

Investigation of Mixed Convection in Cavity With different Aspect Ratio Containing Corrugated Hot Circular Body

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ABSTRACT

This work made a study numerically the effects of corrugated hot circular cylinder fitted in square cavity on steady state laminar mixed convection heat transfer, the analyses assumed the outer walls of cavity as a cold with (T_c), and having a corrugated cylinder with a hot surface (10) repeated sinusoidal, and corrugation amplitude ($\lambda = 0.3$). The mixed convection induced by the constant speed condition to right side was applied only on the upper surface, the rest region between the wall and placed cylinder filled by air which assumed as working fluid with ($Pr=0.71$). The solution is exists for a wide range parameters, the location of cylinder (δ), Richardson number (0.1, 1, and 10), aspect ratio of enclosure changed (0.75,1 and 1.5), in additional $Re(100-300)$, both (Nu_L), (\overline{Nu}) are also examined. The results show the temperature and stream plots are strongly dependent on position of sinusoidal hot cylinder, the cavity aspect ratio. The results also show, for cavity aspect ratio =0.75, (Nu_L) reach to maximum value when the corrugated cylinder placed closed to the blow surface of cavity, and for all values Reynolds number (i.e $100 \leq Re \leq 300$) there are significant effect especially when Richardson Number >1 , by enhancement (\overline{Nu}) which refer to increasing of the transferred heat rate, also it was found, local Nusselt number has a periodic profile, everywhere the hot cylinder for all aspect ratio, and start to fall down at Richardson Number becomes below unity.

KEYWORDS

Numerical Analysis, sinusoidal cylinder, stream plots, Reynolds and Richardson numbers, local and average Nusselt number, Amplitude

INTRODUCTION

“A phenomenon commonly “in physical it referred to combination for different approach in heat transfer the first approach forced convection and the other a natural convection branch known as mixed convection heat transfer. Investigators cited below concerned in this physical phenomenon and all related problems due to is come upon a extensive applications especially in engineering filed such as solar air collectors, some kinds of heat exchanger and thermal design [19-20]. The experimental research methods completed a many full articles in the heat transfer with different directions, add to numerical studies also used to investigate the mixed convection problems in a wide different shapes of cavities having (circular, square ...etc) [14-17]. It was their purpose either increases or decreases the transferred rate of heat. Present work focused clearly in mixed convection heat transfer for square enclosure containing corrugated hot cylinder in detail. **Mei H .C [1]** Calculated for lid-driven case heat transfer by using numerical technique, through a cavity region with an arc-shape design, air has been selected as a fluid inside the cavity. The results were existing for a wide range of Re ($10E1 -20E2$), Gr ($0-10E8^8$), and slop angles ($0 \leq \beta \leq 2 \pi$). **Carlos et al [2]** Examined a lid-driven numerically work in cavity with square shape, by using an governing equations of fluid flow (i.e) Navier-Stokes.

A wide range was covered in this work Where included dependent on the following parameters Re (0.01, 10, 100, 400 and 1000), future more 42 variables and for convergence the simulation done with (1024 x 1024) nodes grid. **Cheng T.S. [3]** studied numerically a two-dimensional heat flow models in a square cavity. The governing parameters varying in different range Pr (0.01- 50), (Re)(10-2200), ($100 \leq Gr \leq 4.84 * 106$), and (Ri) was varied from (0.01 to 100). Nu_L , and \overline{Nu} were also validated to described the heat rate variations and effect of these parameter on it. A lid driven cavity solved numerically, the flow simulation by assumed the case with unsteady

state was also investigated by **Khanafer et al.[4]**. A mixed convection and two – dimensional fluid flow explained by **Oztop and Dagtekin[5]** A systems is created as a lid-driven in a square shape, the walls with differentially heating right and left while the top and bottom assumed adiabatically was the thermal boundary conditions adopted by solution, also its sliding . The study was also included with three elementary categories of cases, by depending on sliding walls direction. Inclined lid-driven for laminar mixed convection in a square cavity was investigated by **Saha et al. [6]**.

The cavity bottom wall assumed kept as a hot boundary condition by practical heat flux with uniform an constant, and the placed cylinder maintained adiabatically . The results proved that the characteristics of both fluid heat flow strongly dependent by slope angle of cavity. **Ouertatani et al. [7]** analyzed numerically a double lid driven cavity with cubic dimensions the simulations were existed for a extensive variety of two dimensional less group Richardson Reynolds and numbers. The calculated exhibited the heat rate improved by (76%) when both Richardson number equal to unity , and $Re = 400$. The mixed convection heat flow a enclosure square shape having a cylinder circular geometry with a lid-driven , and having air as working fluid was done by **Oztop et al. [9]** .The heated left wall which considered as slides with two opposite directions (i.e) (+y) chosen to represented a positive direction while (-y) represented the opposite direction and the other walls were kept stationary . **Zoubair B.etal[15]** carried out an numerical work for natural and mixed convention in square cavity having a circular cylinder the cavity free region filled with nano fluid (Cu-water).

The main equations (Navier–Stokes and energy) in two dimensional formulations are solved by using (F.V.M).The study deals with a wide range of the following nano fluid (0-0.05),(0.01≤ Ri ≤1000)volume fraction ,and Rayleigh number (103-106) .The works proved the Nusselt number enhanced with varying the positioning from horizontal case to vertical direction for the two of circular cylinders arrangement . **Shih et al.[18]** there system predict the transient characteristics of the (\overline{Nu}) .They assumed square cavity having numerous rotating cylinders such as : (circle, square, and equilateral triangle) positioned in the center point. They proved the laminar fluid characteristics oscillated between a quasiperiodic and periodicity when Re number at extreme point and lowest value respectively .The aims of existing work to examine the heat transfer and fluid flow strength when hot corrugated cylinder placed inside square cavity with lid driven case study. The moving wall boundary condition applied only for the upper wall, while the other drop out stationary. All external wall of enclosure kept at minimum temperature to induced the gradient in temperature with hot inner cylinder. The two-dimensional numerical analysis, steady state and no slip as listed condition to simplified the current work. the number of wavy, amplitude, and cylinder base radius generalization constant. An extensive variety is carried out for the Ri number ($0.1 \leq Ri \leq 10$), aspect ratio and the hot cylinder position. The heat transfer and flow strength behavior effected by the inner cylinder position and aspect ratio.

