# Friction

In the preceding lectures, the forces of action and reaction between contacting surfaces are assumed that act normal to the surfaces. This assumption characterizes the interaction between smooth surfaces. Although this ideal assumption often involves only a relatively small error, there are many problems in which we must consider the ability of contacting surfaces to support tangential as well as normal forces. Tangential forces generated between contacting surfaces are called *friction forces* and occur to some degree in the interaction between all real surfaces. Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency.

In some types of machines and processes we want to *minimize* the retarding effect of friction forces. Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere. In other situations we wish to *maximize* the effects of friction, as in brakes, clutches, belt drives, and wedges. Wheeled vehicles depend on friction for both starting and stopping, and ordinary walking depends on friction between the shoe and the ground.

## Types of Friction

(a) Dry Friction.

Dry friction occurs when the *unlubricated* surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always *opposes* the motion or impending motion.

(b) Fluid Friction.

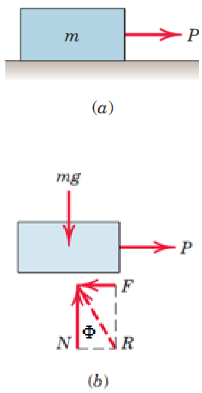
Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction. Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers.

(c) Internal Friction.

Internal friction occurs in all solid materials which are subjected to cyclical loading. For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction. For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this deformation. The mechanism of internal friction is associated with the action of shear deformation, which is discussed in references on materials science.

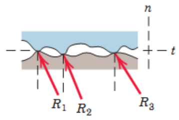
## Dry Friction

Consider a solid block of mass m resting on a horizontal surface, as shown in Fig. a. the contacting surfaces are assumed have some roughness. The *experiment* involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of P is shown in Fig. b, where the tangential friction force exerted by the plane on the block is labeled F. This friction force acting on the body will always be in a direction to *oppose* motion or the tendency toward motion of the body. There is also a normal force N which in this case equals mg, and the total force R exerted by the supporting surface on the block is the resultant of N and F.



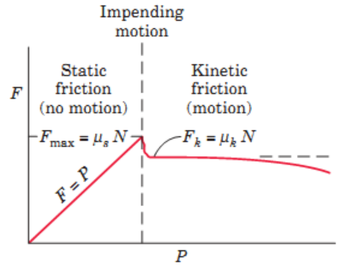

N: normal force
F: tangential friction force
R: the resultant of N and F
Φ: angle of internal friction

A magnified view of the irregularities of the mating surfaces, Fig. c, helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps. The direction of each of the reactions on the block, R1, R2, R3, etc. depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force N is the sum of the n-components of the R's, and the total frictional force F is the sum of the t-components of the R's. When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another. This observation helps to explain the well-known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.



(c)

If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Fig. d. When P is zero, equilibrium requires that there be no friction force. As P is increased, the friction force must be equal and opposite to P as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.



(d)

## Static Friction

The region in Fig. d up to the point of slippage or impending motion is called the range of *static friction*, and in this range the value of the friction force is determined by the equations of equilibrium. This friction force F may have any value from zero up to and including the maximum value $F_{max}$. For a given pair of mating surfaces the experiment shows that this maximum value of static friction $F_{max}$ is proportional to the normal force N. Thus, we may write

$F_{max} = \mu_s N$ Applies only to cases where motion is impending with the friction force at its peak value.

$F_{max}$: maximum frictional force.
$\mu_s$: Coefficient of static friction, $\mu_s = \tan \emptyset_s = \frac{F}{N}$
N: normal force.

For a condition of *static equilibrium* when motion is *not* impending, the static friction force is

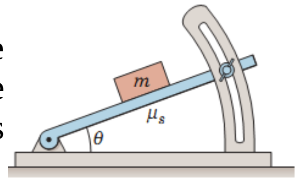
$$F < \mu_s N$$

## Kinetic Friction

After slippage occurs, a condition of *kinetic friction* accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force $F_k$ is also proportional to the normal force. Thus,

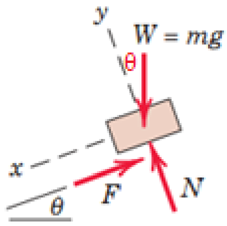
$F_k = \mu_k N$
$\mu_k$: Coefficient of kinetic friction, $\mu_k = \tan \emptyset_k = \frac{F}{N}$, ($\mu_k$ is generally less than $\mu_s$)
$F_k$: Kinetic friction force

## SAMPLE PROBLEM 6/1




Determine the maximum angle $\boldsymbol{\theta}$ which the adjustable incline may have with the horizontal before the block of mass $\boldsymbol{m}$ begins to slip. The coefficient of static friction between the block and the inclined surface is $\mu_s$.

