Journal of Interdisciplinary Mathematics

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjim20

Hpre-closed graph in topological spaces

Zinah Taha Abdulqader, Sarah Haider Khalil, Luma S. Abdalbaqi & Mustafa Hasan Hadi

To cite this article: Zinah Taha Abdulqader, Sarah Haider Khalil, Luma S. Abdalbaqi & Mustafa Hasan Hadi (2022) *Hpre*-closed graph in topological spaces, Journal of Interdisciplinary Mathematics, 25:6, 1823-1828, DOI: 10.1080/09720502.2022.2052572

To link to this article: https://doi.org/10.1080/09720502.2022.2052572

Published online: 26 Sep 2022.

Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
Vol. 25 (2022), No. 6, pp. 1823–1828
DOI : 10.1080/09720502.2022.2052572

Hpre-closed graph in topological spaces

Zinah Taha Abdulqader ‡
Department of Mathematics
College of Education for Woman
University of Tikrit
Tikrit
Iraq

Sarah Haider Khalil †
General Directorate of Education
Hillah
Iraq

Luma S. Abdalbaqi §
Department of Mathematics
College of Education for Woman
University of Tikrit
Tikrit
Iraq

Mustafa Hasan Hadi *
Department of Mathematics
College of Education for Pure Sciences
University of Babylon
Hillah
Iraq

Abstract

This paper aims to define new and consequential types of the closed graph in topology using *Hpre*-closed, we called them *Hpre*-closed graph, and generalizing and deepening them by comparing them with the separation of the chapter and finding types of relationships

‡ *E-mail:* `ztaha@tu.edu.iq`
† *E-mail:* `Sh7459281@gmail.com`
§ *E-mail:* `lumahhany1977@tu.edu.iq`
* *E-mail:* `pure.mustafa.hassan@uobabylon.edu.iq` (Corresponding Author)

among them, and the inverse examples that illustrate the difference between them are mentioned.

Subject Classification: *55N20, 55T05, 57Q05.*

Keywords: *Hpre-closed, Hpre-continuous, Hpre-T_1 space, Hpre-T_2 space.*

1. Introduction

The main element in the formation of any topology is the set, we must implement the idea of generalizing the set in the normal topology, as the topology is concerned with industry, agriculture, medical and engineering sciences, and through this important thing we will present our ideas in this research. The first to know *Hpre*-closed are Hadi, M.H. and Al-Yaseen, M.A.A. K. [1], which is the basis for the construction of our research, where they knew and studied the characteristics of them in terms of closure, interior, complement and the development of schematics that illustrate the relationships between them, and also presented applications about that set and this was shown through illustrative examples and there were many attempts on the development of topological concepts see references [2,3].

Some references were interested in studying other types of sets that are important in the topology, which are very different from our set, but the goal is one and the same i.e. the development of topological applications [4,5]. They clarified them with examples and diagrams through which the researcher can compare them with the regular graph. In [6, 7] they study approximate functions that are considered weaker in terms of application in practice, but are considered useful for building more comprehensive future research. [4,8] They were interested in studying analytic functions, which in turn were constructed from the definition of ordinary groups.

In [1,2,3] they defined types of continuity by $\omega -$ open and studied its characteristics and generalizations and explained that through examples and proofs because of its importance in the development of the various and diverse branches of mathematics. For more information on the subject, see [9,10]. We'll additionally prove and express the connections between *Hpre*-closed and the chapter's separation axioms based on a set of criteria that must be present in the evidence.

2. *Hpre*-Closed Graph

Definition 2.1 : [1] Let $\mathcal{H}\subseteq X$ is *Hpre*-closed set if $Cl_g(\mathrm{Int}(\mathcal{H}))\subseteq\mathcal{H}$. The complement is called *Hpre*-open. $Hpre\,C(X,x)$ and $Hpre\,O(X,x)$ are the means *Hpre*-closed sets and *Hpre*-open respectively. Also, any *Hpre*-T_2 is *Hpre*-T_1 however, this is not the case.

Let us introduce *Hpre*-closed graph and prove some theorems and result around and give some instances of how this notion may be applied.

Definition 2.2 : A function $f:X\to Y$ is called *Hpre*-closed graph if $(x,y)\in\forall(X\times Y)\setminus G(f)$, $\exists A\in$ $Hpre\ O(X,x)$ and $B\in GO\ (Y,y)$, s.t. $(A\times Cl_g\ (B))\bigcap G(f)=\varnothing$.

