2019 First International Conference of Computer and Applied Sciences (CAS),
Mustansiriyah University, Education College, Computer Science Department, Baghdad, Iraq

Three Prior and Double Prior Selection to Comparison Estimate Parameter Rayleigh Distribution under Data Type II Censoring

Abbas Musleh Salman
Ministry of Education
Directorate General of Education in Babylon
Babylon, Iraq
aaladli@yahoo.com

Nabeel Ali Hussein
Ministry of Education
Directorate General of Education in Babylon
Babylon, Iraq
nabeel.ali16@yahoo.com

Ahmed Hadi Hussain
College of Engineering Al-musayab
University of Babylon
Babylon, Iraq
ahmedhadi99@yahoo.com

Sameer Annon Abbas
Ministry of Education
Directorate General of Education in Babylon
Babylon, Iraq
sameeranoon89@yahoo.com

***Abstract*: In this study, we present estimation scale parameter of Rayleigh distribution under data the control of type II by using methods Proposed Bayesian as only prior and Double priors with a new suggestion used three Prior by using distributions Exponential and Gamma and Chi-Square. An experimental study was conducted to compare these methods and to demonstrate the efficiency of the methods proposed in practice by relying on the generated observations from Rayleigh distribution. We compare these methods by using Mean square error (MSE). We did this study by using simulation with different values parameter (θ) and different sample sizes (N=10, 25, 50, 100). It has been shown through computational results that the best way to estimate is the Bayes method proposed (BAY3).**

Keywords: Bayes Estimation, Exponential Distribution, Double Prior, Gamma Distribution, Chi-Square Distribution.

I. INTRODUCTION

Rayleigh distribution is important in various sciences and their applications in life. It has many uses in the medical and industrial field and used widely in wind monitoring and other applications [1]. Usually, when conducting life tests, the experimenter who is studying the tests may not be able to determine and observe the life of all elements tested. This issue is happening because of several different reasons, including time constraints or limitations on the number of failures during a test because the cost of experimenting elements is very expensive. When they are low and directly proportional to the number of failures observed, the experience to be monitored for failures is better and more effective and achieves less time and effort than the time controlled experiment. The researchers also knew the controlled experiment to fail in the name of Type II surveillance. In such a system a type II control system the test is terminated once the number of predefined failures (r) is of observed (n) units tested.

Howlader and Hossain [2] in 1995 studied Bayesian estimation and prediction from Rayleigh based on Type II censored data. Saleem and Aslam [3] in the year 2008 studied the Prior Selection for the Mixture of Rayleigh Distribution using Predictive Intervals. In 2006 Daniels [4] studies Bayesian modelling of several covariance matrices and some results on the propriety of the posterior for linear regression. Moreover, Haq and Aslam [5] in 2009 studied the double prior selection for the parameter of the Poisson distribution.

II. THEORETICAL STUDY

A. Rayleigh Distribution

Suppose T = $t_1, t_2, \ldots, t_m$; is an uncensored observation from a sample of m units or individuals under examination. Also, assume that the uncensored observations data follow the Rayleigh model. Where the one - parameter Rayleigh failure time distribution of θ has a probability density function (pdf) and a cumulative distribution function (cdf) given respectively by

$$f\,(t;\theta) = 2\theta t\, e^{-\theta t^2}\ ,\ 0 < t < \infty,\ \theta > 0 \qquad (1)$$

$$F\,(t;\theta) = 1 - e^{-\theta t^2}\ ,\ \theta > 0 \qquad (2)$$

And reliability function of Rayleigh distribution at time t is given by

$$R_t\,(\theta) = e^{-\theta t^2}\ ,\ 0 < t < \infty,\ \theta > 0 \qquad (3)$$

B. Type II Censoring Data [8]:

Using this type of data mainly in clinical situations and the idea here is to choose (m) of the units such that m < n, n represents the size of the sample under study. Furthermore, the possible function for this type of data is ranked in ascending order by the following formula:

$$L = (\theta|t_1, t_2, \ldots, t_m, m) = \frac{n!}{(n-m)!}\prod_{i=1}^{m} f(t_i;\theta)[1 - F(t_0\,;\theta)]^{n-m} \qquad (4)$$

Such that $1 - F(t_0\,;\theta) = R(t_0\,;\theta)$ represents the reliability function at the time t_0

