Measures of Dispersion (مقاييس التشتت)

When a set of observations has values that are close to their mean, they are less dispersed compared to when they are spread out. Several measures are used to quantify this dispersion, including:

1. The Mean Deviation (الانحراف المتوسط)

The mean deviation, denoted by $M.D$, is calculated by summing the absolute differences between each value and the mean, then dividing by the total number of values. For a dataset with n values $x_1, x_2, \cdots, x_n$, the mean deviation $M.D$ is given by:

$$M.D = \frac{\sum|x_i - \bar{x}|}{n}$$

Example 1: The hemoglobin levels (g/dL) of 6 patients are: 12, 13, 14, 15, 16, 17.

Calculate the mean deviation.

Solution: To find the mean deviation $M.D$, we must know the arithmetic mean

$$\bar{x} = \frac{\sum x_i}{n} = \frac{77}{6} = 14.5$$

Now, we create the following table:

x_i	12	13	14	15	16	17
$\|x_i - \bar{x}\|$	2.5	1.5	0.5	0.5	1.5	2.5

$$M.D = \frac{\sum|x_i - \bar{x}|}{n} = \frac{9}{6} = 1.5$$

For a frequency table, the mean deviation is calculated as the sum of the products of the absolute deviations from the mean and their frequencies, divided by the total frequency.

$$M.D = \frac{\sum f_i|x_i - \bar{x}|}{\sum f_i}$$

Example 2: The following table shows the distribution of systolic blood pressure (mmHg) among a sample of 50 patients with hypertension. Calculate the mean deviation.

Blood pressure	120-129	130-139	140-149	150-159
Frequency	10	15	20	5

Solution: First, you need to find the midpoints x_i of each class, and then the following table is formed:

Class	120-129	130-139	140-149	150-159	Sum
x_i	124.5	134.5	144.5	154.5	
f_i	10	15	20	5	50
$f_i\, x_i$	1245	2017.5	2890	772.5	6925
$\lvert x_i - \bar{x}\rvert$	14	4	6	16	
$f_i\lvert x_i - \bar{x}\rvert$	140	60	120	80	400

$$\bar{x} = \frac{\sum f_i\, x_i}{\sum f_i} = \frac{6925}{50} = 138.5$$

$$M.D = \frac{\sum f_i \lvert x_i - \bar{x}\rvert}{\sum f_i} = \frac{400}{50} = 8$$

2. The Variance (التباين)

The variance of a set of values, which we denote by S^2, is defined as:

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

Example 3: Determine the variance of a sample in example 1.

Solution:

x_i	12	13	14	15	16	17
$(x_i - \bar{x})^2$	6.25	2.25	0.25	0.25	2.25	6.25

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{17.5}{5} = 3.5$$

The variance can be calculated using the following formula:

$$S^2 = \frac{\sum {x_i}^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

Example 4: The concentrations of antibodies in the blood serum of a sample of fourth- year students in the Medical Physics Department were as follows: 2.05, 0.94 ,1.83 ,1.17, 2.16, 1.25. Determine the variance.

Solution:

x_i	2.05	0.94	1.83	1.17	2.16	1.25	$\sum x_i = 9.4$
${x_i}^2$	4.2025	0.8836	3.3489	1.3689	4.6656	1.5625	$\sum {x_i}^2 = 16.032$

$$S^2 = \frac{\sum {x_i}^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

$$= \frac{16.032 - \frac{(9.4)^2}{6}}{5} = 0.259$$

For a frequency table, the variance can be calculated using one of the following formulas:

$$S^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i - 1}$$

$$S^2 = \frac{\sum f_i {x_i}^2 - \frac{(\sum f_i x_i)^2}{\sum f_i}}{\sum f_i - 1}$$

Example 5: Determine the variance of a sample in example 2.

Solution:

Class	120-129	130-139	140-149	150-159	Sum
x_i	124.5	134.5	144.5	154.5	
f_i	10	15	20	5	50
$f_i\, x_i$	1245	2017.5	2890	772.5	6925
$(x_i - \bar{x})^2$	196	16	36	256	
$f_i(x_i - \bar{x})^2$	1960	240	720	1280	4200

$$S^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i - 1} = \frac{4200}{50 - 1} = 85.71$$

Example 6: In 2014, 50 students from the College of Science at the University of Babylon donated blood to the wounded of the Ali al-Akbar Brigade who were wounded during the operations to liberate Talafar from Daeish gangs. The level of hemoglobin in their blood was summarized in the following table. Calculate the variance.

Class	12.8-13.8	13.9-14.9	15.0-16.0	16.1-17.1	17.2-18.2	18.3-19.3
f_i	3	5	15	16	10	1

Solution:

Class	12.8-13.8	13.9-14.9	15.0-16.0	16.1-17.1	17.2-18.2	18.3-19.3	Sum
x_i	13.3	14.4	15.5	16.6	17.7	18.8	
f_i	3	5	15	16	10	1	50
$f_i x_i$	39.9	72	232.5	265.6	177	18.8	805.8
${x_i}^2$	176.89	207.36	240.25	275.56	313.29	353.44	
$f_i {x_i}^2$	530.67	1036.8	3603.75	4408.96	3132.9	353.44	13066.52

$$S^2 = \frac{\sum f_i {x_i}^2 - \frac{(\sum f_i x_i)^2}{\sum f_i}}{\sum f_i - 1} = \frac{13066.52 - \frac{(805.8)^2}{50}}{49} = 1.64$$

3. Standard Deviation (الانحراف المعياري)

Standard deviation is defined as the square root of the variance. It is symbolized by the symbol S.

Example 7: Determine standard deviation of the samples in examples 3, 4, 5 and 6.

Solution: 3. $S = \sqrt{\boldsymbol{S^2}} = \sqrt{\boldsymbol{3.5}} = 1.87$

4. $S = \sqrt{\boldsymbol{S^2}} = \sqrt{\boldsymbol{0.259}} = 0.509$

5. $S = \sqrt{\boldsymbol{S^2}} = \sqrt{\boldsymbol{85.71}} = 9.26$

6. $S = \sqrt{\boldsymbol{S^2}} = \sqrt{\boldsymbol{1.64}} = 1.28$

H.W. Calculate the mean deviation $M.D$, variance S^2 and standard deviation S.

1. Hb% per 100 cc blood of 7 individuals is as follows:

14.7, 13.1, 11.5, 12.9, 13.8, 14.5 and 12.5.

2. The following frequency distribution table

Class	20-24	25-29	30-34	35-39	40-44	45-49
f_i	8	11	21	28	17	15