Q1/ A binary symetric channel having the joint probability matrix of: $\begin{bmatrix} 0.175 & 0.075 \\ 0.225 & 0.525 \end{bmatrix}$ and $p(x_2) = 0.75$. Determine the channel efficiency.

Solution:

P(x,y) = $\begin{bmatrix} 0.175 & 0.075 \\ 0.225 & 0.525 \end{bmatrix}$

P(X)= [0.25   0.75]   from row sum
P(y)= [0.4   0.6]   from column sum

$P(y/x) = \frac{P(x,y)}{P(X)} = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$ (symmetric channel)

Find K

$$K = \sum_{j=1}^{2} P(y/x) log_2 P(y/x)) = 0.7 log_2(0.7) + 0.3\ log_2\ (0.3) = -0.881\ bit/symbol$$

Find H(y) where P(y) =[0.4   0.6]

$$H(y) = -\sum_{j=1}^{2} P(y_j) log_2 P(y_j) = -[\ 0.4\ log_2 0.4 + \ 0.6\ log_2 0.6\ ] = \ 0.97\ bit/\ symbol$$

$$I(x,y) = H(y) + K = 0.97 - 0.881 = 0.09\ bit\ /\ symbol$$

$$C = \ log_2 m + K = \ log_2(2) + K = 1 - 0.881 = 0.119$$

$$\mu = \frac{I(x,y)}{C} = \frac{0.09}{0.119} * 100\ \% = 75.63\ \%$$

Q2/A binary source sending x_1 with a probability of 0.4 and x_2 with 0.6 probability through a channel with a probabilities of errors of 0.1 for x_1 and 0.2 for x_2.Determine:

1- Source entropy.
2- Marginal entropy.
3- Joint entropy.
4- Conditional entropy $H(Y|X)$.
5- Losses entropy $H(X|Y)$.
6- Trans information.

Q4/ A source produces dots"." and dashes"-", if the time duration of a dash is twice as long as dot and has ½ probability of a dot , if a dot intervals is 10ms. What is the average rate of information transmission? $\tau_{dash} = 2\,\tau_{dot}$

$$P_{dash} = \frac{1}{2} P_{dot}$$

$$\tau_{dot} = 10 \text{ msec} \rightsquigarrow \tau_{dash} = 2 * \tau_{dot}$$

$$= 20 \text{ msec}$$

$$P_{dash} + P_{dot} = 1$$

$$\frac{1}{2} P_{dot} + P_{dot} = 1 \rightsquigarrow P_{dot} = \frac{2}{3}$$

and

$$P_{dot} = \frac{1}{3}$$

$$H_{(X)} = -\sum_{i=0}^{n} P_{(Xi)} \text{Log}_2 P_{(Xi)}$$

$$H_{(X)} = \frac{-1}{\text{Ln}2}\left[\frac{2}{3}\text{Ln}\frac{2}{3} + \frac{1}{3}\text{Ln}\frac{1}{3}\right]$$

$$H_{(X)} = 0.918 \text{ bits / symbol}$$

$$\bar{\tau} = \sum_{i=1}^{n} \tau_i P_{(Xi)} = 10 * \frac{2}{3} + 20 * \frac{1}{3}$$

$$\bar{\tau} = 13.3 \text{ msec.}$$

$$R_{(X)} = \frac{H_{(x)}}{\bar{\tau}} = \frac{0.918}{13.3 \text{ msec}} = 69 \text{ bits / sec}$$

Q5/ A channel has joint probability matrix

$$P(X,Y) = \begin{bmatrix} 0.03 & 0.05 & 0.12 \\ 0.1 & 0.24 & 0.06 \\ 0.24 & 0.06 & 0.1 \end{bmatrix}.$$

Find the binary channel efficiency and redundancy.

