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To cite this article: Fouad Hamza Abd Alsharify *et al* 2021 *J. Phys.: Conf. Ser.* **1999** 012083

A modified technique to compute the minimal path sets for the reliability of complex network

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Abstract. Consider a complex network whose components either work or fail, there probabilities are known. Our paper study two methods for the exact computation of the reliability network, the first one is a decomposition method which may be used even after the community can go through no further modular decomposition, is based on a partition the vertices of network into sets that can be analyzed them sequentially. This method involves choosing one component as a key and then calculate the reliability network twice the first when the key work and the other if the key failed. These two probabilities are then combined to obtain the reliability network. The second method is inclusion-exclusion method which is one of the earliest techniques to compute complex network reliability expressions using the probability laws. We get the same polynomial by using two methods. In both methods, we need to calculate all of the minimum paths so we calculated them by use the adjacency matrix.

1. Introduction

The history of the reliability subject goes returned around 1930 AD whilst probability concepts had been implemented to electric electricity technology related issues [1,2]. In 1947 Cornell University carried out a reliability look at of over 100000 electronic tubes. There were several elements that led to the concern for reliability in product planning such as product complexity, the introduction of items related to reliability into planning specifications, competition, attention to cost effectiveness, and prior device errors [2, 3].

The reliability is the probability that an object will carry out its assigned mission satisfactorily for the said time period whilst used below the required situations [1, 4]. Network reliability evaluation gets superb attention for the planning, effectiveness, Network reliability evaluation receives top notch interest for the planning, effectiveness, and safety of many real global networks, which include computer systems, communications, electrical circuits, plane, linear accelerators, and power networks [4, 5, 6]. The devices of a network are subject to random failures, as many groups and institutions become dependent upon networked computing packages. The failure of any component of a network may additionally at once have an effect on the operation of a network, for this reason the probability of every component of a network is a completely important whilst considering the reliability of a network [2, 7, 8].

2. Calculate the minimal path

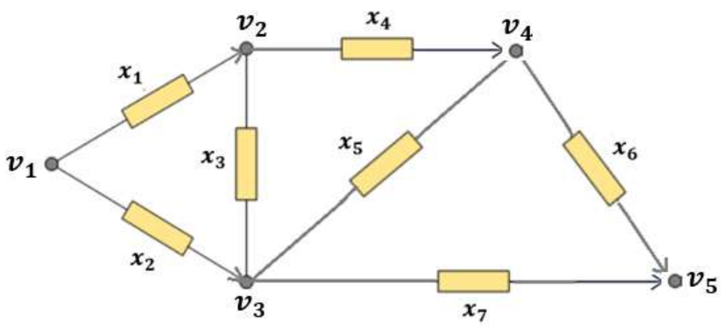
2.1. A minimal path
A path is a sequence of a network components. A minimal path is a path from which no component can be removed without disconnecting the link it creates between the begin vertex and the end vertex. A path is a set of components which, while working, connect the begin vertex with the end vertex through operating components, thereby guaranteeing that the network is an operating state [3, 7, 9].

2.2. Adjacency matrix
Assume that any network has m vertices. An adjacency matrix is $m \times m$ matrix that has factors that indicate whether or not pairs of vertices are adjacent or no longer within the network. The factors of an adjacency matrix are zero or one with zeros on its diagonal, denoted by A [4,8,10].

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ \vdots \\ v_m \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & \cdots & v_m \end{array} \\ \begin{bmatrix} 0 & a_{12} & \cdots & a_{1m} \\ a_{21} & 0 & \cdots & a_{2m} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & 0 \end{bmatrix} \end{array}$$

Where a_{ij} represent the component e_{ij} between two vertices v_i and v_j , if v_i adjacent with v_j, then $a_{ij} = e_{ij}$ and when v_i not adjacent with v_j put $a_{ij} = 0$.

