# *NUMBER SYSTEMS*

## *Introduction*

There are several number systems which we normally use, such as decimal, binary, octal, hexadecimal, etc. Amongst them we are most familiar with the decimal number system. These systems are classified according to the values of the base of the number system. The number system having the value of the base as 10 is called a decimal number system, whereas that with a base of 2 is called a binary number system. Likewise, the number systems having base 8 and 16 are called octal and hexadecimal number systems respectively.
With a decimal system we have 10 different digits, which are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. But a binary system has only 2 different digits: 0 and 1. Hence, a binary number cannot have any digit other than 0 or 1. So to deal with a binary number system is quite easier than a decimal system. Now, in a digital world, we can think in binary nature, e.g., a light can be either off or on. There is no state in between these two. So we generally use the binary system when we deal with the digital world. Here comes the utility of a binary system. We can express everything in the world with the help of only two digits i.e., 0 and 1. For example, if we want to express 2510 in binary we may write $11001_2$. The right most digit in a number system is called the 'Least Significant Bit' (LSB) or 'Least Significant Digit' (LSD). And the left most digit in a number system is called the 'Most Significant Bit' (MSB) or 'Most Significant Digit' (MSD). Now normally when we deal with different number systems we specify the base as the subscript to make it clear which number system is being used.
In an octal number system there are 8 digits 0, 1, 2, 3, 4, 5, 6, and 7. Hence, any octal number cannot have any digit greater than 7. Similarly, a hexadecimal number system has 16 digits 0 to 10 and the rest of the six digits are specified by letter symbols as A,B, C, D, E, and F. Here A, B, C, D, E, and F represent decimal 10, 11, 12, 13, 14, and 15
respectively.

## *CONVERSION BETWEEN NUMBER SYSTEMS*

It is often required to convert a number in a particular number system to any other number system, e.g., it may be required to convert a decimal number to binary or octal or hexadecimal. The reverse is also true, i.e., a binary number may be converted into decimal and so on.

### *Decimal-to-binary Conversion*

Now to convert a number in decimal to a number in binary we have to divide the decimal number by 2 repeatedly, until the quotient of zero is obtained. This method of repeated division by 2 is called the 'double-dabble' method. The remainders are noted down for each of the division steps. Then the column of the remainder is read in reverse order i.e., from bottom to top order. We try to show the method with an example shown in Example 1.1.

***Example 1.1.*** Convert $(26)_{10}$ into a binary number.

***Solution:***

| Division | Quotient | Generated remainder |
|---|---|---|
| $\frac{26}{2}$ | 13 | 0 |
| $\frac{13}{2}$ | 6 | 1 |
| $\frac{6}{2}$ | 3 | 0 |
| $\frac{3}{2}$ | 1 | 1 |
| $\frac{1}{2}$ | 0 | 1 |

Hence the converted binary number is $(11010)_2$.

## *Decimal-to-octal Conversion*

Similarly, to convert a number in decimal to a number in octal we have to divide the decimal number by 8 repeatedly, until the quotient of zero is obtained. This method of repeated division by 8 is called 'octal-dabble.' The remainders are noted down for each of the division steps. Then the column of the remainder is read from bottom to top order, just as in the case of the double-dabble method. We try to illustrate the method with an example shown in Example 1.2.

***Example 1.2.*** Convert $(426)_{10}$ into an octal number.

***Solution:***

| Division | Quotient | Generated remainder |
|---|---|---|
| $\frac{426}{8}$ | 53 | 2 |
| $\frac{53}{8}$ | 6 | 5 |
| $\frac{6}{8}$ | 0 | 6 |

Hence the converted octal number is $(652)_8$.

### *Decimal-to-hexadecimal Conversion*

The same steps are repeated to convert a number in decimal to a number in hexadecimal. Only here we have to divide the decimal number by 16 repeatedly, until the quotient of zero is obtained. This method of repeated division by 16 is called ‘hex-dabble.’ The remainders are noted down for each of the division steps. Then the column of the remainder is read from bottom to top order as in the two previous cases. We try to discuss the method with an example shown in Example 1.3.

***Example 1.3.*** Convert $(348)_{10}$ into a hexadecimal number.

***Solution:***

| Division | Quotient | Generated remainder |
|---|---|---|
| $\frac{348}{16}$ | 21 | 12 |
| $\frac{21}{16}$ | 1 | 5 |
| $\frac{1}{16}$ | 0 | 1 |

Hence the converted hexadecimal number is $(15C)_{16}$.

