Feebly pT(i,k)-spaces in bitopological spaces

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Abstract

In this research we studied a PT(i,k)-spaces by using feebly sets which defined by maheshwari and we find a relation between these spaces and we called it a feebly PT(i,k)-spaces.

Introduction

S.N Maheshwari (1990) define a feebly open set in a topological space . A set A is said to be feebly open (Gyn and Lee ,1984) if there exist an open set O in X such that $O\subset A\subset scl(O)$ where scl denotes the closure set in the topological space .

In bitopological space (X,T_1,T_2) M. Jelic (1994) give a new definition of pair wise T(i,k))-spaces . A bitopological space X is said to be a pair wise T(i,k) –space if for every $x\in X$ and every pT_k-open cover U of X there exist a pTi –open V of X and a $u\in U$ such that $st(x,v)\subset U$, $i,k\in\{1,2,3\}$,and it is denoted by pT(i, k)-space .

In this paper we shall introduce a new definition of pT(i, k)-space by using feebly open set and we shall investigate the relation between these spaces .

2-preliminaries

In this section we shall investigate some properties of feebly open sets in bitopological spaces and give a new definition of pTi-cover by using a feebly open set and discuss a relation between them.

Remark(2-1) (Gyn and Lee ,1984)

Every open set is feebly –open set and the converse is not true.

Theorem (2-2) (Gyn and Lee ,1984)

Any union of feebly –open sets is feebly open.

Definition(2-3) (Gyn and Lee ,1984)

A point p in X is said to be feebly interior point of A if A is feebly neighborhood of p , and the set of all feebly interior points of A is denoted by int(A)

Definition(2-4) (Gyn and Lee ,1984)

A set A in a topological space is said to be feebly–closed if it is complement is feebly open.

Remark(2-5) S.N Maheshwari (1990)

A set A in a topological space is feebly–open iff fint(A)= A

Remark(2-6) (Gyn and Lee ,1984)

The smallest feebly –closed set containing A is called feebly –closure of A and it is denoted by fcl(A) and fcl(A)=fcl(fcl(A).

Theorem(2-7) S.N Maheshwari (1990)

A subset A of a topological space X is called feebly closed iff fcl(A)=A

Remark(2-8)

A collection of all feebly open set is denoted by F.O(X) and we mean by $F.O_{Ti}(X)$ to be a set of all feebly open set with respect to T_1 and T_2 respectively .

Theorem(2-9) (Gyn and Lee ,1984)

If $B\subset X$ then $int(B)\subset fint(B)\subset sint(B)\subset B\subset scl(B)\subset cl(B)$ where sint denotes semi–interior in X and fint denotes feebly interior inX.

Theorem(2-10) (Gyn and Lee ,1984)

If A is feebly –open in a space X and $A\subset B\subset scl(A)$ then B is feebly open

Theorem(2-11)

If V and W are open in X and A is feebly –open and $W\subset A\subset scl(A)$ then $(V \cap w)\neq\emptyset$ and then $(V\cap A)\neq\emptyset$ is feebly –open

Proof: see (Gyn and Lee ,1984)

3- Feebly pT(I , k)-space.

In bitopological space (X,T_1,T_2) a cover U of X is pair wise open if $U\subset T_1\subset T_2$ and if U contains anon –empty member of T_1 and T_2 (Flether, and *et al.,* 1969) .

This pair wise open cover is called pT_1-open and is pT_2-open if for each $u\in U$. $int_{Ti}(X/u)\neq\emptyset$ for i=1 or 2 where int_{Ti} is the interior with respect to T_i and it is called pT_3-open if for each $w\in U$ whenever $w\in T_i$, there exist anon –empty T_i-open sets V_1,V_2 such that $V_1\subset cl_{Ti}(V_1)\subset V_2\subset(X/w)$ for $i\neq j$ and i=1,2 and cl_{Ti} is the closure with respect to T_i [3]in this section we shall define these covers by using feebly –open set and define pT(i,k)-spaces also by feebly–open and investigate areolation between these spaces.

Definition(3-1)

A cover U of a bitopological space (X, T_1,T_2) is called feebly pair wise open if $U\subset F.O_{T1}(X)\subset F.O_{T2}(X)$ and U contains anon –empty member of $F.O_{T1}(X)$ and a non empty member of

$F.O_{T2}(X)$ then a feebly pair wise open cover is called feebly pair wise T_1-open and it is dented by FpT_1-open.

Definition (3-2)

A feebly pair wise open cover U of abitopological space (X,T_1,T_2) is said to be feebly pT_2-open if for each $u\in U$, $fint_{Ti}(X/u)\neq\emptyset$, for i=1,2 and it is denoted by FpT_2-open , where $fint_{T2}$ denotes a feebly interior with respect to Ti, i=1,2 .

Definition(3-3)

A feebly pair wise open cover W of a bitopological space (X,T_1,T_2) is said to be feebly pT_3-open if for each $w\in W$ whenever $w\in T_I$, there exist T_I –feebly open sets V_1,V_2 such that $V_1,V_2\neq\emptyset$, $V_1\subset fcl_{Ti}(V_1)\subset V_2\subset(X/w)$ and i=1,2 and it is denoted by FpT_3-open, where fcl_{Ti} is a feebly closure with respect to T_i , i=1,2.

Definition(3-4)

A bitopological (X,T_1,T) is said to be feebly pair wise $T(i,k)$ –space ,$i,k\in\{1,2,3\}$ if for every $x\in X$ and every FpT_k-open cover U of X there exist a $FpT_i(i,k)$ –open cover V of X and U such that $F.st(X,V)\subset U$ and it is denoted by FPT(i,k)-space ,where F.st is a feebly Neighborhood system.