2.3. Traversing through adjacency matrix
Taking the adjacency matrix as the starting point, the method begins with scanning the first row corresponding to source vertex and collects the non-zeros entries in its all columns, signifying source vertices connectivity with other vertices, to form incomplete paths by appending each adjacent vertex as a separate path. Now, for each incomplete path, its last element is checked for whether it is terminal vertex [11, 12]. If so, then a path is found else every column corresponding to the row of last element (recently appended vertex in the list) in the incomplete path are checked for adjacent vertices. Each non-zero entry is appended in the incomplete path, if not already exiting to provide several other paths [13]. If no non-zero entry is found corresponding to the last entry in an incomplete path or a repeat of an element occurs, then that path is discarded whereas if the last entry is corresponding to the terminal vertex, it constitutes a path and is stored. This process is applied to all incomplete paths till there is no incomplete path remained [9, 14].
Now, apply the traversing through adjacency matrix method to the figure 1



$\{v_1, v_2, v_3, v_4, v_5\}$ is the set of vertices.
$\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is the set of components.

Figure 1. Complex network

An adjacency matrix is $A = \begin{matrix} & \begin{matrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{matrix} & \begin{bmatrix} 0 & x_1 & x_2 & 0 & 0 \\ 0 & 0 & x_3 & x_4 & 0 \\ 0 & x_3 & 0 & x_5 & x_7 \\ 0 & 0 & x_5 & 0 & x_6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$

Step 1. Taking the first vertex and compare it with the first row, we get the following incomplete paths: $v_1 \to v_2$ and $v_1 \to v_3$.(i.e. the source vertex v_1 is connected with vertex v_2 and v_3 respectively. It has no direct adjacency to terminal vertex v_5, entry being 0 (no first order path)).

Step 2. Taking incomplete path $v_1 \to v_2$ and scanning row 2, we get two incomplete paths $v_1 \to v_2 \to v_3$ and $v_1 \to v_2 \to v_4$ (no path has its last vertex entry v_5 the terminal vertex).

Similarly taking incomplete path $v_1 \to v_3$ and scanning row 3, we get also two incomplete paths $v_1 \to v_3 \to v_2$ and $v_1 \to v_3 \to v_4$ (no path has its last vertex entry v_5 the terminal vertex). While we found one complete path $P_1: v_1 \to v_3 \to v_5$ has last entry v_5.

Step 3. Proceeding as in step 2 above and appending these incomplete paths one by one, we taking incomplete path $v_1 \to v_2 \to v_3$ and scanning row 3, we get one incomplete path $v_1 \to v_2 \to v_3 \to v_4$ and one complete path $P_2: v_1 \to v_2 \to v_3 \to v_5$ has last entry v_5. Similarly, taking incomplete path $v_1 \to v_2 \to v_4$ and scanning row 4, we get one incomplete path $v_1 \to v_2 \to v_4 \to v_3$ and one complete path $P_3: v_1 \to v_2 \to v_4 \to v_5$ has last entry v_5. Now taking incomplete path $v_1 \to v_3 \to v_2$ and scanning row 2 to get incomplete path $v_1 \to v_3 \to v_2 \to v_4$. Taking incomplete path $v_1 \to v_3 \to v_4$ and scanning row 4, we found the complete path $P_4: v_1 \to v_3 \to v_4 \to v_5$ and taking incomplete path $v_1 \to v_2 \to v_3 \to v_4$ and scanning row 4, we found the complete path $P_5: v_1 \to v_2 \to v_3 \to v_4 \to v_5$. Also taking incomplete path $v_1 \to v_2 \to v_4 \to v_3$ and scanning row 3, we found the complete path $P_6: v_1 \to v_2 \to v_4 \to v_3 \to v_5$. Finally, taking incomplete path $v_1 \to v_3 \to v_2 \to v_4$ and scanning row 4, we found the complete path $P_7: v_1 \to v_3 \to v_2 \to v_4 \to v_5$.