$f(t_i;\theta)$ represents the density function of failure

$(n-m)$ the number of units is a failed after time t_0

For Rayleigh distribution, the "(4)" becomes as follows

$$L(\theta|t_1,t_2,\dots,t_m,m) = \frac{n!}{(n-m)!}\,(2\,\theta)^m \prod_{i=1}^{m} t_i\; e^{-\theta\sum_{i=1}^{m} t_i^2}\left[e^{-\theta\, t_0^2}\right]^{n-m}$$

$$= W\,(2\,\theta)^m \prod_{i=1}^{m} t_i\; e^{-\theta(\sum_{i=1}^{m} t_i^2 + (n-m)\, t_0^2)} \quad (5)$$

When we take Log the two parties, and from the derivative second we get maximum likelihood estimator such that $W = \frac{n!}{(n-m)!}$

$$\therefore \hat{\boldsymbol{\theta}}_{MLE} = \frac{m}{\sum_{i=1}^{m} t_i^2 + (n-m)\, t_0^2} \quad (6)$$

III. METHDES BAYESIAN ESTIMATION

A. GAMMA $G(a_1,b_1)$ DISTRIBUTION AS ONLY PRIOR (BAY1)

In this method, we will use one prior is a Gamma $G(a_1,b_1)$ distribution for parameter θ, its density function of Gamma is given by $V_1(\theta) = \frac{\theta^{a_1-1}\; b_1^{a_1}\; e^{-\theta\, b_1}}{\Gamma(a_1)}$,

$$\theta > 0\,;\ a_1 \text{ and } b_1 > 0 \quad (7)$$

and by using "(5)", and "(7)", Then joint density function of T and θ described in will be as follows

$$f^*(t_1,t_2,\dots,t_m,\theta) = L(t_1,t_2,\dots,t_m|\theta)\, V_1(\theta)$$

$$= W\,(2\,\theta)^m \prod_{i=1}^{m} t_i\; e^{-\theta(\sum_{i=1}^{m} t_i^2 + (n-m)\, t_0^2)} * \frac{\theta^{a_1-1}\; b_1^{a_1}\; e^{-\theta\, b_1}}{\Gamma(a_1)}$$

Let $K = \sum_{i=1}^{m} t_i^2 + (n-m)\, t_0^2$

Which is a Gamma distribution $G(\omega_1,\beta_1)$ with parameters $\omega_1 = a_1 - 1,\ \beta_1 = K + b_1$

$$\therefore\ f^*(t_1,t_2,\dots,t_m,\theta) = 2^m\, W\, \theta^{m+\boldsymbol{\omega_1}} \prod_{i=1}^{m} t_i\; e^{-\theta\,\boldsymbol{\beta_1}} * \frac{b_1^{a_1}\; e^{-\theta\, b_1}}{\Gamma(a_1)} \quad (8)$$

$$f^*(t_1,t_2,\dots\dots t_m) = \int_0^\infty f^*(\underline{t},\theta)\, d\theta$$

$$= \int_0^\infty 2^m\, W\, \theta^{m+\boldsymbol{\omega_1}} \prod_{i=1}^{m} t_i\; e^{-\theta\,\boldsymbol{\beta_1}} * \frac{e^{-\theta\,\boldsymbol{\beta_1}}}{\Gamma(a_1)}\, d\theta$$

by using transformation $u = \theta\,\beta_1 \Rightarrow \theta = \frac{u}{\beta_1} \Rightarrow d\theta = \frac{1}{\beta_1}\, du$

Hence $f^*(t_1,t_2,\dots\dots t_m) = \frac{2^m\, W\; b_1^{a_1} \prod_{i=1}^{m} t_i}{\Gamma(a_1)} \int_0^\infty \frac{u^{m+\boldsymbol{\omega_1}}}{\boldsymbol{\beta_1}^{m+\boldsymbol{\omega_1}}}\, e^{-u}\, \frac{1}{\boldsymbol{\beta_1}}\, du$

$$= \frac{2^m\, W\; b_1^{a_1} \prod_{i=1}^{m} t_i\, \Gamma(m+\omega_1+1)}{\Gamma(a_1)\;\; \beta_1^{m+\boldsymbol{\omega_1}+1}} \quad (9)$$