PROBLEM DESCRIPTION

Figure (1) shows physical description, of square enclosure with corrugated cylinder, the dimensions of ($W \times L$) and its walls at (T_c) while leave the upper surface slides towards (+x) direction. Whereas the inner body surface wall fixed at (T_h) , The sinusoidal equations in (x,y) coordinates denoted as follows equations (I, and II) :

$$X = [R + L \sin(m * \phi * \frac{\pi}{180})/m] * (\cos(\phi * (\frac{\pi}{180}))) \quad (I)$$

$$Y = [R + L \sin(m * \phi * \frac{\pi}{180})/m] * (\sin(\phi * (\frac{\pi}{180}))) \quad (II)$$

The cavity length taken ($L=0.1m$), radius of circular body has ($0.2 L$). Also ($\delta=0.25L$) is the distance that the cylinder moves in right, left, up and down relative the cavity center, the amplitude of sinusoidal is taken as ($0.3L$). With constant fluid physical Properties except fluid density and assumed laminar Newtonian, steady state, with Neglecting of radiation with no effect viscous dissipation. Richardson number varying between ($Ri=0.1, 10$). Stream and isothermal lines are obtainable, by both local and average Nusselt numbers the heat transfer in cavity was characterized.

MATHEMATICAL DEFINITION

The fundamentals equations that described the fluid motion in forced and natural convection are mass, momentum, and energy equations, its described in dimensionless form: [12]

"Continuity Equation "

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

"Momentum Equations:"

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri \cdot \theta \quad (3)$$

"Energy Equation:"

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

Defined dimensionless variables are:

$$X = \frac{x}{L} \quad (5)$$

$$Y = \frac{y}{L} \quad (6)$$

$$U = \frac{u}{U_{lid}} \quad (7)$$

$$\theta = \frac{(T - T_c)}{(T_h - T_c)} \quad (8)$$

$$P = \frac{p}{\rho U^2 lid} \quad (9)$$

The (U, and V) in above equations are referring to velocities in (x ,y) directions ,respectively .To find out the meaning of the rest(i.e non-dimensional terms) ,see the below nomenclature .The rate of heat transfer everywhere the sinusoidal body carried out by calculations of both local and average Nusselt number:

$$h = -k (T_s - T_o) \frac{\partial T}{\partial n} \Big|_{wall} , \text{ and } Nu_{Local} = \frac{hs}{k} \quad (10)$$

Esitamination the average Nusselt number around corrugated cylinder described by equation below:

$$\bar{h} = \frac{1}{2\pi} \int_0^{2\pi} h d\theta , \text{ and } Nu_{average} = \frac{\bar{h}S}{k} \quad (11)$$

Numerical Technical and Boundary Conditions

(Mass balance, Momentum and Energy) fundamentals equations are solved by (Ansys 16.0) based on Fluent. The grid generation are shown in figure around sinusoidal hot cylinder inside a three different enclosure aspect ratio. To resolve the flow and temperatures distributions a fine mesh is required. In the FLUENT solution control, for both momentum and energy second order upwind are confirm, while for solution a pressure-velocity coupling a SIMPLEC algorithm scheme has been set, the least squares cell based for gradient. The standard setup of absolute convergence was verified for precise solution for continuity 10^{-6} , X-velocity, Y-velocity and energy. The tested

mesh ($50 \times 50, 60 \times 60, 80 \times 80, 90 \times 90, 100 \times 100$, and 120×120). Figure (2-A) illustrates convergence of the \overline{Nu} , this validation done around a central corrugated hot cylinder, for finish the work the calculations of a fine mesh employed with (120×120) have been accepted for minimize the expected error in current simulation. For more investigation of stream lines and isothermal contours, the present work is checked for two cases of square enclosure, the first a mixed convection approach by using a square cylinder as a fitted body Akand et al. [12], Figure (3), and table(1) show the comparison result with excellent agreement, the second case for square cavity containing rotation circular cylinder Chuan et al. [13].

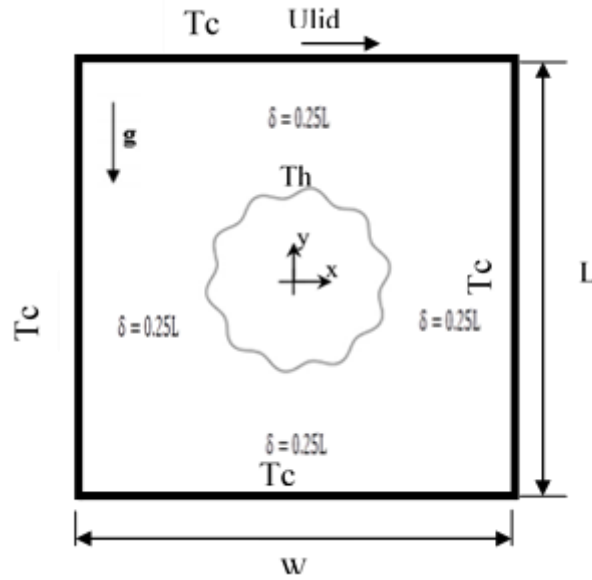


Figure 1. Geometric System

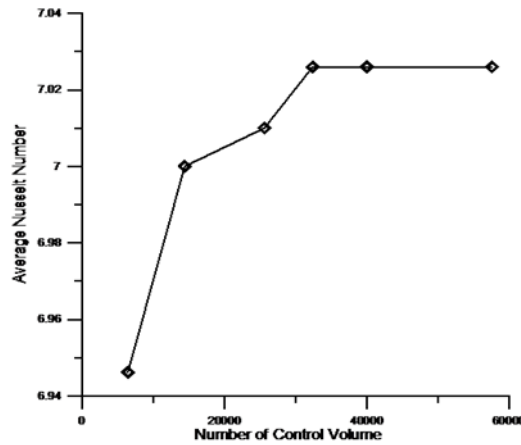


Figure 2. Represent Average Nusselt Number convergence of the at the Cylinder surface