Solution /

$$P(X,Y) = \begin{bmatrix} 0.03 & 0.05 & 0.12 \\ 0.1 & 0.24 & 0.06 \\ 0.24 & 0.06 & 0.1 \end{bmatrix}$$

P(X)= [0.2 0.4 0.4] from row sum
P(y)= [0.37 0.35 0.28] from column

$$P(y/x) = \frac{P(x,y)}{P(X)} = \begin{bmatrix} 0.15 & 0.25 & 0.6 \\ 0.25 & 0.6 & 0.15 \\ 0.6 & 0.15 & 0.25 \end{bmatrix}$$

Find K

$$K = \sum_{j=1}^{3} P(y/x) log_2 P\left(\frac{y}{x}\right)) = 0.15 log_2(0.15) + 0.25\ log_2\ (0.25) + 0.6 log_2(0.6)$$
$$= -1.352\ bit/symbol$$

$$H(y) = -\sum_{j=1}^{3} P(\,y_j) log_2 P(y_j) = -[\,0.37\ log_2 0.37 +\ 0.35 log_2 0.35 + 0.28\ log_2 0.28]$$
$$= 1.575\ \ bit/\ symbol$$

$$I(x,y) = H(y) + K = 1.575 - 1.352 = 0.223\ bit\ /\ symbol$$

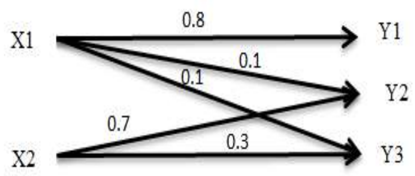
$$C = log_2 m + K = log_2(3) + K = 1.585 - 1.352 = 0.232\ bit\ /\ symbol$$

$$\mu = \frac{I(x,y)}{C} = \frac{0.223}{0.232} * 100\ \% = 96.12\ \%$$

$$R = 1 - \mu = 1 - 0.9612 = 0.038$$

Q7/ The channel module shown below $P(x_1) = 0.6$ and $P(x_2) = 0.4$. Find

a) Marginal entropy H(X) and H(Y)
b) Joint entropy H(X,Y).
c) Noise entropy H(Y/X).
d) Losses entropyH(X/Y).



Solution /

$P(x) = [\ 0.6 \ \ 0.4]$

$$P(y/x) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

$$P(x\ ,\ y) = P(y/x).P(x) = \begin{bmatrix} 0.48 & 0.06 & 0.0.6 \\ 0 & 0.28 & 0.12 \end{bmatrix}$$

$P(y) = [\ 0.48 \quad 0.34 \quad 0.18]$

a) $H(X) = -\sum_{i=1}^{n} p(x_i) \log_2 p(x_i) \ = \ 0.9710 \, bit/symbol$

b) $H(Y) = -\sum_{j=1}^{m} p(y_j) \log_2 p(y_j) = 1.4828$ bit/symbol

$$H(X, Y) = -\sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i, y_j) \log_2 P(x_i, y_j) \ = 1.8766 \text{ bit/ symbol}$$

c) $H(Y/X) = H(X,Y) - H(X) = 0.4050$ bit/ symbol

d) $H(X/Y) = H(X,Y) - H(Y) = 0.3946$ bit/ symbol

Q8/ The channel model shown $P(Y/X) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.7 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$

The source information $I(X_1) = 2$ and $I(X_2) = 4$. Calculate

a- the noise(H(Y/X)).
b- loss entropy and (H(X/Y)).

Sol: $P(x_1) = 2^{-2} = 0.25$ $P(x_2) = 2^{-4} = 0.0625$ $P(x_3) = 0.6875$

$P(x) = [\ 0.25 \quad 0.0625 \quad 0.6875]$

$$P(x\,,\,y) = P(y/x).P(x) = \begin{bmatrix} 0.2 & 0.05 & 0 \\ 0.00625 & 0 & 0.05625 \\ 0.20625 & 0.48125 & 0 \end{bmatrix}$$

$P(y) = [\ 0.4125 \quad 0.53125 \quad 0.05625]$

H(X) = bit/symbol

H(Y) = bit/symbol

H(X,Y)= bit/symbol

H(Y/X) = H(X, Y) – H(X) = bit/symbol

H(X/Y) = H(X, Y) – H(Y) = bit/symbol

Q9 / Having the text (* * * # * $ # $ $ # # * * * & # # # $ &) .Find:

a- The text probability P(x) and Entropy H(x).

b-If τ(*) = τ(#) = τ($) = 0.1 μsec and τ(D) = 0.2 μsec. Calculate average source entropy rate R(x).

c- The source information of A and B [I($), I(&)]

d- Find maximum information rate that a transmitted over channel 3.3KHz bandwidth and SNR(dB) = 18 dB.