By using (8) and (9), then density function of the posterior distribution of $\boldsymbol{\theta}$ is given by

$$H_1^*(\theta\,|t_1,\dots,t_m) = \frac{f^*(t_1,\dots t_m,\theta_1)}{f^*(t_1,\dots\dots t_m)}$$

$$= \frac{\beta_1^{m+\boldsymbol{\omega_1}+1}\;\; \theta^{m+\boldsymbol{\omega_1}}\;\; e^{-\theta\,\boldsymbol{\beta_1}}}{\Gamma(m+\omega_1+1)} \quad (10)$$

By using the quadratic loss function $c(\hat{\theta}-\theta)^2$. Then Bayes' estimator will be the estimator that minimises the posterior risk given by

$$Risk\,(\theta) = E\left[c\,(\hat{\theta},\theta)\right] = \int_0^\infty (\hat{\theta}-\theta)^2\;\; H_1^*(\theta\,|t_i)\; d\theta_1 \quad ,$$

$Let\ \frac{\partial\, Risk\,(\theta)}{\partial\theta} = 0$

Then $\hat{\theta}_{BAY1} = \int_0^\infty \theta\;\; H_1^*(\theta\,|t_i)\; d\theta = \int_0^\infty \frac{\beta_1^{m+\boldsymbol{\omega_1}+1}\;\theta^{m+\boldsymbol{\omega_1}+1}\;\; e^{-\theta\,\boldsymbol{\beta_1}}}{\Gamma(m+\boldsymbol{\omega_1}+1)}\, d\theta$

by using transformation $u = \theta\,\beta_1 \Rightarrow \theta = \frac{u}{\beta_1}$

$$\Rightarrow d\theta = \frac{1}{\beta_1}\, du$$

$$\hat{\theta}_{BAY1} = \frac{\beta_1^{m+\boldsymbol{\omega_1}+1}}{\Gamma(m+\omega_1+1)} \int_0^\infty \left[\frac{u}{\beta_1}\right]^{m+\boldsymbol{\omega_1}+1} \frac{e^{-u}}{\beta_1}\, du$$

$$\therefore \hat{\theta}_{BAY1} = \frac{\Gamma(m+\omega_1+2)}{\Gamma(m+\omega_1+1)\;\beta_1} = \frac{m+\omega_1+1}{\beta_1} \quad (11)$$

B Double Priors by Using Exponential and Gamma Distributions (BAY 2)

We will here use two density functions for two different distributions, the first prior distribution for Exponential Distributions of θ with a parameter with hyperparameter c_1, where it possesses (pdf)

$$V_{21}(\theta) = c_1\; e^{-\theta\, c_1}\,, \qquad \theta > 0\,;\ c_1 > 0 \quad (12)$$

As second Prior Distributions of θ is density functions pdf for Gamma Distributions with hyperparameters a_2 *and* b_2, where it possesses (pdf)

$$V_{22}(\theta) = \frac{\theta^{a_2-1}\;\; b_2^{a_2}\;\; e^{-\theta\, b_2}}{\Gamma(a_2)}\,,$$

$$\theta > 0\,;\ a_2 \text{ and } b_2 > 0 \quad (13)$$

By combining the two previous functions (12) and (13), we will define double prior for θ as follows

$$V_{T1}(\theta) \propto V_{21}(\theta) * V_{22}(\theta) = \theta^{a_2-1}\;\, e^{-\theta\,(c_1+\,b_2)} \quad (14)$$

And by using "(5)" and "(14)", Then joint density function of **T** and $\boldsymbol{\theta}$ described in will be as follows

$$f^*(t_1,t_2,\dots,t_m,\theta) = L(t_1,t_2,\dots,t_m|\theta)\, V_{T1}(\theta)$$

$$= W\,(2\,\theta)^m \prod_{i=1}^{m} t_i\;\, e^{-\theta\, K} * \theta^{a_2-1}\;\, e^{-\theta\,(c_1+\,b_2)} \quad (15)$$

$$= W\, 2^m\, \theta^{m+\boldsymbol{\omega_2}} \prod_{i=1}^{m} t_i\;\; e^{-\theta\,(K+\,c_1+\,b_2)} \quad (16)$$

Where $\boldsymbol{K} = \sum_{i=1}^{m} t_i^2 + (n-m)\, t_0^2$ *and* $\boldsymbol{\omega_2} = a_2 - 1$ *and* $\boldsymbol{\beta_2} = c_1 + b_2$