Remark 2.3 : Obviously any closed graph is *Hpre*-closed in general, but the inverse is not truly the case. See example:

Example 2.4 : Let $X=\{\varkappa_1,\varkappa_2,\varkappa_3\}$ with the topology $\tau=\{X,\varnothing,\{\varkappa_2\},\{\varkappa_2,\varkappa_3\}\}$ and $Y=\{\yen_1,\yen_2,\yen_3\}$ with the topology $\sigma=\{Y,\varnothing,\{\yen_1\},\{\yen_1,\yen_2\}\}$. A function $f:X\to Y$ be defined by $f(\varkappa_1)=f(\varkappa_3)=\yen_1$ and $f(\varkappa_2)=\yen_2$. Now $\tau^c=\{X,\varnothing,\{\varkappa_1\},\{\varkappa_1,\varkappa_3\}\}$, $pre\,O(X)=\{X,\varnothing,\{\varkappa_2\},\{\varkappa_2,\varkappa_3\},\{\varkappa_1,\varkappa_2\}\}$, $Pre\,C(X)=\{X,\varnothing,\{\varkappa_1\},\{\varkappa_3\},\{\varkappa_1,\varkappa_3\}\}$, $GO(X)=\{X,\varnothing,\{\varkappa_2\},\{\varkappa_2,\varkappa_3\},\{\varkappa_3\}\}$, $GC(X)=\{X\varnothing,\{\varkappa_1\},\{\varkappa_1,\varkappa_2\},\{\varkappa_1,\varkappa_3\}\}$, $Hpre\,O(X)=\{X,\varnothing,\{\varkappa_2\},\{\varkappa_3\},\{\varkappa_2,\varkappa_3\},\{\varkappa_1,\varkappa_2\}\}$, $Hpre\,C(X)=\{X,\varnothing,\{\varkappa_1\},\{\varkappa_3\},\{\varkappa_1,\varkappa_2\},\{\varkappa_1,\varkappa_3\}\}$, $\sigma^c=\{Y,\varnothing,\{\yen_3\},\{\yen_2,\yen_3\}\}$, $pre\,O(Y)=\{Y,\varnothing,\{\yen_1\},\{\yen_1,\yen_2\},\{\yen_1,\yen_3\}\}$, $pre\,C(Y)=\{Y,\varnothing,\{\yen_2\},\{\yen_3\},\{\yen_2,\yen_3\}\}$, $GO(Y)=\{Y,\varnothing,\{\yen_1\},\{\yen_2\},\{\yen_1,\yen_2\}\}$, $GC(Y)=\{Y,\varnothing,\yen_3\},\{\yen_2,\yen_3\},\{\yen_1,\yen_3\}\}$, $Hpre\,O(Y)=\{Y,\varnothing,\{\yen_1\},\yen_2\},\{\yen_1,\yen_2\},\{\yen_1,\yen_3\}\}$, $Hpre\,C(Y)=\{Y,\varnothing,\{\yen_2\},\{\yen_3\},\{\yen_1,\yen_3\},\{\yen_2,\yen_3\}\}$. Since $\{\varkappa_1,\varkappa_2\}\in Hpre\,O(X,\varkappa_1)$ and $\{\yen_2\}\in GO(Y,\yen_2)$, but $\{\{\varkappa_1,\varkappa_2\}\notin O(X)$ and $\{\yen_2\}\notin O(Y)$. Then $G(f)$ is *Hpre*-closed but not closed.

Theorem 2.5 : *A function $f:X\to Y$ is Hpre-closed graph iff $\forall(x,y)\in(X,Y)\setminus G(f)$, $\exists A\in Hpre\ O(X,x)$ and $B\in GO(Y,y)$ such that $f(A)\bigcap Cl_g(B)=\varnothing$.*

***Proof* :** Suppose that f is *Hpre*-closed graph. So $\forall(x,y)\in(X\times Y)\setminus G(f)$, $\exists A\in Hpre\ O(X,x)$ and $B\in GO(Y,y)$ s.t. $(A\times Cl_g\ (B))\bigcap G\ (f)=\varnothing$. therefore, for each $f(x)\in f(A)$ and $y\in Cl_g(B)$. Since $y\neq f(x)$, $f(A)\bigcap Cl_g\ (B)=\varnothing$. Now, for the converse, let $(x,y)\in(X\times Y)\setminus G(f)$. This implies that there exists $A\in Hpre\,O\ (X,x)$ and $B\in GO\ (Y,y)$ such that $f(A)\bigcap Cl_g(B)=\varnothing$. Therefore $f(x)\neq y\ \forall x\in A$ and $y\in Cl_g(B)$. Hence $(A\times Cl_g\ (B))\bigcap G\ (f)=\varnothing$.

