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Successive Approximation for the Inhomogeneous Burgers Equation

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The inhomogeneous Burgers equation is a simple form of the Navier-Stokes equations. From the analytical point of view, the inhomogeneous form is poorly studied, the complete analytical solution depending closely on the form of the nonhomogeneous term.

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1. Introduction and preliminaries

The inhomogeneous Burgers equation is the simplest nonlinear model equation for diffusive waves in fluid dynamics. It reads

$$u'_t - \nu u''_{xx} + u u'_x = f, \tag{1.1}$$

where u stands generally for the fluid velocity, x the space variable, t the time variable, ν is the kinematic viscosity, or the diffusion coefficient, and f a given forcing term. The inverse $R = 1/\nu$ of the diffusion coefficient is known as the Reynold number. Burgers [3] first developed this equation primarily to shed some light on turbulence described by the interaction of two opposite effects of convection and diffusion. However, turbulence is more intricate in the sense that it is both three-dimensional and statistically random in nature. Note that equation (1.1) is parabolic, if $\nu > 0$, whereas (1.1) with $\nu = 0$ is hyperbolic. More importantly, the properties of solutions to parabolic equations are significantly different from those of hyperbolic equations.

The mathematical structure of equation (1.1) includes a nonlinear convection term uu'_x which makes the equation more interesting, and a viscosity term of higher order u''_{xx} which regularises the equation and produces a dissipation effect of the solution near a shock. When the viscosity

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