Hence from (16), we find marginal density function of $\boldsymbol{T}$ is given by

$$f^*(t_1,t_2,\dots\dots t_m)$$

$$= 2^m\, W \prod_{i=1}^{m} t_i \int_0^\infty \theta^{m+\boldsymbol{\omega_2}}\;\;\; e^{-\theta(\,K+\boldsymbol{\beta_2}\,)}\; d\theta$$

by using transformation $u = \theta\,(K+\beta_2) \Rightarrow \theta = \frac{u}{K+\beta_2} \Rightarrow d\theta = \frac{du}{K+\beta_2}$

Hence $f^*(t_1,t_2,\dots\dots t_m) = 2^m\, W\;\; \prod_{i=1}^{m} t_i \int_0^\infty \frac{u^{m+\boldsymbol{\omega_2}}}{(K+\beta_2)^{m+\boldsymbol{\omega_2}}}\, e^{-u}\; \frac{1}{K+\beta_2}\, du$

$$= \frac{2^m\, W \quad \prod_{i=1}^{m} t_i \ \Gamma\,(m + \omega_2 + 1)}{(K\ +\ \beta_2)^{m+\boldsymbol{\omega_2}+\mathbf{1}}} \qquad (17)$$

By using (16) and (17), then pdf of the posterior distribution of $\boldsymbol{\theta}$ is given by

$$H_2^{\,*}(\theta\,|t_1,\dots,t_m) = \frac{f^*(t_1\,,\dots t_m\,,\theta_1)}{f^*(t_1\,,\dots\dots t_m\,)}$$

$$= \frac{\theta^{\,m+\boldsymbol{\omega_2}+\mathbf{1}} \quad e^{-\,\theta(K\,+\,\beta_2)}}{\Gamma\,(m + \boldsymbol{\omega_2} + \mathbf{1})}\ ,\theta > 0 \qquad (18)$$

By using the quadratic loss function $c\big(\ \hat{\theta} - \theta\ \big)^2$. Then Bayes' estimator will be the estimator that minimises the posterior risk given by

$$Risk\ (\theta) = E\big[c\,\big(\hat{\theta},\theta\big)\big]\ = \int_0^\infty \big(\ \hat{\theta} - \theta\ \big)^2 \quad H_2^{\,*}\ (\theta\,|t_i)\ d\theta_1 \quad ,$$

$Let\ \frac{\partial\, Risk\ (\ \theta\ )}{\partial \theta} = 0$

Then

$$\hat{\theta}_{BAY2} = \int_0^\infty \theta \quad H_2^{\,*}\ (\theta\,|t_i)\ d\theta = \int_0^\infty \frac{\theta^{\,m+\boldsymbol{\omega_2}+\mathbf{1}} \quad e^{-\,\theta(\,K\,+\,\beta_2\,)}}{\Gamma\,(\,m+\boldsymbol{\omega_2}+\mathbf{1}\,)}\ d\theta$$

Using the same previous conversion

$$\hat{\theta}_{BAY2} = \frac{(K\ +\ \beta_2\,)^{\,m+\boldsymbol{\omega_2}+\mathbf{1}}}{\Gamma\,(m + \omega_2 + 1)} \int_0^\infty \left[\frac{u}{K\ +\ \beta_2}\right]^{m+\boldsymbol{\omega_1}+\mathbf{1}} \quad \frac{e^{-\,u}}{K\ +\ \beta_2} \quad du$$

$$\therefore \hat{\theta}_{BAY2} = \frac{\Gamma\,(m + \omega_2 + 2)}{\Gamma\,(m + \omega_2 + 1)\ \ K\ +\ \beta_2}$$

$$= \frac{m + \omega_2 + 1}{K\ +\ \beta_2} \qquad (19)$$

C. Using Exponential and Chi-squarere Distributions as Double Priors (BAY 3)

Consider the first prior distribution for Exponential Distributions of θ with hyperparameter c_2 , where it possesses (pdf)

$$V_{31}(\theta) = \quad c_2\ e^{-\,\theta\, c_2}\ , \qquad \theta > 0\,;\ c_2\ > 0 \qquad (20)$$

The second Prior Distributions of θ is density functions pdf for Chi-squarere Distributions with hyperparameters a_3 , where it possesses (pdf)

$$V_{32}(\theta) = \frac{\theta^{\frac{a_3}{2}-1}\ \ e^{-\frac{\theta}{2}}}{2^{\frac{a_3}{2}}\ \Gamma\left(\frac{a_3}{2}\right)}\ , \qquad \theta > 0\ and\ a_3 > 0 \qquad (21)$$