Theorem 2.6 : *A function is Hpre-closed graph* $f: X \to Y$ if $\forall (x, y) \in (X, Y) \setminus G(f)$, $\exists A \in HpreO\ (x, y)$, $B \in HpreO\ (Y, y)$, s.t.$(A \times Cl\, pre \wedge H(B)) \cap G(f) = \varnothing$.

Theorem 2.7 : *Let* $f: X \to Y$ *is Hpre-closed if* $\forall (x, y) \in (X \times Y)\ \Gamma(f)$, $\exists\ A \in HpreO\ (X, x)$ *and* $B \in HpreO\ (Y, \mathrm{y})$ s.t. $f(A) \cap Cl^{H}_{pre}(B) = \varnothing$.

Proof **:** Suppose f is *Hpre*-closed, then $\forall (x, y) \in (X \times Y) \setminus G(f)$, $\exists A \in Hpre\ O(X, x)$ and $B \in GO(Y, y)$ s.t. $f(A) \cap Cl_g(B) = \varnothing$. Since $Cl^{H}_{pre}(B) \subseteq Cl_g$, $f(A) \cap Cl^{H}_{pre}(B) \subseteq f(A) \cap Cl_g(B) = \varnothing$.

Theorem 2.8 : *Any surjection function* $f: X \to Y$ *and* $G(f)$ is *Hpre-closed. Then Y is* $g - T_1$.

Proof **:** Let $c \neq d$ in Y. Since f is a surjection, this means $f(x_1) = d$ for some $x_1 \in X$ and $(x_1, c) \in (X \times Y) \setminus G(f)$. Since the *Hpre*-closed of $G(f)$ equipping $A_1 \in HpreO\ (X, x_1), B_1 \in GO(Y\ , c)$ s.t. $(A_1) \cap Cl_g(B_1) = \varnothing$. Now, $x_1 \in A_1$ this implies $f(x_1) = d \in f(A_1)$. This means that $f(A_1) \cap Cl_g(B_1) = \varnothing$, warranty that $d \notin B_1$. Also from the subjectivity of f we obtain $x_2 \in X$ such that $f(x_2) = c$. Now, $(x_2, d) \in (X \times Y) \setminus G(f)$ and the *Hpre*-closedness of $G(f)$ provides $A_2 \in HpreO\ (X, x_2)$, $B_2 \in GO\ (Y, d)$ such that $f(A_2) \cap Cl_g(B_2) = \varnothing$. Now $x_2 \in A_2$ this implies that $f(x_2) = c \in f(A_2)$, then $c \notin B_2$ therefore, we obtain sets B_1, $B_2 \in GO(Y)$ such that $c \in B_1$ but $d \notin B_1$, while $d \in B_2$ but $c \notin B_2$, therefore Y is $g - T_1$.

Corollary 2.9 : *Every surjection function* $f: X \to Y$ *and* $G(f)$ *Hpre-closed. So* Y *is Hpre-*T_1.

Proof **:** Direct from Theorem 2.5 and Theorem 2.8.

Theorem 2.10 : *Every injective function* $f: X \to Y$ *and* $G(f)$ *Hpre-closed. So X is Hpre-*T_1.

Proof **:** Let $a \neq b$ in X. Since f is injective this mean $f(a) \neq f(b)$ when of obtains that $(a, f(b)) \in (X \times Y) \setminus G(f)$. The *Hpre* –closedness of $G(f)$ provides $A_1 \in HpreO\ (x, a), B_1 \in GO(y, f(b))$ such that $f(A_1) \cap Cl_g(B_1) = \varnothing$. So $f(b) \notin f(A_1)$ this means $b \notin A_1$. Also $(b, f(a)) \in (X \times Y) \setminus G(f)$ and *Hpre*-closedness of $G(f)$ then $A_2 \in HpreO\ (X, b)$, $B_2 \in GO\ (Y, f(a))$ with $f(A_2) \cap Cl_g(B_2) = \varnothing$, warranty that $f(a) \notin f(A_2)$ and therefore $a \notin A_2$. Hence A_1 and $A_2 \in HpreO(X)$ such that $a \in A_1$ but $b \notin A_1$ while $b \in A_2$ but $a \notin A_2$. Then X is *Hpre*-T_1.