Now we finish this procedure because there are no other incomplete paths. Therefore, there are seven minimal paths between two vertices v_1 and v_5 are generated from this procedure. Now we find the minimal paths from these paths by extracting the components between these paths:

$P_1: v_1 \to v_3 \to v_5 \equiv \{x_2x_7\}$, $P_2: v_1 \to v_2 \to v_3 \to v_5 \equiv \{x_1x_3x_7\}$,
$P_3: v_1 \to v_2 \to v_4 \to v_5 \equiv \{x_1x_4x_6\}$, $P_4: v_1 \to v_3 \to v_4 \to v_5 \equiv \{x_2x_5x_6\}$,
$P_5: v_1 \to v_2 \to v_3 \to v_4 \to v_5 \equiv \{x_1x_2x_3x_4x_5\}$, $P_6: v_1 \to v_2 \to v_4 \to v_3 \to v_5 \equiv \{x_1x_4x_5x_7\}$ and
$P_7: v_1 \to v_3 \to v_2 \to v_4 \to v_5 \equiv \{x_2x_3x_4x_6\}$.

3. The decomposition method

The decomposition method is used to find the reliability of complex networks, which it is divided into two simple subnetworks by applying the conditional probability theory. Posteriorly, the network reliability is determined by summation the subnetworks reliability.

The basis for the approach includes choosing the key component used to decompose a given network. The efficiency of the approach depends on the selection of this key component. The past experience usually plays an important role in its selection.

The method starts with the assumption that the key component, say k, is replaced by another component that is 100% reliable or never fails and then it assumes the key component k is completely removed from the network [5,7]. Thus, the total network reliability R_N is expressed by

$$R_N = \mathbb{P}(N|k)\mathbb{P}(k) + \mathbb{P}(N|\bar{k})\mathbb{P}(\bar{k}) \quad (1)$$

where $\mathbb{P}(k)$ is the probability of success or reliability of the key component k.

$\mathbb{P}(\bar{k})$ is the failure probability or unreliability of k $\left(\text{i. e } \mathbb{P}(\bar{k}) = 1 - \mathbb{P}(k)\right)$.

This approach is demonstrated by find reliability of the network in figure 1

If we choose x_5 as a key, then the network reliability is given by

$$R_N = \mathbb{P}(N|x_5)\mathbb{P}(x_5) + \mathbb{P}(N|\overline{x_5})\mathbb{P}(\overline{x_5}) \quad (2)$$

When x_5 works the network $(N|x_5)$ becomes as in figure 2

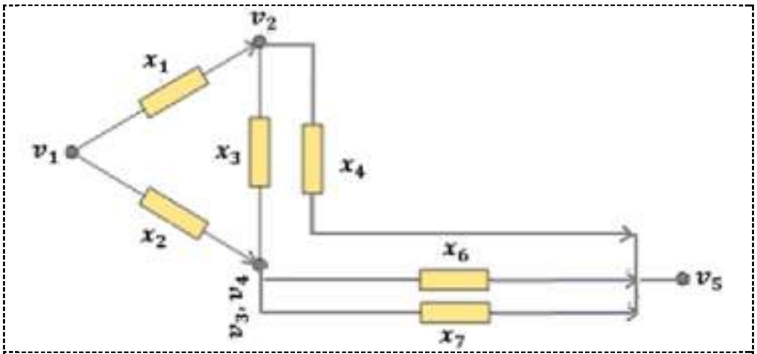


Figure 2. Simple network $(N|x_5)$.

The minimal paths of the simple network $(N|x_5)$ are $\overline{P_1} = \{x_1x_4\}$, $\overline{P_2} = \{x_1x_3x_6\}$, $\overline{P_3} = \{x_1x_3x_7\}, \overline{P_4} = \{x_2x_6\}, \overline{P_5} = \{x_2x_7\}, \overline{P_6} = \{x_2x_3x_6\}$ and $\overline{P_7} = \{x_2x_3x_7\}$.
Hence, the conditional probability $\mathbb{P}(N|x_5)$ is:

$$\mathbb{P}(N|x_5) = 1-(1-R_1R_4)(1-R_1R_3R_6)(1-R_1R_3R_7)(1-R_2R_6)(1-R_2R_7)(1-R_2R_3R_6)(1-R_2R_3R_7) \quad (3)$$

If x_5 fails, then the network $(N|\overline{x_5})$ becomes as in figure 3.