Lemma(3-5)

Let (X,T_1,T_2) be a bitopological space then

1-every FpT_3-open cover is FpT_2-open cover

2- every FpT_2-open cover is FpT_1-open cover.

Proof(1):-

Let U be a FpT_3-open cover , then for each $u\in U$ whenever u is feebly open with respect to T_i, there exist two non empty T_i-feebly open sets V_1,V_2 such that $V_1\subset Fcl_{Ti}\subset V_2\subset X/U$

$Fint_{Ti}(V_1)\subset(fint(fcl))_{Ti}(V_1)\subset fint_{Ti}(V_2)\subset fint_{Ti}(X/U)$ which is mean $fint_{Ti}(X/U)\neq\varnothing$ then U is FpT_2-open cover

Proof(2):-

Let U be a FpT_2-open cover then for each $u\in U$ $fint_{Ti}(X/u)\neq\varnothing$, for i=1or2 Let $w\in T_i/int_{Ti}(X/u)$ for i=1 or 2 then $w\in T_I$ and $w\notin fint(X/u)$ $w\in T_i$ and then $w\in fint_{Ti}(u)$ from that we get u is feebly –open and then $fint_{Ti}(u)=u$ then $w\in T_i$ and $w\in u$ then U contains anon empty member T_i and i=1 or 2 there for U is FpT_1-open cover .

The following examples show that the converse of the above lemma is not true for (1)and (2) respectively.

<u>**Example(3-6):**</u>- let $X=\{a,b,c\}$, $T_1=\{X,\varnothing,\{a\},\{b,c\}\}$,

$T_2=\{X,\varnothing,\{a\},\{b\},\{c\},\{a,b\},\{b,c\}\{a,c\}\}$

$F.OT_1(X)=\{X,\{a\},\{b,c\},\{a,b\},\{a,c\}\}$

$F.OT_2(X)=\{X,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$

Let $U=\{X,\{a\},\{b\},\{b,c\},\{c\}\}$

Then U is feebly open cover since $U\subset F.OT_1\cup F.OT_2$

Now $\{a\}\in U$, $X/\{a\}=\{b,c\}$, $fintT_1\{b,c\}=\{b,c\}\neq\varnothing$

$fintT_2\{b,c\}=\{b\}\neq\varnothing$, FpT_2-open but not FpT_3-open.

Example(3-7):- let $X=\{a,b,c\}$\

$T_1=\{X,\varnothing,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\}\}$

$T_2=\{X,\varnothing,\{a\},\{b\},\{a,b\}\}$

$F.OT_1(X)=\{X,\{a\},\{b\},\{c\},\{b,c\}\{a,c\}\}$

$F.OT_2(X)=\{X,\{a\},\{b\},\{c\},\{a,b\}\}$

Let $U=\{X,\{a\},\{b\},\{a,b\},\{c\},\{b,c\}\}$

Then U is FpT-open cover since U contains anonempty member of $F.OT_1(X)$ and anon empty member of $F.OT_2(X)$

But U is not FpT_2 –open cover

Theorem(3-8)

let (X,T_1,T_2) be a bitopological space then FpT(3,3) –space is FpT(2,3)-space.

then for every x in X and FpT_3-open cover U of X , there exist FpT_3-open cover of X and $u\in U$ such that $F.st(X,V)\subset U$ now since V is FpT_3-open cover then V is FpT_2-open cover by

lemma(3-5)part (10).Then for every FpT_3-open cover ,there exist FpT_2-open cover satisfying the condition then X is FpT(2,3).

Corollary(3-9)

1- every FpT(2,k) –space is FpT(j,k)-space provided i>j and k constant i,j ,k, $\in\{1,2,3\}$

Proof: the proof exist by using lemma (3-6) part (1)and (2) .

2- every FpT(i,k)-space provided k<j and i constant $i,j,k\in\{1,2,3\}$.and the follow diagram is easy to prove

Remark(3-10):-

the convers of (1) and (2) of aremark above need not to be true , since from remark(3-6) and it is example we can make sure an example of each one of FpT(i,k) and FpT(i , k) –spaces by change of the value of i,j,k =1,2,3 and using the same covers of an examples remark(3-6).

FpT(3,3)-space$\Rightarrow$ FpT(2,3)-space $\Rightarrow$ FpT(1,3)-space

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FpT(3,2)-space $\Rightarrow$FpT(2,2)space $\Rightarrow$FpT(1,2)-space

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FpT(3,1)-space $\Rightarrow$ FpT(2,1)-space $\Rightarrow$FpT(1,1)-space

References

S.N. Maheshwari and U. Tapi (1990) “note oon some application feebly open sets” M.B.,J. University of Sauger (to appear).

Gyn Inn Chae and D.W. Lee (1984) "feebly –pen sets and feebly-continues in topological spaces" dep. F mathematics ,UIT reprt , 15, No.2, 367-371.

M. Jelic (1989) ”T_I-pairwise continues function and bitopological separation axioms mat”. Vesuik 41,3,155-159.

M. Jelic (1994) “nearly pT_I- continues mapping” MATEM,46,25-28.

P. Flether, H.B. Hoyle and C.W. Puth (1969) "the comparison of topologies" pure math. J.36,2,325-331.

الخلاصة

ان هذا البحث تناول فضاءات PT(I,k) الثنائية التبولوجي وذلك باستخدام المجموعة الواهنة (feebly set) والتي قام بتعريفها العالم Maheshwari حيث قمنا بايجاد العلاقة بين هذه الفضاءات واطلقنا عليها تسمية فضاءات PT(I,k) الواهنة .