Corollary 2.11 : *Every bijection* $f : X \to Y$ *and* $G(f)$ *Hpre-closed. So* X *and* Y *are Hpre-*T_1.

Proof : Direct from Theorem 2.10.

Theorem 2.12 : *Every surjection function* $f : X \to Y$ *and* $G(f)$ *Hpre-closed. Then* Y *is g-*T_2.

Proof **:** Let $a \neq b$ in Y. Since f is surjective this means $f(x_1) = a$, for some $x_1 \in X$ and $(x_1, b) \in (X \times Y) \setminus G(f)$. The *Hpre*-closed of $G(f)$ provides $A \in HpreO(X, x_1)$ and $B \in GO(Y, b)$ such that $f(A) \cap Cl_g(B) = \varnothing$. Now $x_1 \in A$ this implies $f(x_1) = a \in f(A)$. This means that $f(A) \cap Cl_g(B) = \varnothing$, warranty that $a \notin Cl_g(B)$. Then there exists $M \in GO(Y, a)$ s.t. $M \cap B = \varnothing$. So Y is g-T_2.

Corollary 2.13 : *Every surjection function* $f : X \to Y$ *and* $G(f)$ *Hpre-closed. So* Y *is Hpre-*T_2 .

Proof : Direct from Theorem 2.12.

3. Discussion and Conclusion

1. By defining *Hpre*-closed graph in this paper, it is possible to introduce new types of graph, develop them using the definitions found in the [2,9,10,11] papers, generalize them and mention the relationships and concepts between them and the definition found in our work because of its great importance in the development of topology in particular and mathematics in general.
2. The definition of *Hpre*-closed graph can be used in the field of Neutrosophic and its applications through the first type to define Neutrosophic and obtain a new type of graph called Neutrosophic graph.

References

[1] Hadi, M.H. and Al-Yaseen, M.A.A.K., Study of Hpre -open sets in topological spaces, AIP Conference Proceedings 2292, (2020).

[2] Hadi, M.H., AL-Yaseen, M.A.A.K. and Al-Swidi, L.A., Forms weakly continuity using weak ω–open sets, *Journal of Interdisciplinary Mathematics*, 24(5), pp. 1141-1144, (2021).

[3] Alswidi, L.A. andAlhosaini, A.M.A., Weak forms of ω-open sets in bitopological spaces and connectedness, *European Journal of Scientific Research,* 52(2), pp. 204-212, (2011).

[4] Cammaroto, F. andNoiri, T., A note on weakly-compact spaces, Indian *J. pure appl. Math.,* 16(12), pp. 1472-1477, (1985).

[5] Kar, A. and Bhattacharyya, P., Some weak separation axioms, Bull. Calcutta Math.Soc. 82, pp. 415-422, (1990).

[6] Maheshwari, S. N. and Prasad, R., Some new separation axioms, Ann. Soc. Sci. Bruxelles 89, pp. 395-402, (1975).

[7] Jankovic, D. and Hamlett ,T. R., New topologies from old via ideals, *The American Mathematical Monthly,* vol. 97, pp. 295–310, (1990).

[8] Noiri, T., Al-Omari, A. A. and Noorani, M. S. M., Weak forms of ω-open sets and decomposition of continuitym E.J.P.A.M., 2(1), pp.73-84, (2009).

[9] Mashhour, A. S., Abd El- Monsef, M. E. and El- Deeb, S. N., On pre-continuous and weak pre-continuous functions, *Proc. Math. Phys. Soc.* Egypt 51, pp. 47-53, (1982).

[10] Mohammad Reza Molaei and TahereNasirzadeh, Oncocycles, *Journal of Interdisciplinary Mathematics,* 22(1), pp. 91-99, (2019).

[11] Sen, A. K. and Bhattacharyya, P., On weakly α-continuous functions, *Tamkang J. Math.,* 24 (4), pp. 445-460, (1993).

Received October, 2021

Revised December, 2021