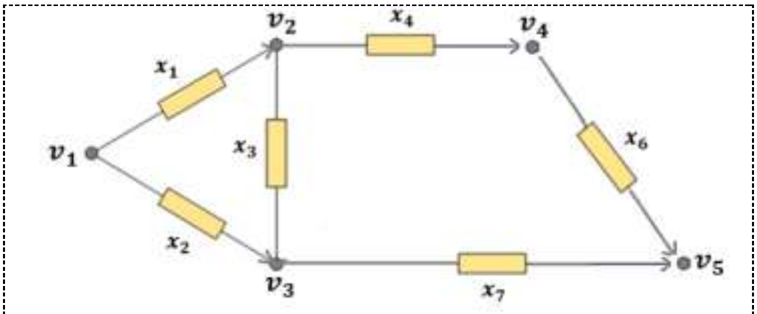


Figure 3. Network $(N|\overline{x_5})$.

The minimal paths of a network $(N|\overline{x_5})$ are $\overline{\overline{P_1}} = \{x_1x_4x_6\}, \overline{\overline{P_2}} = \{x_1x_3x_7\}, \overline{\overline{P_3}} = \{x_2x_7\}$ and $\overline{\overline{P_4}} = \{x_2x_3x_4x_6\}$. So, the probability of $(N|\overline{x_5})$ is

$$\mathbb{P}(N|\overline{x_5}) = 1-(1-R_1R_4R_6)(1-R_1R_3R_7)(1-R_2R_7)(1-R_2R_3R_4R_6) \quad (4)$$

But we have $\mathbb{P}(x_5) = R_5$ and $\mathbb{P}(\overline{x_5}) = 1-R_5$
Therefore, the reliability of our network is giving by:

$$R_N = R_5[(1-R_1R_4)(1-R_1R_3R_6)(1-R_1R_3R_7)(1-R_2R_6)(1-R_2R_7)(1-R_2R_3R_6)(1-R_2R_3R_7)] + (1-R_5)[1-(1-R_1R_4R_6)(1-R_1R_3R_7)(1-R_2R_7)(1-R_2R_3R_4R_6)] \quad (5)$$

Note: If the component i work, then $R_i = 1$ and $R_i = 0$ when the component i failed $\forall\, i = 1, \cdots, 7$, these impels to $R_i^n = R_i$.
By use the note above we get the polynomial

$$\begin{aligned} R_N = {} & R_1R_2 + R_6R_7 + R_2R_3R_4 + R_4R_5R_7 + R_1R_3R_5R_7 + R_2R_3R_5R_6 + R_1R_2R_3R_4R_5R_6 \\ & +2R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + R_1R_2R_4R_5R_6R_7 \\ & +R_1R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - R_1R_2R_3R_4 - R_1R_2R_6R_7 - R_4R_5R_6R_7 \\ & -R_1R_2R_3R_5R_6 - R_1R_2R_3R_5R_7 - R_1R_2R_4R_5R_7 - R_2R_3R_4R_5R_6 - R_2R_3R_4R_5R_7 \\ & -R_1R_3R_5R_6R_7 - R_2R_3R_4R_6R_7 - R_1R_3R_4R_5R_7 - R_2R_3R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7 \end{aligned} \quad (6)$$

4. Inclusion-exclusion method

The inclusion-exclusion method is one of the earliest techniques to compute complex network reliability expressions using the probability laws. For example, taking an example of two-terminal reliability measures for instance, the performance of all components in at least one minimal path set guarantees the success of the network [4,8]. Thus, if all minimal paths between the source and sink of a network are $P_1, P_2, \cdots, P_n$, then the reliability of this network is given by:

$$R_N = \mathbb{P}(\cup_{i=1}^{n} P_i) \quad (7)$$

The principle of inclusion–exclusion states that for finite sets $P_1, P_2, \cdots, P_n$ is

$$\cup_{i=1}^{n} P_i = \sum_{k=1}^{n} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le k} \left(P_{i_1} \cap P_{i_2} \cap \cdots \cap P_{i_k} \right) \right) \quad (8)$$

So,

$$R_N = \mathbb{P}\left(\sum_{k=1}^{n} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le k} \left(P_{i_1} \cap P_{i_2} \cap \cdots \cap P_{i_k} \right) \right) \right) \quad (9)$$

$$R_N = \left(\sum_{k=1}^{n} (-1)^{k+1} \left(\sum_{1 \le i_1 < \cdots < i_k \le k} \mathbb{P}\left(P_{i_1} \cap P_{i_2} \cap \cdots \cap P_{i_k} \right) \right) \right) \quad (10)$$