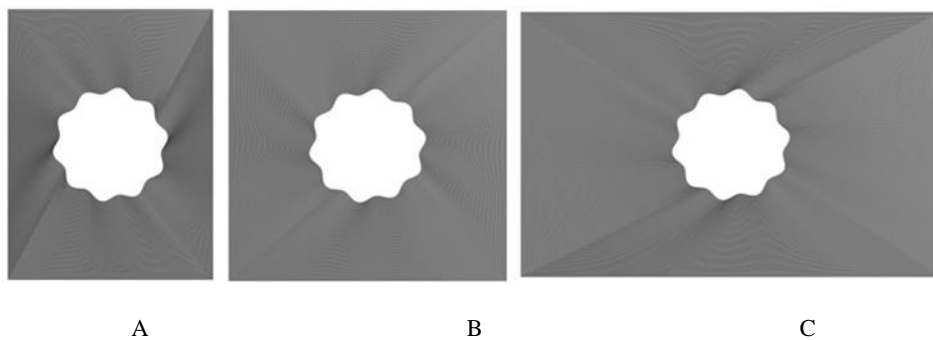


Figure 3. Mesh Generation A- Aspect ratio =0.75, B- Aspect ratio =1, C- Aspect ratio =1.5

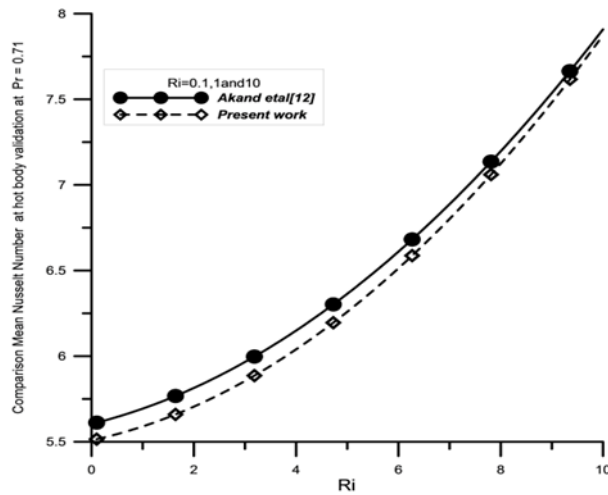


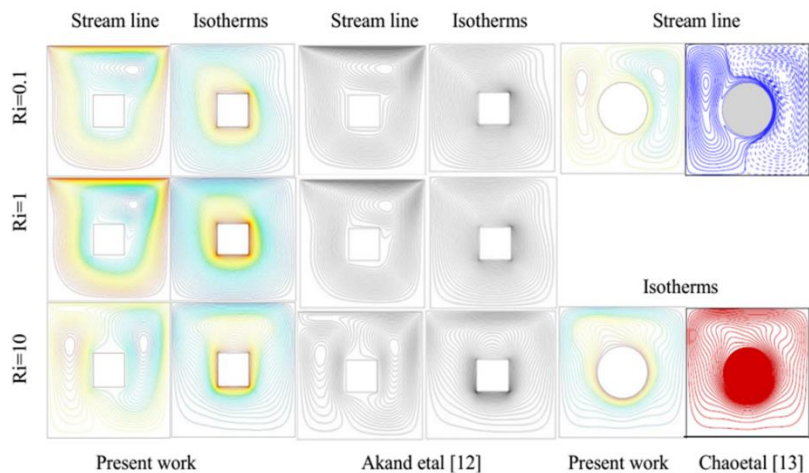
Figure 4. Mean Nusselt Number At Hot Body Comparison Between The Present Solution And Ref [12]

Table 1. Comparison of $(Nu)_{\bar{}}$ Validation at Pr=0.71 Mean Nusselt Number At Hot Body

<i>Ri</i>	<i>Akand etal[12]</i>	<i>Present work</i>	<i>Error (%)</i>
0.1	5.6118	5.5153	1.71
1	5.6935	5.5896	1.82
10	7.9083	7.8741	0.432
	<i>Chuan et al[13]</i>	<i>Present work</i>	<i>Error (%)</i>
<i>Ri= 100</i>	9.73	9.75	-0.205

RESULTS AND DISCUSSION

A lid-driven subjected on the upper surface of square cavity to induced the mixed convection heat flow, “while the natural convection generated by the difference in walls temperatures (i.e) the four walls of enclosure kept at (T_c) , while inserted body fixed wall temperature at (T_h) . The cavity aspect ratio effects, locations of centric and eccentric of cylinder with ten corrugation, the effect of changing both (Ri) and (Re) on the characteristics of heat transfer inside the cavity have been checked. The fluid flow with $(Pr=0.71)$ assumed filling the cavity.



Flow and Temperature profiles

The preparation of figures (4, and 5) for both streams and isotherm lines, shows the variation cavity aspect ratio in horizontal direction and (Ri) number in vertical position. At ($Ri=0.1$) and enclosure aspect ratio $=0.75$, and $Re=100$, the main flow streams appears in uni-circulation cell and in right side of cavity, with small cores and elliptical shape; this is because of the effects of buoyancy and forced convection, will makes the confirmed fluid to mixing by those different forces, the streams lines becomes crowded there due to flow strength and the smallest area existence between the walls, leading to appears streams line in this profile and Squeezed. As aspect ratio becomes unity, and Richardson Number kept the elliptical behavior of uni-vortex twitch to converted slowly to circular profile it is clear the uni-cell size increased due to phenomena of separation of flow lines take place. Precisely as ($AS=1.5$) there is a change of performance in flow stream lines (i.e) its flux states translation to cavity flow. When Richardson Number become unity as downward direction in mentioned above figures the domination of natural convection inside the cavity becomes clear.

The main flow will be contained four cells recirculation different in strength this corresponding when ($AS=0.75$, $Ri=10$) with opposite directions developed. As increasing cavity aspect ratio (i.e $AS=1$) merge in upper and lower cells in the right side, with a rotation in direction clockwise, there is identical incident in left eddies its rotate in oppositely. When cavity aspect ratio set at 1.5, the heat transfer by natural convection mechanism inside enclosures enhanced, also the lines of flow sight with irregularity scattering, this will repeated at Richardson number $=10$. Due to the free region between the internal and external wall increases, hence all area will filling by bi-large eddies. The temperatures lines are displaced in (Figure 5), as expected there is a thicker thermal boundary layers, the profile of this layer taking the shape of corrugated and crowded at hot surface ended, the figure also exhibited stepper gradient in temperature between the colder and hottest walls, at cavity aspect ratio $=0.75$.