Solution /

Variable	Number	P(x)	$\tau(x)$
*	7	0.35	$0.1\mu sec$
#	7	0.35	$0.1\mu sec$
$	4	0.2	$0.1\mu sec$
&	2	0.1	$0.2\mu sec$

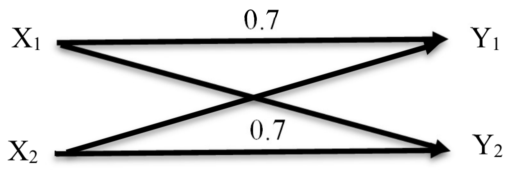
$H(x) = -\sum_{i=1}^{4} P(xi)Log_2\, p(xi) = 1.857$ bits/symbol

$\bar{\tau} = \sum_{i=1}^{4} \tau_i\, P_{(Xi)} = 0.7\ msec$

$R_{(X)} = \frac{H(X)}{\bar{\tau}} = 2.65$ Kb/sec

Q10 / A binary symmetric channel (BSC) with the model of Fig. has been connected with a binary source of information whose symbol probabilities are $p(x_1) = 0.75$ and $p(x_2) = 0.25$.

Find the channel capacity C with the mutual information I(x,y)



$$P(Y \mid X) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

Since the channel is symmetric then

$C=\log_2 m + K$ and $n = m$

where n and m are number row and column repestively

$$K = 0.7\log_2 0.7 + 0.3\log_2 0.3 = -0.88129$$

$$C = 1 - 0.88129 = 0.1187 \text{ bits/symbol}$$

The channel efficiency $\eta = \frac{I(X,Y)}{C}$

$I(x_1) = -\log_2 P(x_1) = 2$

$$P(x_1) = 0.75 \quad \text{then } P(X2) = 0.25$$

And we have $P(x_i, y_j) = P(x_i)P(y_j \mid x_i)$ so that

$$P(X,Y) = \begin{bmatrix} 0.75 \times 0.7 & 0.25 \times 0.3 \\ 0.75 \times 0.3 & 0.25 \times 0.7 \end{bmatrix} =$$

$$P(Y) = [\ 0.6 \quad 0.4] \rightarrow H(Y) = 0.97095 \text{ bits/symbol}$$

$$I(X,Y) = H(Y) + K = 0.97095 - 0.88129 = 0.0896 \text{ bits/symbol}$$

Then $\eta = \frac{0.0896}{0.1187} = 75.6\%$

Q17/ Having the text [MMMAAAHHMMMMOOAAAAHH] Find:

1- The text probability p (x) and entropy H (x).
2- The self information of A and B.
3- If the $\tau(M) = \tau(A) = \tau(H) = 0.1\ \mu sec\ and\ \tau(O) = 0.2\ \mu sec.$ Calculate the source entropy rate R (x).

Variable	Number	P(x)	$\tau(x)$
M	7	0.35	$0.1\mu sec$
A	7	0.35	$0.1\mu sec$
H	4	0.2	$0.1\mu sec$
O	2	0.1	$0.2\mu sec$

$$H(x) = -\sum_{i=1}^{4} P(xi) Log_2\ p(xi) = 1.857 \text{ bits/symbol}$$

$$\bar{\tau} = \sum_{i=1}^{4} \tau_i\ P_{(Xi)} = 0.7\ msec$$

$$R_{(X)} = \frac{H(X)}{\bar{\tau}} = 2.65 \text{ Kb/sec}$$

Q18/ A system having the following joint probability matrix:

$$P\ (x, y) = \begin{bmatrix} 0.0625 & 0.0625 \\ 0 & 0.125 \\ 0.5 & 0.25 \end{bmatrix} \quad \text{Find:}$$