Solution.




The free-body diagram of the block shows its weight W=mg, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present.

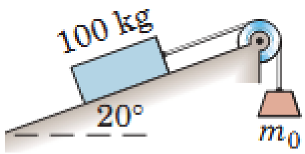
$$\nearrow \sum F_x = 0 \Rightarrow F - mg\sin\theta = 0 \Rightarrow F = mg\sin\theta$$

$$\nwarrow \sum F_y = 0 \Rightarrow N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$$

$$F_{max} = \mu_s N = \mu_s mg\cos\theta$$

Max. Angle θ occurs ⇒impending motion⇒ $F = F_{max} \Rightarrow mg\sin\theta = \mu_s mg\cos\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \mu_s \Rightarrow \tan\theta = \mu_s \Rightarrow \theta = \tan^{-1}\mu_s$

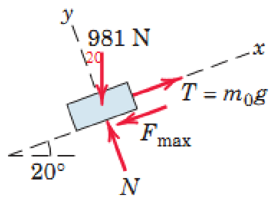
## SAMPLE PROBLEM 6/2




Determine the range of values which the mass $m_0$ may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.

Solution.

The maximum value of $m_0$ will be given by the requirement for motion impending *up* the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure.



Case I

$$W = mg = 100 * 9.81 = 981N$$

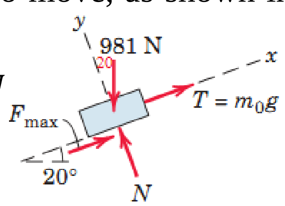
$$\nwarrow \sum F_y = 0 \Rightarrow N - W\cos 20 = 0 \Rightarrow N - 981\cos 20 = 0 \Rightarrow N = 922N$$

$$F_{max} = \mu_s N = 0.3 * 922 = 277N$$

$$\nearrow \sum F_x = 0 \Rightarrow m_0 g - F_{max} - W\sin 20 = 0$$

$$\Rightarrow m_0 * 9.81 - 277 - 981\sin 20 = 0 \Rightarrow m_0 = 62.4kg \quad \text{maximum value}$$

The minimum value of $m_0$ is determined when motion is impending *down* the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II.



Case II

$$\nwarrow \sum F_y = 0 \Rightarrow N - W\cos 20 = 0 \Rightarrow N - 981\cos 20 = 0 \Rightarrow N = 922N$$

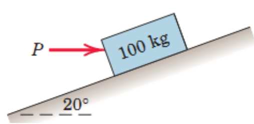
$$F_{max} = \mu_s N = 0.3 * 922 = 277N$$

$$\nearrow \sum F_x = 0 \Rightarrow m_0 g + F_{max} - W\sin 20 = 0$$

$$\Rightarrow m_0 * 9.81 + 277 - 981\sin 20 = 0 \Rightarrow m_0 = 6kg \quad \text{minimum value}$$

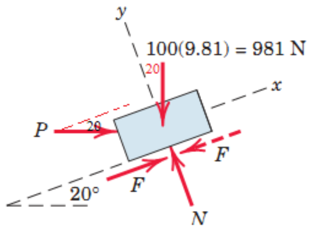
Thus, $m_0$ may have any value from 6 to 62.4 kg, and the block will remain at rest.

## SAMPLE PROBLEM 6/3




Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, P= 500 N and, second, P=100 N. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.

Solution.