As mentioned earlier the aspect ratio increases to (1, and 1.5), this leading to ascending the free area this leads to regularly reduction in the density of line there, at ($0.1 \leq Ri \leq 10$) the bifurcation of plume distributions of temperature patterns become significant with clearly at ($Ri = 10$). The different hot surface position, see Figure (6) its proves the flow lines at Richardson number $=0.1$, its layers adjusted to lid driven wall will exposed to strength forced convective, with pattern seems as clutched and uni-bubble. At Richardson number $=1$, add to main flow, also the secondary flow appears in the left side of cavity because of the buoyancy effect, which will causes the separation phenomena. The air closed to inner hot body become light; and as expected started to moves towards upward direction and trapped with the right vortex, also the left cell generation due to the sinusoidal hot surface which hinders the fluid rising drive and forces it to move beside the vertical cold wall when ($Ri=10$). At ($Ri=0.1$) and the upper situation the main flow consists of bi-eddies due to lid will slide with high velocity, with small difference in strength present below the lid driven one occupied the right of cylinder and the other in the left, nothing worth while ($Ri=1$).

Change becomes important when natural convention become strongly associated when ($Ri=10$), the flow included two cells with expanded, and occupied the cavity. By putting the hot body closed to right wall and (Ri) set minimum the flow action will come back with little distortion, at ($Ri=10$). At forced convection case (i.e, $Ri=0.1$) and insert body fixed at this time closed to right wall. It is detected the flow contain of a cell correspondingly, the weak cell founded down the hot body and rotate in direction anti-clockwise due to a little quantity of fluid detached, this amount of air cannot mixing and come nearly the faster hot air. The lines of isothermal matching to eccentric cylinder position see (figure7), the thermal boundary layers seems very thin, and temperature lines become more squeezed due to decreases in available area there, moreover violent gradient in temperature. Its clear modification in temperature distribution individually due to the formation of plume bifurcation this occurs as (Ri) reach the maximum. A decreases in temperature from the wall of hot corrugated cylinder and cold walls of cavity, where the bifurcation of plume distribution in neighborhood area, because of the flow become motionless. The velocity distribution inside the enclosure and isothermal lines has been estimated a shown in Figures (8 and 9) respectively, with $Ri = 0.1$ and up left case.

The single flow recirculation filled the corresponding area (i.e.) upper right corner region due to lid-driven speed there are some change in streamline in which its containing additional idle circulation cell especially at ($Ri=10$). For the top right corrugated cylinder position one recirculation bubbles procedure at top left position on the

empty place side and the other un-effective will appears distinctively below the hot cylinder. For the down-left location of the cylinder flow lines having single vortex filled the enclosure for ($Ri = 0.1$) in which the force convection dominating with no change in flow patterns is created at the value of ($Ri = 1$) pure mixed convection situation, also its containing bi-vortex for extent $Ri = 10$ this is occurred subsequently due to buoyancy effect rises. For the downright case no modification in flow lines for ($Ri = 0.1$) with the comparison with mention earlier however, two circulation cells seem at $Ri = 1$ with different strength one a counter rotating below cell while the second upper cell rotating in opposite direction, and when Richardson number = 10 its becomes expands and approximately equally in strength because of natural convection effects. Figure(9) explains the isothermal patterns inside the enclosure. For all corrugated hot cylinder cases i.e.(up left, up right, down left and down right). It is clearly that a temperature lines adjusted with streamlines. It's almost concentrated near the cylinder occupied area. On the other hand, clustering of the heat lines distinctively toward the heated corrugated cylinder. One may notice a high temperature gradient there also the shape of a thin formulated thermal layer taken the corrugated cylinder profile for all convective regimes of ($Ri 0.1- 10$).

Local and Average Nusselt Number

Figures (10,11 and 12), show the behavior of (Nu_L) for Reynolds number, (100-300), different corrugated cylinder position, and all values of cavity aspect ratio are existing. Figure (13) defined the (\overline{Nu}) around the hot surface ($0.1 \leq Ri \leq 10$), and ($100 \leq Re \leq 300$). Starting from Figures (10A, B, C and D), its explained that the fluctuations style, the maximum of local Nusselt number associated with cavity aspect ratio = 0.75, and this indicated more enhancement in heat flow by convection mode if compared to the other ratio. However, when ($Re = 300$ and $Ri = 10$), figure (10D) the values of (Nu_L) is significantly larger than its values when ($Re = 100$ and $Ri = 10$). On the other hand, the location of corrugated wall influenced on the heat transfer, as in figures (11 A, B, C and D) in which the figures (A, B and C) showed the profile of local Nusselt number with Richardson number, for different wall four positions matching ($Re = 100$), while (D) refers to ($Re = 300$), as Richardson number = 0.1 the development looking sinusoidal inequality when hot surface occupied closed lower wall. for down cylinder position, and the peak between ($20^\circ < \theta < 50^\circ$). When ($Re = 300$) the (Nu_L) curve behavior around hot cylinder remains unaltered but all points shifted up. Figure (12 A and B) described the variation of local Nusselt number with corresponding value of ($Ri = 10$) when the hot corrugated located in different positions (down left, downright, up left and up right), while Reynolds number changing values ($Re = 100-300$) respectively, as mention above (Nu_L) also appears in sinusoidal shape and the lowest value (Nu_L) for the considered up right position. Finally, the average Nusselt number for ($Re = 300$) shows in figure (13) for corrugated cylinder positions (eccentric). As shown with increasing (Ri) the curves registered ascending. As Ri varying from (0.1 to 1) in sensitive changing (\overline{Nu}), but at ($Ri = 10$) a noticeable (\overline{Nu}) and reaches a peak value (14.03)

CONCLUSIONS

From the above results we can concluded the following points:

- 1- Streams, temperatures plots and transferred of heat are strongly dependents on sinusoidal inner cylinder position, and the aspect ratio of cavity. With high sensitive by varying the aspect ratio.
- 2- The \overline{Nu} fluctuation between the highest value when the sinusoidal body located near the lower surface of cavity and the lowest value when placed in upper situation.
- 3- The flow strength increases by decreases of (Ri), The critical aspect ratio was initiate smallest free area. The inverse relationship between heat rates and (AS)(i.e) the maximum value corresponding with minimum aspect ratio
- 4- At peak value of (Re), the value of ($Ri = 10$) and aspect ratio = 0.75, more increasing in heat transfer rate
- 5- Nu_L has a wavy profile around hot surface, when (Ri) increasing the start value and curve points forced to displacement up, and break down when ($Ri < 1$).
6. The heat transfer reach to highest value as documented for the down left case of corrugated cylinder and it maximum with ($Ri = 10$). Also, heat rate recorded numerical difference is not significant for down left and down right case, while the difference is at the peak if compared with up right case.