By assuming that $P_I = \left(P_{i_1} \cap P_{i_2} \cap \cdots \cap P_{i_k} \right)$ we get

$$R_N = \left(\sum_{k=1}^{n} (-1)^{k+1} \left(\sum_{I \subseteq \{1, i_1, \cdots, i_k, k\}} \mathbb{P}(P_I) \right) \right) \quad (11)$$

Now we going to find the reliability of our network in figure 1 with all minimal paths

$$R_N = \left(\sum_{k=1}^{7} (-1)^{k+1} \left(\sum_{I \subseteq \{1,2,\cdots,7\}} \mathbb{P}(P_I) \right) \right) \quad (12)$$

$$\mathbb{P}_1 = \mathbb{P}(P_1) + \mathbb{P}(P_2) + \mathbb{P}(P_3) + \mathbb{P}(P_4) + \mathbb{P}(P_5) + \mathbb{P}(P_6) + \mathbb{P}(P_7)$$
$$\mathbb{P}_1 = R_2R_7 + R_1R_3R_7 + R_1R_4R_6 + R_2R_5R_6 + R_1R_2R_3R_4R_5 + R_1R_4R_5R_7 + R_2R_3R_4R_6$$
$$\mathbb{P}_2 = \mathbb{P}(P_1)\mathbb{P}(P_2) + \mathbb{P}(P_1)\mathbb{P}(P_3) + \mathbb{P}(P_1)\mathbb{P}(P_4) + \cdots + \mathbb{P}(P_1)\mathbb{P}(P_7) + \mathbb{P}(P_2)\mathbb{P}(P_3) + \cdots + \mathbb{P}(P_6)\mathbb{P}(P_7)$$
$$\mathbb{P}_2 = R_1R_2R_3R_7 + R_1R_2R_4R_6R_7 + \cdots + R_1R_2R_3R_4R_5R_6R_7$$
$$\vdots$$
$$\mathbb{P}_7 = \mathbb{P}(P_1)\mathbb{P}(P_2)\mathbb{P}(P_3)\mathbb{P}(P_4)\mathbb{P}(P_5)\mathbb{P}(P_6)\mathbb{P}(P_7)$$
$$\mathbb{P}_7 = R_1R_2R_3R_4R_5R_6R_7$$
$$R_N = \mathbb{P}_1 - \mathbb{P}_2 + \mathbb{P}_3 - \mathbb{P}_4 + \mathbb{P}_5 - \mathbb{P}_6 + \mathbb{P}_7$$
$$R_N = R_1R_2 + R_6R_7 + R_2R_3R_4 + R_4R_5R_7 + R_1R_3R_5R_7 + R_2R_3R_5R_6 + R_1R_2R_3R_4R_5R_6 + 2R_1R_2R_3R_4R_5R_7 + R_1R_2R_3R_4R_6R_7 + 2R_1R_2R_3R_5R_6R_7 + R_1R_2R_4R_5R_6R_7 + R_1R_3R_4R_5R_6R_7 + 2R_2R_3R_4R_5R_6R_7 - R_1R_2R_3R_4 - R_1R_2R_6R_7 - R_4R_5R_6R_7 - R_1R_2R_3R_5R_6 - R_1R_2R_3R_5R_7 - R_1R_2R_4R_5R_7 - R_2R_3R_4R_5R_6 - R_2R_3R_4R_5R_7 - R_1R_3R_5R_6R_7 - R_2R_3R_4R_6R_7 - R_1R_3R_4R_5R_7 - R_2R_3R_5R_6R_7 - 3R_1R_2R_3R_4R_5R_6R_7$$

Which is the same as equation (6).

5. Conclusions

Two different methods are studied for calculating the reliability of the complex network, both of them based on the laws of probability and the reliability was calculated based on the set of minimal paths that were found by use the traversing through adjacency matrix method. The polynomial for complex network reliability were the same using both methods and this indicates that our results were correct and that the two methods can be relied upon to calculate the reliability of complex networks.

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