There is no way of telling from the statement of the problem whether the block will remain in equilibrium or whether it will begin to slip following the application of P. It is therefore necessary that we make an assumption, so we will take the *friction force to be up the plane*, as shown by the solid arrow. From the free-body diagram a balance of forces in both x- and y-directions gives

$W = mg = 100 * 9.81 = 981N$

$$\nearrow \sum F_x = 0 \Rightarrow P\cos 20 + F - W\sin 20 = 0 \Rightarrow P\cos 20 + F - 981\sin 20 = 0 \ldots..1$$

$$\nwarrow \sum F_y = 0 \Rightarrow N - P\sin 20 - W\cos 20 = 0 \Rightarrow N - P\sin 20 - 981\cos 20 = 0 \ldots..2$$

Case I. P= 500 N

$P\cos 20 + F - 981\sin 20 = 0 \ldots..1 \Rightarrow 500\cos 20 + F - 981\sin 20 = 0 \Rightarrow F = -134N$

The negative sign tells us that *if* the block is in equilibrium, the friction force acting on it is in the direction opposite to that assumed and therefore is down the plane, as represented by the dashed arrow.

$N - P\sin 20 - 981\cos 20 = 0 \ldots..2 \Rightarrow N - 500\sin 20 - 981\cos 20 = 0 \Rightarrow N = 1093N$

$F_{max} = \mu_s N = 0.2 * 1093 = 219N$

Since $F_{max} = 219N > F = 134N$(required for equilibrium), we conclude that the assumption of static equilibrium was correct.

Case II. P= 100 N

$P\cos 20 + F - 981\sin 20 = 0 \ldots..1 \Rightarrow 100\cos 20 + F - 981\sin 20 = 0 \Rightarrow F$
$= +242N \textit{friction force to be up the plane is correct assumption.}$

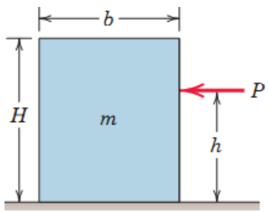
$N - P\sin 20 - 981\cos 20 = 0 \ldots..2 \Rightarrow N - 100\sin 20 - 981\cos 20 = 0 \Rightarrow N = 956N$

$F_{max} = \mu_s N = 0.2 * 956 = 191N$

Since $F = +242N > F_{max} = 191N$ ⇒Therefore, static equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane.

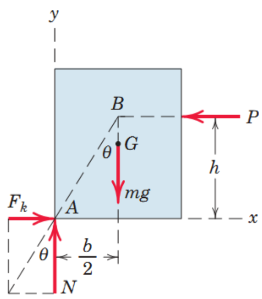
$F_k = \mu_k N = 0.17 * 956 = 163N \textit{up the plane}$

SAMPLE PROBLEM 6/4




The homogeneous rectangular block of *mass m*, *width b*, and *height H* is placed on the horizontal surface and subjected to a horizontal force *P* which moves the block along the surface with a constant velocity. The coefficient of kinetic friction between the block and the surface is $\mu_k$. Determine (a) the greatest value which *h* may have so that the block will slide without tipping over and (b) the location of a point C on the bottom face of the block through which the resultant of the friction and normal forces acts if h=H/2.

Solution.



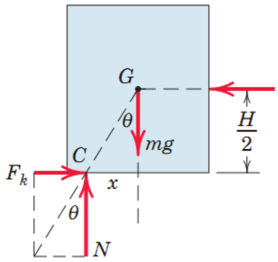

(a) With the block on the verge of tipping, we see that the entire reaction between the plane and the block will necessarily be at A. The free-body diagram of the block shows this condition. Since slipping occurs, the friction force is the limiting value $F_k = \mu_k N$, and the angle θ becomes $\theta = \tan^{-1}\mu_k$. The resultant of $F_k$ and N passes through a point B through which P must also pass, since three coplanar forces in equilibrium are concurrent. Hence, from the geometry of the block

$$\tan\theta = \mu_k = \frac{b/2}{h} \Rightarrow h = \frac{b}{2\mu_k} \quad ans.$$

If $h > \frac{b}{2\mu_k}$, moment equilibrium about A would not be satisfied, and the block would tip over.

