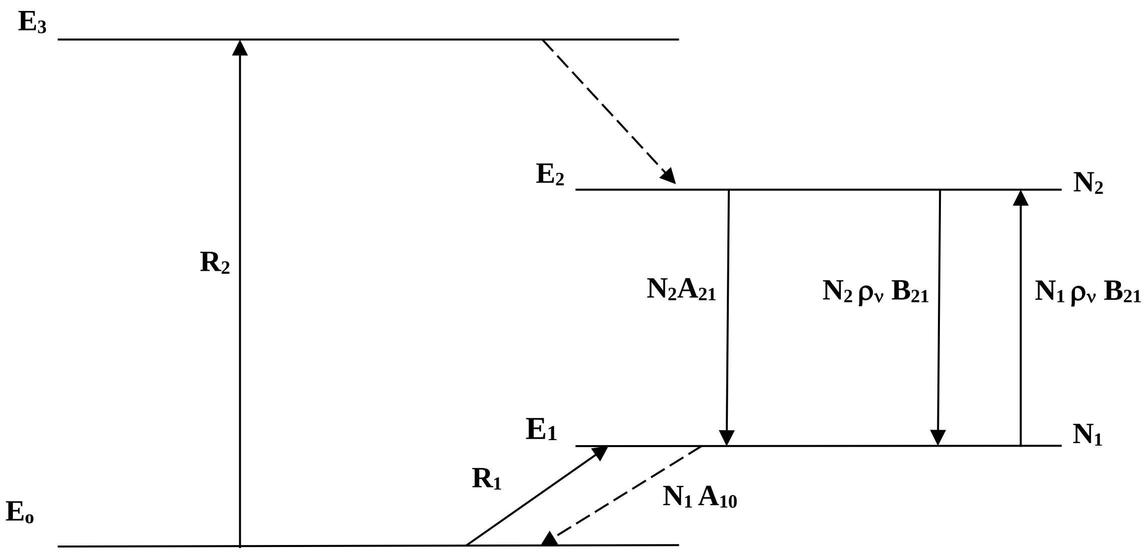


Figure (8 ) Transitions within an ideal four-level system

We assume that $E_1 >>$ kT so that the thermal population of level 1 is negligible; we also assume that the threshold population density $(N_{th})$ is very small compared with the ground state population so that during lasing the latter is hardly affected. If we let $(R_2)$ and $(R_1)$ be the rates at which atoms are pumped into levels 2 and 1 respectively, we can write the rate equations for these levels (assuming $g_1 = g_2$ for simplicity, and hence $B_{21} = B_{12}$) as

$$\frac{dN_2}{dt} = R_2 - N_2 A_{21} - \rho_\nu B_{21}(N_2 - N_1) \quad \text{......................... (20)}$$

and

$$\frac{dN_1}{dt} = R_1 + \rho_\nu B_{21}(N_2 - N_1) + N_2 A_{21} - N_1 A_{10} \quad \text{................. (21)}$$

Process $(R_1)$, which populates the lower laser level 1, is detrimental to laser action as it clearly reduces the population inversion. Although such pumping is unavoidable in many lasers, for example, gas lasers pumped via an electrical discharge. We shall henceforth ignore $(R_1)$. If we assume that the system is being pumped at a steady state then we have

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0 .$$

Hence, we may solve Eqs. (1.24) and (1.25) for ($N_1$) and ($N_2$).

$$N_1 = \frac{R_2}{A_{10}} \qquad\qquad N_2 = R_2\left(1+\frac{\rho_\nu\, B_{21}}{A_{10}}\right)\left(A_{21}+\rho_\nu\, B_{21}\right)^{-1}$$

and hence

$$N_2 - N_1 = R_2\left(\frac{1-\frac{A_{21}}{A_{10}}}{A_{21}+\rho_\nu\, B_{21}}\right) \quad \text{.......................................................} \ (22)$$

We can see that unless A $_{2I}$ $<A_{10}$, the numerator will be negative and no population inversion can take place. As the Einstein *(A)* coefficients are the reciprocals of the spontaneous lifetimes, the condition $A_{21} < A_{10}$ is equivalent to the condition $\tau_{10}<\tau_{21}$. that is the upper lasing level has a longer spontaneous emission lifetime than the lower level. In most lasers $\tau_{21}>>\tau_{10}$ and $\left(1-\frac{A_{21}}{A_{10}}\right)\approx 1$ .

Now, below the threshold we may neglect ($p_v$) since lasing has not yet commenced and most of the pump power appears as spontaneous emission; thus Eq. (26) can be written as

$$N_2 - N_1 = R_2\left(\frac{1-\frac{A_{21}}{A_{10}}}{A_{21}}\right)$$

That is, there is a linear increase in population inversion with pumping rate but insufficient inversion to give amplification.