### *Binary-to-decimal Conversion*

Now we discuss the reverse method, *i.e.*, the method of conversion of binary, octal, or hexadecimal numbers to decimal numbers. Now we have to keep in mind that each of the binary, octal, or hexadecimal number system is a positional number system, *i.e.*, each of the digits in the number systems discussed above has a positional weight as in the case of the decimal system. We illustrate the process with the help of examples.

***Example 1.4.*** Convert $(10110)_2$ into a decimal number.

***Solution.*** The binary number given is 1 0 1 1 0

Positional weights *4 3 2 1 0*

The positional weights for each of the digits are written in italics below each digit.

Hence the decimal equivalent number is given as:

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 0 + 4 + 2 + 0$

$= (22)_{10}$.

Hence we find that here, for the sake of conversion, we have to multiply each bit with its positional weights depending on the base of the number system.

### *Octal-to-decimal Conversion*

***Example 1.5.*** Convert $3462_8$ into a decimal number.

**Solution.** The octal number given is 3 4 6 2

Positional weights 3 2 1 0

The positional weights for each of the digits are written in italics below each digit. Hence the decimal equivalent number is given as:

$3 \times 8^3 + 4 \times 8^2 + 6 \times 8^1 + 2 \times 8^0$

$= 1536 + 256 + 48 + 2$

$= (1842)_{10}$.

### *Hexadecimal-to-decimal Conversion*

***Example 1.6.*** *Convert $42AD_{16}$ into a decimal number.*

**Solution.** The hexadecimal number given is 4 2 A D

Positional weights *3 2 1 0*

The positional weights for each of the digits are written in italics below each digit. Hence the decimal equivalent number is given as:

$4 \times 16^3 + 2 \times 16^2 + 10 \times 16^1 + 13 \times 16^0$

$= 16384 + 512 + 160 + 13$

$= (17069)_{10}$.

### *Fractional Conversion*

So far we have dealt with the conversion of integer numbers only. Now if the number contains the fractional part we have to deal in a different way when converting the number from a different number system (*i.e.*, binary, octal, or hexadecimal) to a decimal number system or vice versa. We illustrate this with examples.

***Example 1.7.*** Convert $1010.011_2$ into a decimal number.

**Solution.** The binary number given is 1 0 1 0. 0 1 1

Positional weights *3 2 1 0 -1-2-3*

The positional weights for each of the digits are written in italics below each digit. Hence the decimal equivalent number is given as:

$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$

$= 8 + 0 + 2 + 0 + 0 + 0.25 + 0.125$

$= (10.375)_{10}$.

***Example 1.8.*** *Convert $362.35_8$ into a decimal number.*

**Solution.** The octal number given is 3 6 2. 3 5

Positional weights *2 1 0 -1-2*

The positional weights for each of the digits are written in italics below each digit. Hence the decimal equivalent number is given as:

$3 \times 8^2 + 6 \times 8^1 + 2 \times 8^0 + 3 \times 8^{-1} + 5 \times 8^{-2}$

$= 192 + 48 + 2 + 0.375 + 0.078125$

$= (242.453125)_{10}$.

***Example 1.9.*** *Convert 42A.12$_{16}$ into a decimal number.*

***Solution.*** The hexadecimal number given is 4 2 A. 1 2

Positional weights *2 1 0 -1-2*

The positional weights for each of the digits are written in italics below each digit. Hence the decimal equivalent number is given as:

$4 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 + 1 \times 16^{-1} + 2 \times 16^{-2}$

$= 1024 + 32 + 10 + 0.0625 + 0.00390625$

$= (1066.06640625)_{10}$.

***Example 1.10.*** Convert *25.625*$_{10}$ into a binary number.

| **Solution.** Division | Quotient | Generated remainder |
|---|---|---|
| $\frac{25}{2}$ | 12 | 1 |
| $\frac{12}{2}$ | 6 | 0 |
| $\frac{6}{2}$ | 3 | 0 |
| $\frac{3}{2}$ | 1 | 1 |
| $\frac{1}{2}$ | 0 | 1 |

Therefore, $(25)_{10} = (11001)_2$.

Fractional Part:

| | | |
|---|---|---|
| 0.625 | 0.250 | 0.500 |
| ×2 | ×2 | ×2 |
| 1.250 | 0.500 | 1.000 |
| ↓ | ↓ | ↓ |
| 1 | 0 | 1 |

i.e., $(0.625)_{10} = (0.101)_2$

Therefore, $(25.625)_{10} = (11001.101)_2$