1- Marginal entropies
2- Joint entropy
3- Conditional entropies
4- Transinformation

Sol: P(x) = [0.125 0.125 0.75]

P(y) = [0.5625 0.4375]

H(x,y)=

H(x) =

H(y) =

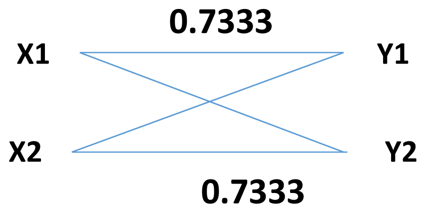
H(x/y) = H(x,y) – H(y)

H(y/x)= H(x,y) – H(x)

Q19/ Consider the joint probability is

$$P\,(x,\,y) = \begin{bmatrix} 0.183 & 0.067 \\ 0.2 & 0.549 \end{bmatrix}$$

Find the channel capacity and channel efficiency of this channel.



Solution /

$$P\,(x,\,y) = \begin{bmatrix} 0.183 & 0.067 \\ 0.2 & 0.55 \end{bmatrix}$$

P(X)= [0.25 0.75] from row sum
P(y)= [0.383 0.617] from column sum

$$P(y/x) = \begin{bmatrix} 0.7333 & 0.2667 \\ 0.2667 & 0.7333 \end{bmatrix}$$

$$K = \sum_{j=1}^{2} P(y/x)log_2P(y/x)) = 0.7333log_2(0.7333) + 0.2667\;log_2\;(0.2667)$$

$$= -0.836\;bit/symbol$$

Find H(y) where P(y) =[0.383 0.617]

$$H(y) = -\sum_{j=1}^{2} P(y_j)log_2P(y_j) = -[\;0.383\;log_2 0.383 + \;0.617log_2\;0.617\;]$$

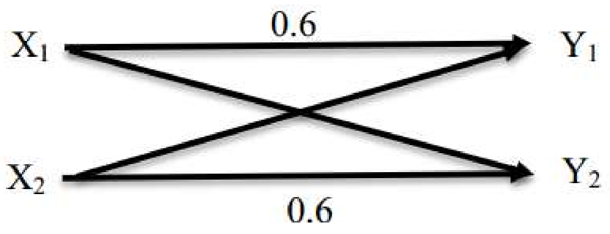
$$= 0.96\;bit/\;symbol$$

$$I(x,y) = H(y) + K = 0.96 - 0.836 = 0.124\;bit\;/\;symbol$$

$$C = log_2 m + K = log_2(2) + K = 1 - 0.836 = 0.164\;bit\;/\;symbol$$

$$\mu = \frac{I(x,y)}{C} = \frac{0.124}{0.164} * 100\;\% = 75.6\;\%$$

Q25/ For the BSC shown:



Find the channel capacity and efficiency if I(X_2) =2bits.

Sol:

$$P(Y\mid X)=\begin{bmatrix}0.6 & 0.4\\ 0.4 & 0.6\end{bmatrix}$$

- Since the channel is symmetric then

  C=$\log_2 m + K$ and $n = m$

where n and m are number row and column repestively

$K = 0.6\log_2 0.6 + 0.4\log_2 0.4 =$

$C = \log_2 m + K =$ bits/symbol

The channel efficiency $\eta = \frac{I(X,Y)}{C}$

$I(x_1) = -\log_2 P(x_1) = 2$

$$P(x_1) = 2^{-2} = 0.25 \quad \text{then } P(X2) = 0.75$$

And we have $P(x_i, y_j) = P(x_i)P(y_j \mid x_i)$ so that

$P(X,Y) = \begin{bmatrix}0.6 \times 0.25 & 0.4 \times 0.25\\ 0.4 \times 0.75 & 0.6 \times 0.75\end{bmatrix} =$