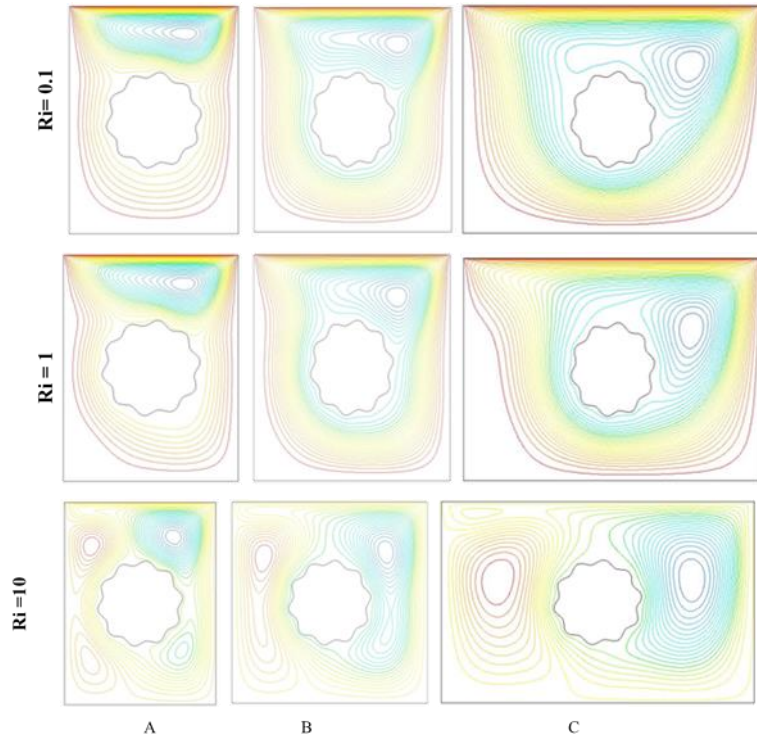


Figure 4. Streamlines A-Aspect ratio=0.75 B-Aspect ratio =1 C- Aspect ratio = 1.5, $Re=100$, For Different Richardson Number

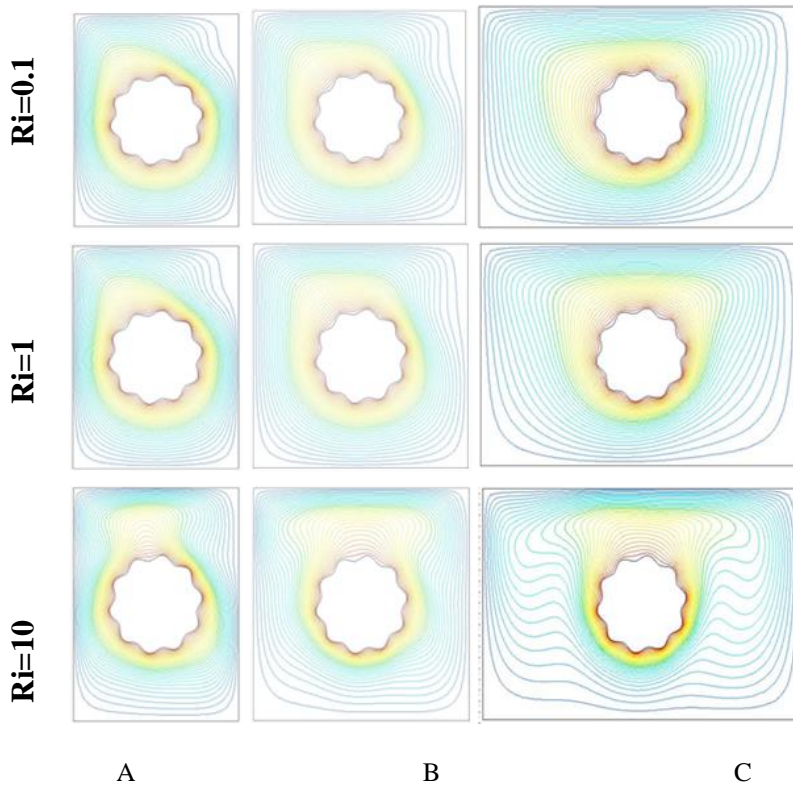


Figure 5. Temperature Lines A-Aspect ratio =0.75 B- Aspect ratio =1 C- Aspect ratio =1.5, $Re=100$, For Different Richardson Number