OR

$$\circlearrowright\sum M_B=0\Rightarrow N\frac{b}{2}-F_kh=0\Rightarrow h=\frac{Nb}{2F_k}\Rightarrow h=\frac{Nb}{2\mu_kN}\Rightarrow h=\frac{b}{2\mu_k}\quad ans.$$



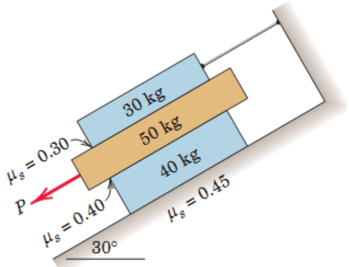

(b) With h=H/2 we see from the free-body diagram for case (b) that the resultant of $F_k$ and N passes through a point C which is a distance x to the left of the vertical centerline through G. The angle θ is still $\theta=\emptyset=\tan^{-1}\mu_k$ as long as the block is slipping. Thus, from the geometry of the figure

$$\tan\theta=\mu_k=\frac{x}{H/2}\Rightarrow x=\frac{\mu_kH}{2}\quad ans.$$

OR

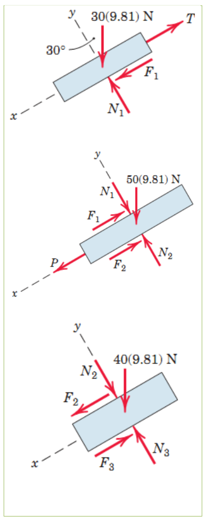
$$\circlearrowright\sum M_G=0\Rightarrow Nx-F_k\frac{H}{2}=0\Rightarrow x=\frac{F_kH}{2N}\Rightarrow x=\frac{\mu_kNH}{2N}\Rightarrow x=\frac{\mu_kH}{2}\quad ans.$$

## SAMPLE PROBLEM 6/5




The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the *maximum* value which P may have before any slipping takes place.

<u>Solution:</u>




$$\nwarrow\sum F_y=0$$

$30kg\ block\Rightarrow N_1-mg\cos 30=0\Rightarrow N_1-30*9.81\cos 30=0\Rightarrow N_1=255N$

$50kg\ block\Rightarrow N_2-N_1-mg\cos 30=0\Rightarrow N_2-255-50*9.81\cos 30=0$

$\Rightarrow N_2=680N$

$40kg\ block\Rightarrow N_3-N_2-mg\cos 30=0\Rightarrow N_3-N_2-40*9.81\cos 30=0$

$\Rightarrow N_3=1019N$

There are two possible conditions for *impending* motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50-and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

1. The 50-kg block slips, so that the 40-kg block remains in place (i.e. $F_1=F_{1max},F_2=F_{2max},F_3<F_{3max}$).

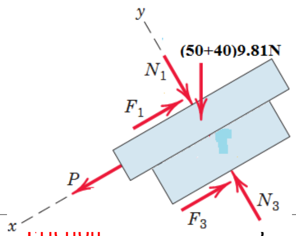
For *impending* slippage at both surfaces of the 50-kg block,

$F_{max}=\mu_sN$

$F_{1max}=\mu_sN_1=0.3*255\Rightarrow F_{1max}=76.5N$

$F_{2max}=\mu_sN_2=0.4*680\Rightarrow F_{2max}=272N$

$\swarrow\sum F_{x)_{50kg}}=0\Rightarrow P-F_1-F_2+mg\sin 30=0\Rightarrow P-76.5-272+50*9.81\sin 30=0\Rightarrow P=103.5N$




2. The 50-and 40-kg blocks move together with *impending* slippage occurring between the (50-and 40-kg blocks) and the (incline and 30-kg block)(i.e. $F_1=F_{1max},F_3=F_{3max},F_2<F_{2max}$)

$$\nwarrow \sum F_{y)}{}_{(40+50)kg} = 0 \Rightarrow N_3 - N_1 - (50+40) * 9.81 \cos 30 = 0$$

$$\Rightarrow N_3 - 255 - (50+40) * 9.81 \cos 30 = 0 \Rightarrow N_3 = 1019N$$

$$F_{3max} = \mu_s N_3 = 0.45 * 1019 \Rightarrow F_{3max} = 459N$$

$$\swarrow \sum F_{x)}{}_{(40+50)kg} = 0 \Rightarrow -F_3 - F_1 + P + (50+40)9.81 \sin 30 = 0$$

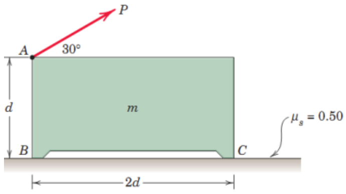
$$\Rightarrow -459 - 76.5 + P + (50+40)9.81 \sin 30 = 0 \Rightarrow P = 94N$$