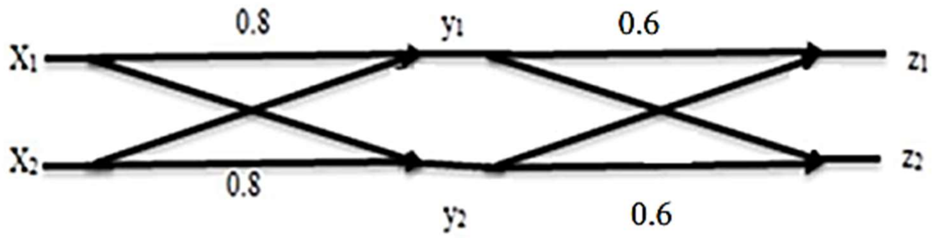
$P(Y) =$

$\rightarrow H(Y) =$ bits/symbol

$I(X,Y) = H(Y) + K =$ bits/symbol

Then $\eta =$

Q25/ Two BSC is cascaded as shown:



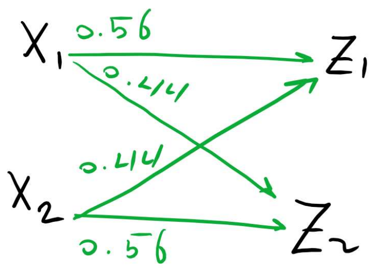
a- Find the resultant channel matrix; then, plot the final model

b- Find the channel capacity and efficiency of resultant channel matrix if $I(X_2) = 2$bits

Solution / a/

$$(Z/X) = P(Y/X) * P(Z/Y)$$

$$= \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} * \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.56 & 0.44 \\ 0.44 & 0.56 \end{bmatrix}$$



b/ $I(X_2) = 2bits =$ ⋙ $P(X_2) = 2^{-2} = 0.25$

$$P(X_1) = 1 - P(X_2) = 1 - 0.25 = 0.75$$

$$P(X) = [\,0.75 \quad 0.25\,]$$

$$P(x, z) = P(z/x).P(x) = \begin{bmatrix} 0.56 & 0.44 \\ 0.44 & 0.56 \end{bmatrix} * [\,0.75 \quad 0.25\,]$$

$$P(x,z) = \begin{bmatrix} 0.42 & 0.33 \\ 0.11 & 0.14 \end{bmatrix}$$ from Colum sum we get

$$P(Z) = [\,0.53 \quad 0.47\,]$$

Find K from $P(z/x)$

$$K = \sum_{j=1}^{2} P(z/x) log_2 P\left(\frac{z}{x}\right)) = 0.56 log_2(0.56) + 0.44\, log_2\,(0.44)$$

$$= -0.989\; bit/symbol$$

Find H(Z) from $P(Z) = [\,0.53 \quad 0.47\,]$

$$H(z) = -\sum_{j=1}^{2} P(z_j) log_2 P(z_j) = -[\,0.53\, log_2 0.53 + \;0.47\, log_2 0.47\,]$$

$$= 0.997\; bit/\; symbol$$

$$I(x,z) = H(z) + K = 0.997 - 0.989 = 0.008\; bit\,/\,symbol$$

$$C = log_2 m + K = log_2(2) + K = 1 - 0.989 = 0.011$$

$$\mu = \frac{I(x,y)}{C} = \frac{0.008}{0.011} * 100\,\% = 72.7\%$$

Q27/ Find the code efficiency of a fixed length code for 15 equiprobable messages