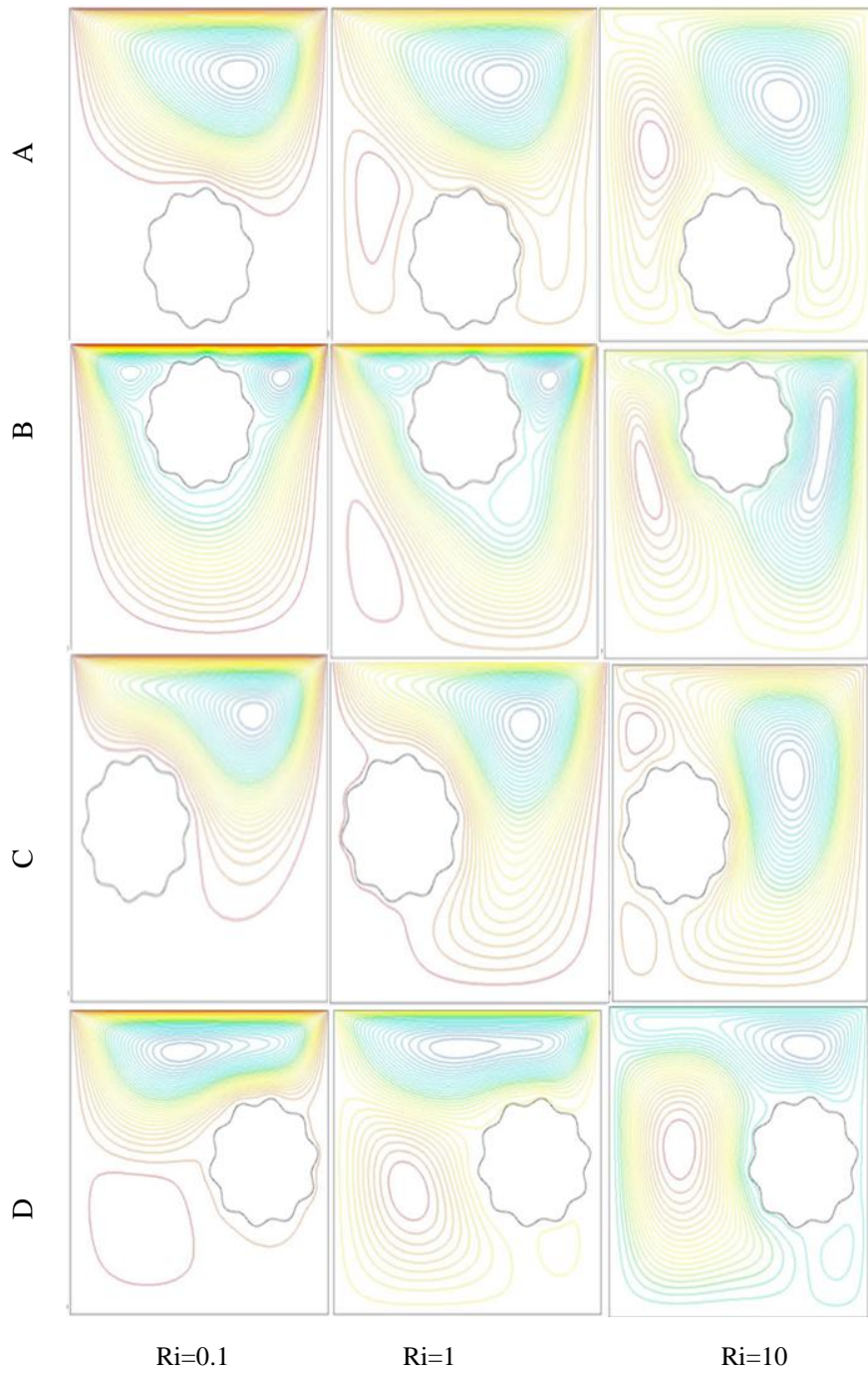


Figure 6. Stream Lines A –Down B-Up C-Left D-Right ,Re=100 , For Different Richardson Number

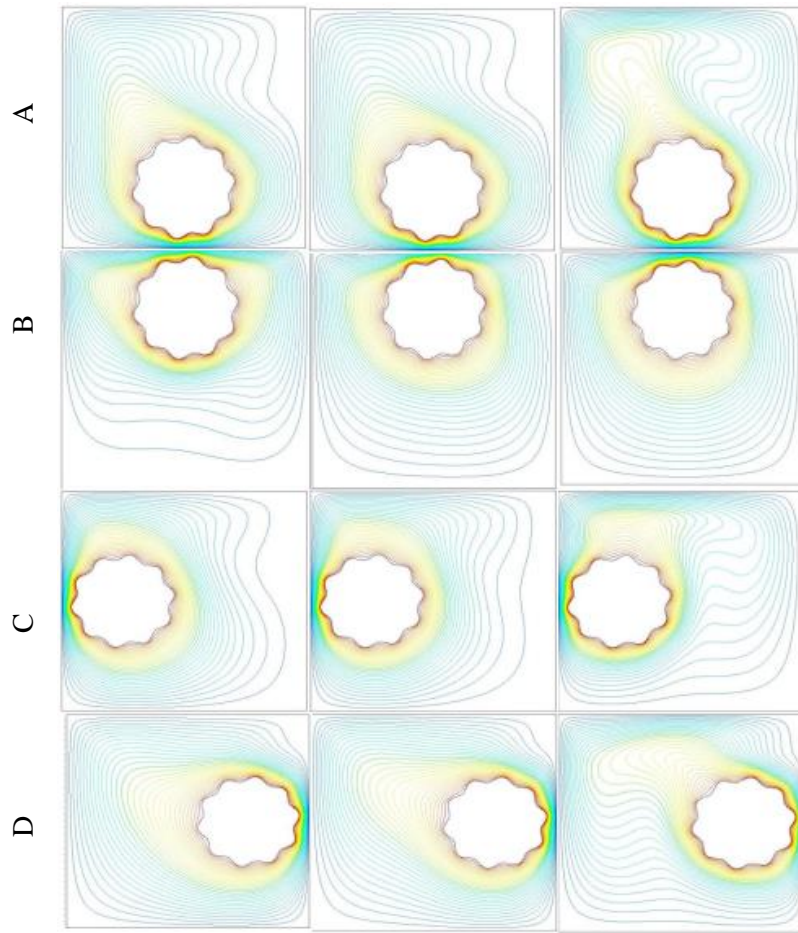


Figure 7. Temperature Lines A-Down B- Up C-Left D-Right, $Re=100$, For Different Richardson Number