Choose minimum value of $P(103.5, 94)N$,

$\therefore P = 94N \quad ans.$

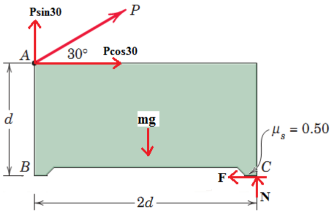
## Prob. 6/5

The magnitude of force **P** is slowly increased. Does the homogeneous box of mass ***m*** slip or tip first? State the value of **P** which would cause each occurrence. Neglect any effect of the size of the small feet.



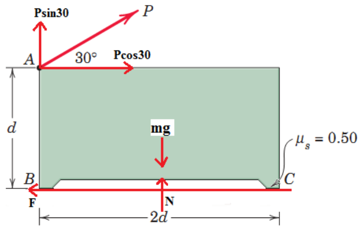

Solution:

1. Assume tipping occur first about point c.




$$\circlearrowright \sum M_C = 0 \Rightarrow P \sin 30 * 2d + P \cos 30 * d - mg\frac{2d}{2} = 0 \Rightarrow P_{\text{tipping}} = 0.536mg$$

2. Assume sliding occur first.




$$\uparrow \sum F_y = 0 \Rightarrow N + P \sin 30 - mg = 0 \Rightarrow N = mg - P \sin 30$$

$$\leftarrow \sum F_x = 0 \Rightarrow F - P \cos 30 = 0 \Rightarrow F = P \cos 30$$

$F_{max} = \mu_s N = 0.5(mg - P \sin 30)$

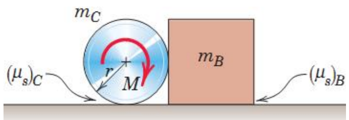
For slip beginning, $F = F_{max} \Rightarrow$

$P \cos 30 = 0.5(mg - P \sin 30) \Rightarrow P_{slipping} = 0.448mg$

Since $P_{slipping} = 0.448mg < P_{tipping} = 0.536mg \Rightarrow \therefore$ *slipping occur first Ans.*

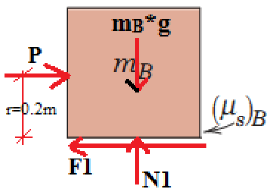
## Prob. 6/24

A clockwise couple **M** is applied to the circular cylinder as shown. Determine the value of **M** required to initiate motion for the conditions **$m_B$=3kg, $m_C$=6kg, $(\mu_s)_B$=0.5, $(\mu_s)_C$=0.4, and r = 0.2 m**. Friction between the cylinder C and the block B is negligible.




Solution:

Body B:



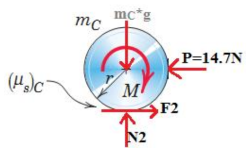

$$\uparrow \sum F_y = 0 \Rightarrow N_1 - m_B g = 0 \Rightarrow N_1 - 3 * 9.81 = 0 \Rightarrow N_1 = 29.43N$$

$F_{1max} = \mu_{B)s} N_1 = 0.5 * 29.43 = 14.7N$

For slipping of body B, $F_1 = F_{1max} = 14.7N$

$$\leftarrow \sum F_x = 0 \Rightarrow F_1 - P = 0 \Rightarrow P = F_1 = 14.7N$$

Body C:




$$\uparrow \sum F_y = 0 \Rightarrow N_2 - m_C g = 0 \Rightarrow N_2 - 6 * 9.81 = 0 \Rightarrow N_2 = 59N$$

$$\rightarrow \sum F_x = 0 \Rightarrow F_2 - P = 0 \Rightarrow F_2 - 14.7 = 0 \Rightarrow F_2 = 14.7N$$

$F_{2max} = \mu_{C)s} N_2 = 0.4 * 59 = 23.5N$

$F_2 = 14.7N < F_{2max} = 23.5N$ O.K (for motion, not slipping)

$$\circlearrowright \sum M_{Center} = 0 \Rightarrow M - F_2 * r = 0 \Rightarrow M - 14.7 * 0.2 = 0 \Rightarrow M = 2.94N.m \circlearrowright$$