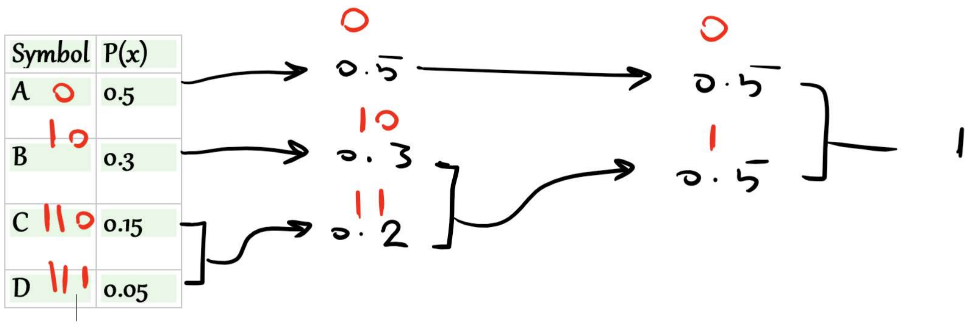
n = 15

Lc = int [$log_2 15$] + 1 = 4 bits/message

$H(x) = log_2 15 = 3.906$ bits/message

$$\mu = \frac{H(X)}{Lc} = \frac{3.906}{4} * 100\,\% = 97.6\,\%$$

Q29/ Design Huffman code for the following text AABBAABAAACCBCBBDAAA” then find code efficiency, p(0), and p(1).



Symbol	Number	P(x)	Codeword	li	0i	1i
A	10	0.5	0	1	1	0
B	6	0.3	10	2	1	1
C	3	0.15	110	3	1	2
D	1	0.05	111	3	0	3

$$H(X) = -\sum_{i=1}^{4} P(Xi) log_2 P(Xi) = 1.64 \text{bit/symbol}$$

$$Lc = \sum_{i=1}^{5} P(xi) Li = 1.7 \ bit/symbol$$

$$\eta = \frac{H(x)}{L_c} = 96.4\ \%$$

$$p(0) = \frac{\sum_{i=1}^{7} 0_i \, p(x_i)}{L_C} = 0.558 \quad , P_{(1)} = 1 - P_{(0)} = 0.442$$

Q29/ For the following joint probability matrix:

$$p(x,y) = \begin{bmatrix} 0.32 & 0.08 & 0 \\ 0 & 0.32 & 0.08 \\ 0.04 & 0 & 0.16 \end{bmatrix}$$

Find the channel efficiency if p(x1) =0.4 and p(x2) =2p(x3).

Solution /

$$p(x,y) = \begin{bmatrix} 0.32 & 0.08 & 0 \\ 0 & 0.32 & 0.08 \\ 0.04 & 0 & 0.16 \end{bmatrix}$$

$P(x_1) = 0.4$

$P(x_2) = 0.4$

$P(x_3) = 0.2$

جمع الصفوف

$P(y) = [0.4 \quad 0.4 \quad 0.2]$ ← جمع الأعمدة

find $P(y/x) = \dfrac{P(x,y)}{P(x)}$ } نقسم كل سطر على x التابع له

بعدها نجد K و $I(X,Y)$

و $\underline{\underline{C}}$

Q15) Having the text [A A B B A A B A A A C C B C B B D A A A], what is the minimum codeword using Huffman code. What is the code efficiency?

Solution:

A	0.5 →	0.5 →	0.5 0	0	1
B	0.3 →	0.3 0 →	0.5 1	10	2
C	0.15 0 →	0.2 1		110	3
D	0.05 1			111	3

$H(x) = \sum_{i=1}^{n} P(xi) \quad \log_2 \; P(Xi) \; = 0.5 \log_2 0.5 + 0.3 \log_2 0.3 +$ ------
$=1.64$bit/symbol

$L_c = \sum_{i=1}^{4} li \; P(xi) = 1.7$bits/symbol

$\eta \quad = \frac{H(x)}{L_c} = 96.4\ \%$

Q7) Develop Huffman code for A=[a_1,a_2,a_3,a_4,a_5] having the corresponding probabilities [0.05 , 0.4, 0.3, 0.15, 0.1] find code efficiency and redundancy.