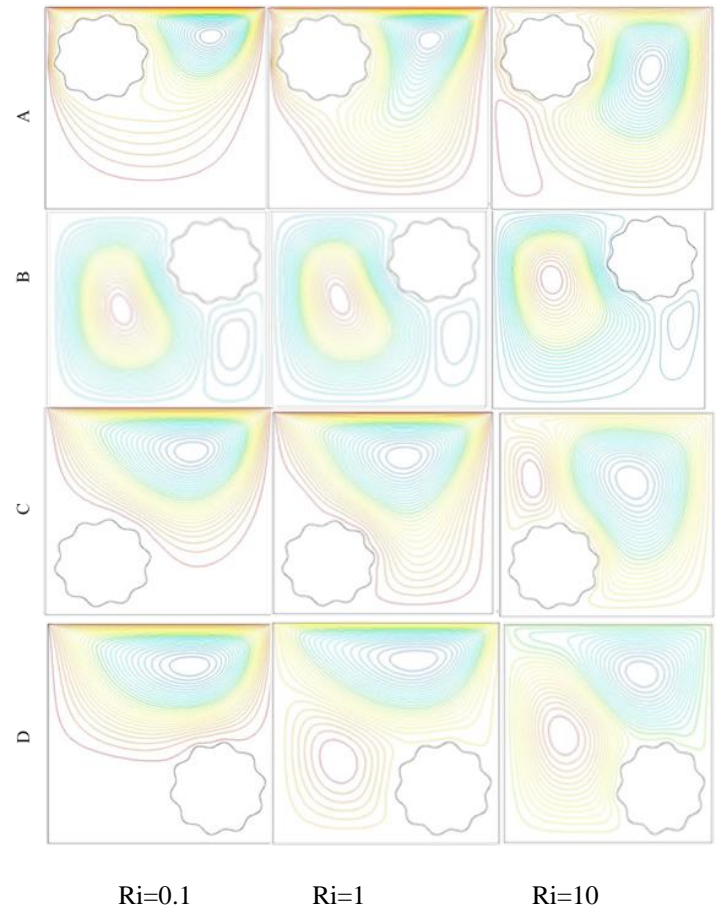


Figure 8. Stream Lines A –Up Left B-Up Right C-Down Left D-Down Right , $Re=100$, For Different Richardson Number

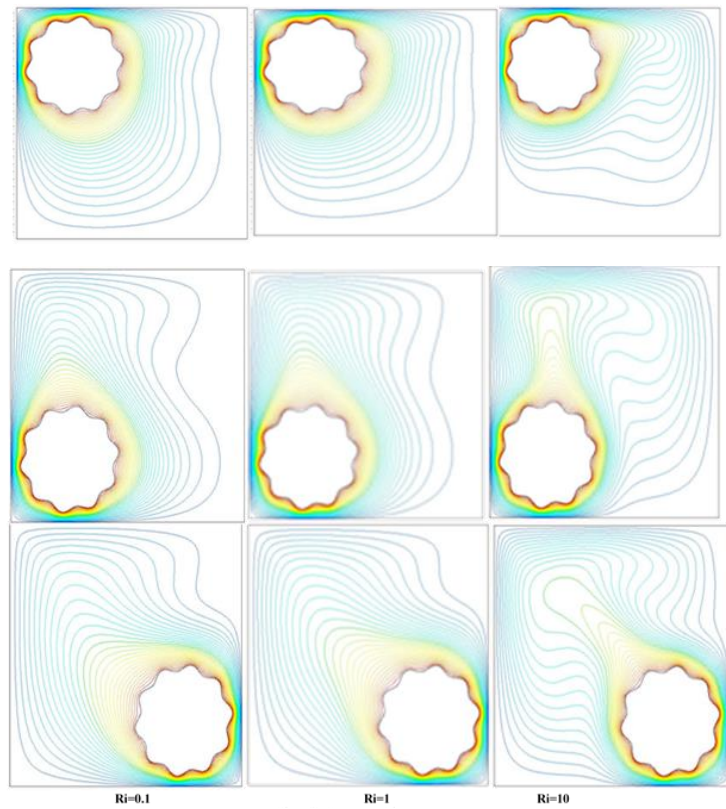


Figure 9. Temperature lines A-Up Left B-Up Right C-Down Left D-Down Right, $Re=100$, For Different Richardson Number

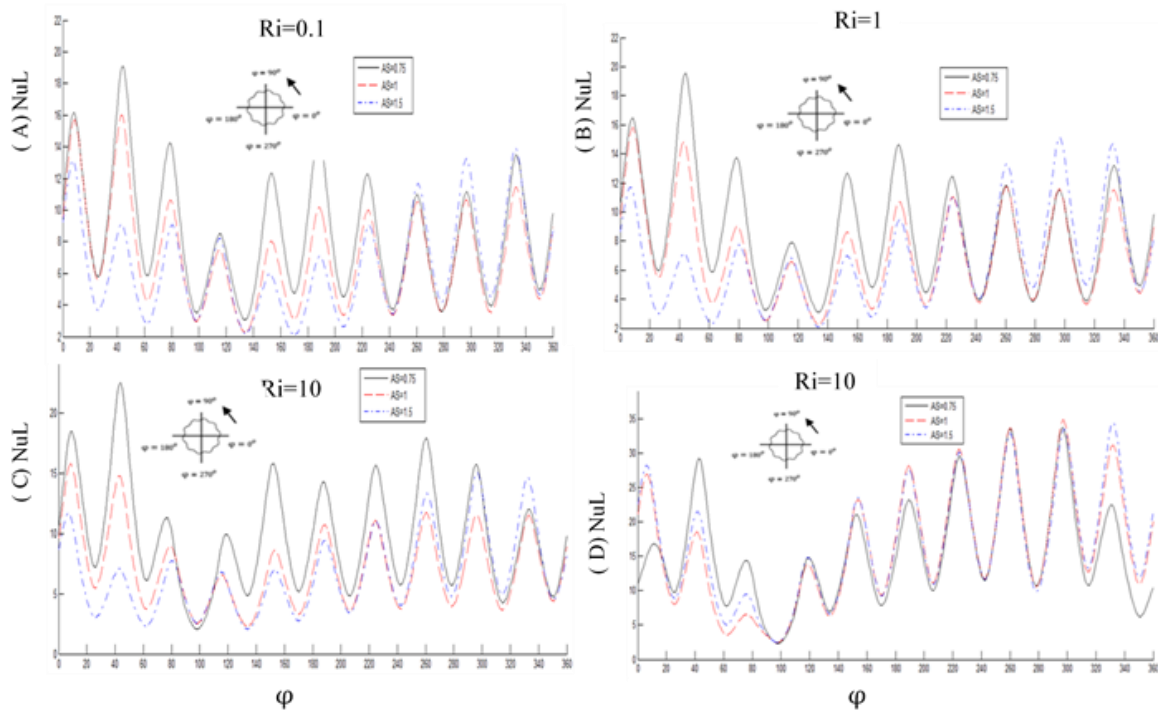


Figure 10. NuL around Hot Cylinder For A- $Ri=0.1$ B- $Ri=1$ C- $R=10$, $Re=100$ and D- $Ri=10$, $Re=300$