At the threshold, (*pv*) is still small and assuming $g_1 = g_2$ we can express the threshold population inversion in terms of the threshold pump rate, that is

$$\left(N_2 - N_1\right)_{th} = N_{th} = R_{th}\left(\frac{1-\frac{A_{21}}{A_{10}}}{A_{21}}\right) \quad \text{................. (23)}$$

or inserting the above approximation that $\left(1-\frac{A_{21}}{A_{10}}\right) \approx 1$

$R_{th} = N_{th}\,A_{21}$

or

$$R_{th} = \frac{N_{th}}{\tau_{21}}$$

Each atom raised into level 2 requires an amount of energy ($E_3$) so that the total pumping power per unit volume ($P_{th}$) required at threshold may be written as

$$P_{th} = \frac{E_3\,N_{th}}{\tau_{21}}$$

We may substitute for ($N_{th}$) from eq.(23) to give

$$P_{th} = \frac{E_3\;8\;\pi\;\nu_0^2\;k_{th}\;\Delta\nu\;n^2}{c^2} \quad \text{.......................................... (24)}$$

This is the point at which the gain due to the population inversion exactly equals the cavity losses. Further increase of the population inversion with pumping is impossible in a steady state situation since this would result in a rate of induced energy emission, which exceeds the losses. Thus, the total energy stored in the cavity would increase with time in violation of the steady-state assumption (this is the phenomenon of gain saturation described earlier).

This argument suggests that [ $N_2$ —$N_1$ ] must remain equal to $N_{th}$ regardless of the amount by which the threshold pump rate is exceeded. Equation (26) shows that this is possible providing ($p_\nu\,B_{21}$) increase (once $R_2$ exceeds its threshold value given by Eq. (27) so that the equality

$$N_{th} = R_2 \left( \frac{1 - \frac{A_{21}}{A_{10}}}{A_{21} + \rho_\nu \, B_{21}} \right)$$

is satisfied. Now combining this equation with Eq. (27) we have

$$\frac{R_{th}}{A_{21}} = \frac{R_2}{A_{21} + \rho_\nu \, B_{21}} \quad \text{.................................} \quad (25)$$

Since the power output (***W***) of the laser will be directly proportional to the optical power density within the laser cavity and the pump rate into level 2 (i.e. $R_2$) will be proportional to the pump power ***(P)*** delivered to the laser, we may rewrite eq.(29) as

$$W = W_0 \left( \frac{P}{P_{th}} - 1 \right) \quad \text{....................................} \quad (26)$$

Where ($W_0$) is a constant.

Thus if the pump rate is increased above the value ($P_{th}$) the beam irradiance is expected to increase linearly with the pump rate. This is borne out in practice and plots of population inversion and laser output as a function of pump rate are of the form shown in Fig. (8).

The additional power above the threshold is channeled into a single (or a few) cavity mode(s). Spontaneous emission still appears above the threshold but it is extremely weak in relation to the laser output as it is emitted in all directions and has a much greater frequency spread.

## 9-Stability of Laser Resonators

For certain combinations of $r_1$, $r_2$, and $L$, the equations summarized in the previous subsection give nonphysical solutions (that is, imaginary spot sizes). Rays that bounce back and forth between the spherical mirrors of a laser resonator experience a periodic focusing action. The effect on the rays is the same as in a periodic sequence of lenses.

Rays passing through a stable sequence of lenses are periodically refocused. For unstable systems, the rays become more and more dispersed the further they pass through the sequence. In an optical resonator operated in the stable region, the waves propagate between reflectors without spreading appreciably. This fact can be expressed by a stability criterion

$0 \leq g_1 \; g_2 \leq 1$ ……………… (52)

or

$$0 \leq \left(1 - \frac{L}{r_1}\right)\left(1 - \frac{L}{r_2}\right) \leq 1 \qquad \text{……………… (53)}$$

To show graphically which type of resonator is stable and which is unstable, it is useful to plot a stability diagram on which each particular resonator geometry is represented by a point. This is shown in Fig. (17), where the parameters $g_1 = 1 - \frac{L}{r_1}$ , $g_2 = 1 - \frac{L}{r_2}$ are drawn as the coordinate axes.

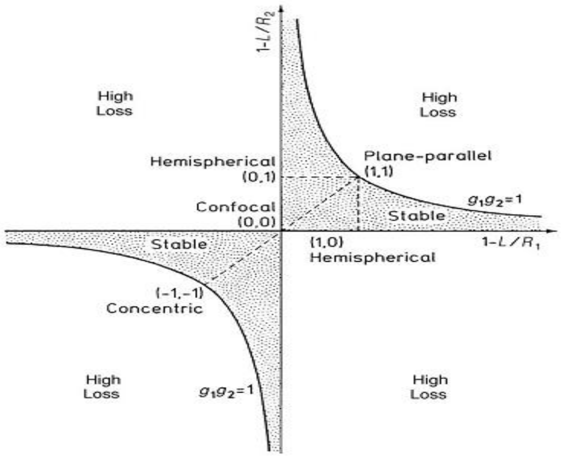


**Figure (17). Stability diagram for the passive laser resonator.**

All cavity configurations are unstable unless they correspond to points located in the area enclosed by a branch of the hyperbola $g1 \, g_2 = 1$ and the coordinate axes. The origin of the diagram represents the confocal system. The diagram is divided into positive and negative branches defining quadrants for which ($g_1 \, g_2$) is either positive or negative. The reason for this classification becomes clear when we discuss unstable resonators.