Solution:

symbol	Pro.	Step1	Step2	Step3	Step4	Code	Li
a_2	0.4	0.4	0.4	0.6	1	1	1
a_3	0.3	0.3	0.3	0.4		00	2
a_4	0.15	0.15	0.3			010	3
a_5	0.1	0.15				0110	4
a_1	0.05					0111	4

$$H(X) = -\sum_{i=1}^{n} p(x_i)\, log_2 p(x_i)$$

$$= -\left[\frac{0.4ln0.4 + 0.3ln0.3 + 0.15ln0.15 + 0.1ln0.1 + 0.05ln0.05}{ln2}\right]$$

$$= 2.008\ bit/symbol$$

$$L_C = 0.4 * 1 + 0.3 * 2 + 0.15 * 3 + 0.1 * 4 + 0.05 * 4$$

$$= 2.05\ bits/symbol$$

$$\eta = \frac{H(X)}{L_C} \times 100 = \frac{2.008}{2.05} \times 100 = 97.95\%$$

$$redunduncy = 100 - efficiency\% = 100 - 97.95\% = 2.05\%$$

Q8)

A) Design Shannon Fano for A=[a_1, a_2, a_3, a_4, a_5, a_6, a_7] having the corresponding probability [0.4, 0.08, 0.16, 0.12, 0.1, 0.1, 0.04] find code efficiency.

B) Find code efficiency for 32 equiprobable messages coded using fixed length code.

Solution:

Symbol	Probability	Code				Li
a1	0.4	0	0			2
a3	0.16	0	1			2
a4	0.12	1	0	0		3
a5	0.10	1	0	1		3
a6	0.10	1	1	0		3
a2	0.08	1	1	1	0	4
a7	0.04	1	1	1	1	4

$$H(X) = -\sum_{i=1}^{n} p(x_i)\, log_2 p(x_i)$$

$$= -\left[\frac{0.4ln0.4 + 0.16ln0.16 + 0.12ln0.12 + 2 * 0.1ln0.1 + 0.08ln0.08 + 0.04ln0.04}{ln2}\right]$$

$$= 2.460\ bit/symbol$$

$$L_C = 0.4 * 2 + 0.16 * 2 + 0.12 * 3 + 2 * 0.1 * 3 + 0.08 * 4 + 0.04 * 4$$

$$= 2.56\ bits/symbol$$

$$\eta = \frac{H(X)}{L_C} \times 100 = \frac{2.460}{2.56} \times 100 = 96.093\%$$

B) $L_c = log_2 n = log_2 32 = 5bit$

$$H(x) = log_2 n = log_2 32 = 5\ bit/symbol$$

$$\eta = \frac{H(X)}{L_C} \times 100 = 100\%$$

Q2) A symbols (x_1, x_2, x_3, x_4, x_5, x_6 and x_7), with a prpbabilities of (0.11, 0.04, 0.12, 0.13, 0.45, 0.1 and 0.05) respectively. Develop Hufmman code to obtain the binary code for each symbol. Calculate the code efficincy.

<u>Solution</u>

$$\eta = \frac{2.26279}{2.23} \times 100 = 97.75\%$$

Q18) For the following symbols for {a_1,a_2,a_3,a_4,a_5,a_6,a_7} having the probabilities (0.25, 0.35, 0.03, 0.12, 0.07, 0.18}:

(a) Design Shannon-Fano code.
(b) Calculate the code efficiency

Solution:

a.

A	P(A)	Code			l_i
a2	0.35	0	0		2
a1	0.25	0	1		2
a6	0.18	1	0	0	3
a4	0.12	1	0	1	3
a5	0.07	1	1	0	3
a3	0.03	1	1	1	3

b.

$$\eta = \frac{H(x)}{L_c} \times 100$$

$$H(x) = \sum \log_2 P(x_i) \times P(x_i)$$

$$H(x) = \frac{(0.35\times\ln(0.35)+0.25\times\ln(0.25)+0.18\times\ln(0.18))+0.12\times\ln(0.12)+ 0.07\times\ln(0.07)+0.03\times\ln(0.03)}{\ln(2)} = 2.26279 \frac{bits}{symbol}$$

$$L_C = \sum_i l_i \times P(x_i)$$

$$= 2 \times (0.35 + 0.25) + 3 \times (0.18 + 0.12 + 0.07 + 0.03) = 2.4 \frac{bits}{message}$$

$$\eta = \frac{2.26279}{2.4} \times 100 = 94.28\%$$