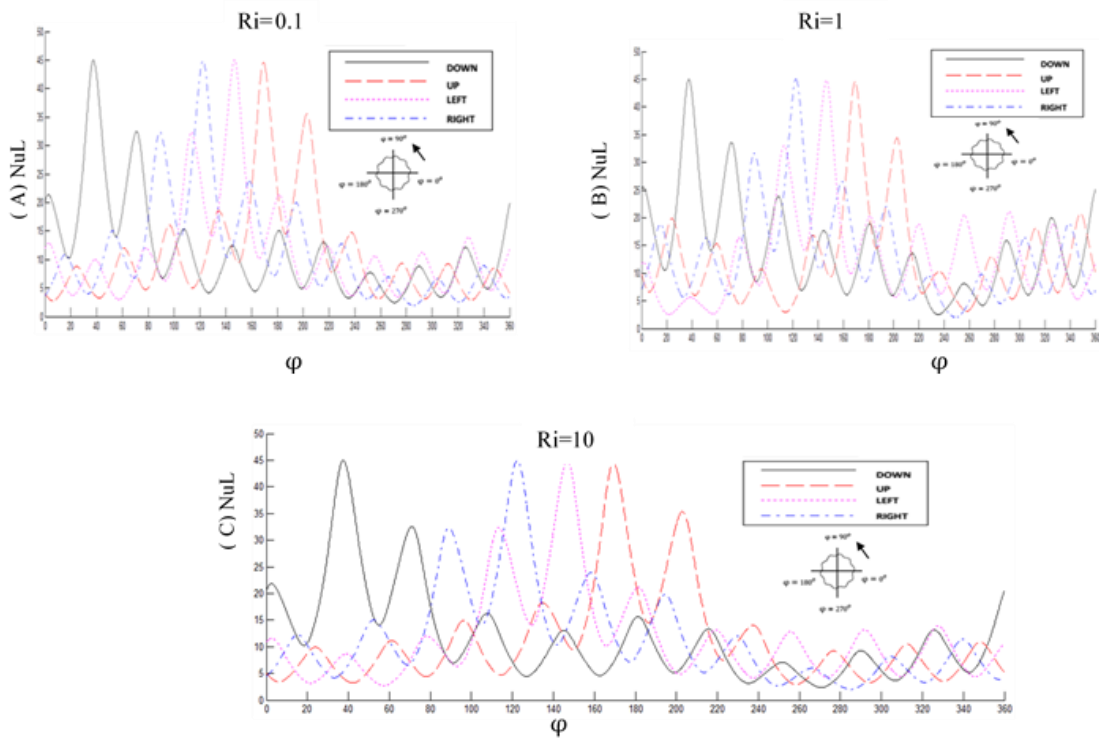


Figure 11. NuL Around Hot Cylinder Located at Different Locations for $Re=100$, A- $Ri=0.1$.B- $Ri=1$,C- $Ri=10$

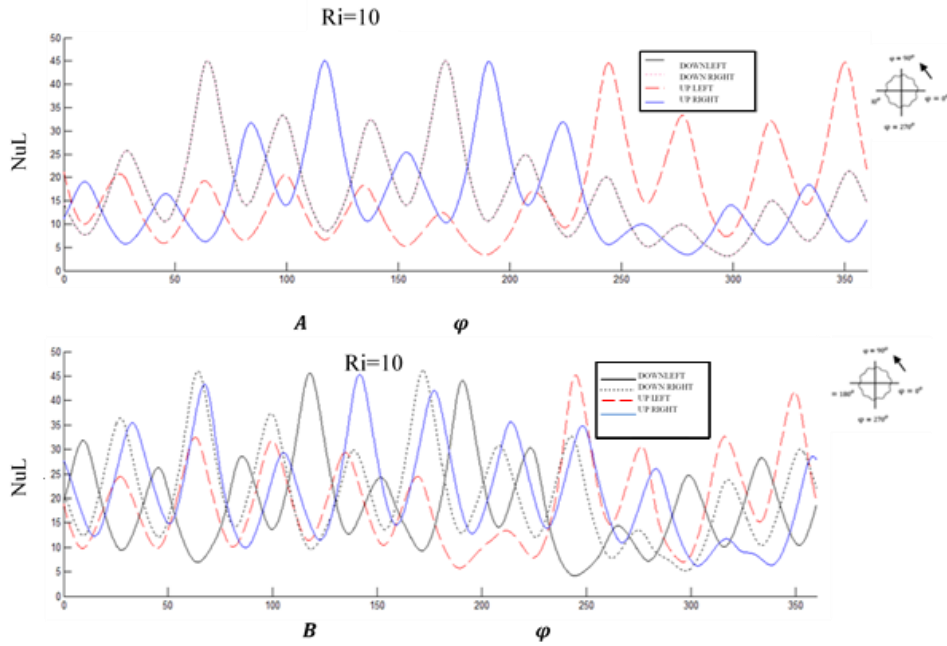


Figure 12. NuL around hot cylinder located at different Locations A-Re=100,Ri=10, B-Re=300, Ri=10

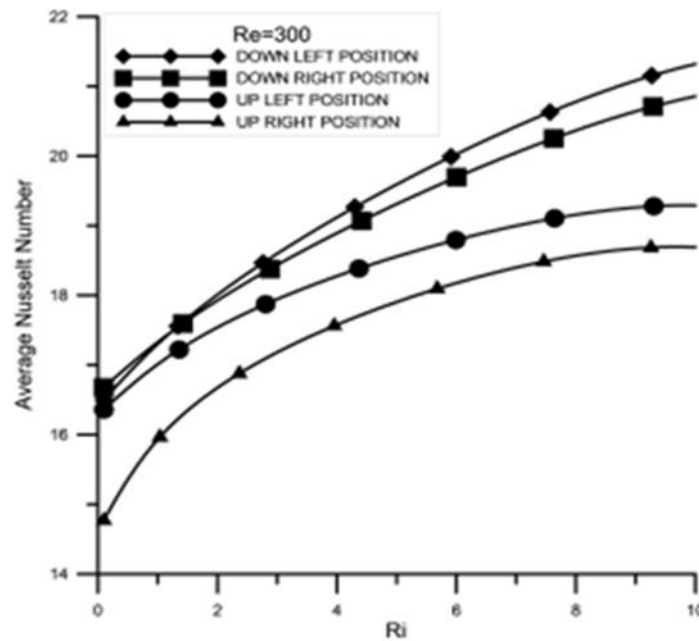


Figure 13. $(Nu)^{-}$ Around Hot Cylinder Located at Different Position for Re= 300

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