We will define double prior for θ as follows

$$V_{T2}(\theta) \propto V_{31}(\theta) * V_{32}(\theta) = \theta^{\frac{a_3}{2}}\ \ e^{-\,\theta(c_2+\frac{1}{2}\ )} \qquad (22)$$

And by using “ (5)”, and “(22)”, Then joint density function of **T** and $\boldsymbol{\theta}$ described in will be as follows

$$f^*(t_1,t_2,\dots,t_m,\theta) = L(t_1,t_2,\dots,t_m|\theta)\ V_{T2}(\theta)$$

$$= W\ (\ 2\ \theta\ )^m\ \prod_{i=1}^{m} t_i\ \ e^{-\,\theta\, K}\ \ * \theta^{\frac{a_3}{2}}\ \ e^{-\,\theta(c_2+\frac{1}{2}\ )}$$

$$= W\ 2^m\ \theta^{m+\boldsymbol{\omega_2}} \prod_{i=1}^{m} t_i\ \ e^{-\,\theta(\ K+\boldsymbol{\beta_3}\ )} \qquad (23)$$

Where $K = \sum_{i=1}^{m} {t_i}^2 + (n-m)\ {t_0}^2\ \ and\ \ \boldsymbol{\omega_3} = \frac{a_3}{2}\ \ and$

$$\beta_3 = c_2 +\ \frac{1}{2}$$

Hence from (23), we find marginal density function of T is given by

$$f^*(t_1\,,t_2\,,\dots\dots t_m\,)$$

$$= W\ 2^m \prod_{i=1}^{m} t_i \int_0^\infty \ \theta^{m+\boldsymbol{\omega_2}} \quad e^{-\,\theta(\ K+\boldsymbol{\beta_3}\ )}\ d\theta$$

by using transformation $u = \theta\ (\ K +\ \beta_3\ ) \Rightarrow\ \theta = \frac{u}{K\ +\ \beta_3}\ \Rightarrow\ d\theta = \frac{du}{K\ +\ \beta_3}$

Hence

$$f^*(t_1\,,t_2\,,\dots\dots t_m\,) = 2^m\ W \quad \prod_{i=1}^{m} t_i\ \int_0^\infty \frac{u^{\,m+\omega_3}}{(\,K+\beta_3\,)^{\,m+\omega_3}}\ e^{-\,u}\ \ \frac{1}{K+\beta_3}\ du$$

$$= \frac{2^m\ W \quad \prod_{i=1}^{m} t_i\ \ \Gamma\,(m + \boldsymbol{\omega_3} + 1)}{(K\ +\ \beta_3)^{m+\boldsymbol{\omega_3}+\mathbf{1}}} \quad (24)$$

By using (23) and (24), then pdf of the posterior distribution of θ is given by

$$H_3^{\,*}\big(\theta\ |\underline{t}\,\big) = \frac{f^*(t_1\,,\dots t_m\,,\theta_1)}{f^*(t_1\,,\dots,t_m\,)}$$

$$= \frac{\theta^{\,m+\boldsymbol{\omega_3}}\ (K+\beta_3)^{m+\boldsymbol{\omega_3}+\mathbf{1}}\ e^{-\,\theta(K\,+\,\beta_3)}}{\Gamma\,(m + \omega_3 + 1)} \qquad (25)$$

By using the quadratic loss function $c\big(\ \hat{\theta} - \theta\ \big)^2$

Then

$$\hat{\theta}_{BAY3} = \int_0^\infty \theta \quad H_3^{\,*}\ (\theta\,|t_i)\ d\theta = \int_0^\infty \frac{\theta^{\,m+\omega_3+1}\ \ e^{-\,\theta(K\,+\,\beta_3)}}{\Gamma\,(m+\omega_3+1)}\ d\theta$$

Using the same previous conversion

$$\hat{\theta}_{BAY3} = \frac{(K\ +\ \beta_3\,)^{\,m+\omega_3+1}}{\Gamma\,(m + \omega_3 + 1)} \int_0^\infty \left[\frac{u}{K\ +\ \beta_2}\right]^{m+\omega_3+1} \quad \frac{e^{-\,u}}{K\ +\ \beta_3} \quad du$$

$$\therefore \hat{\theta}_{BAY3} = \frac{\Gamma\,(m + \omega_2 + 2)}{\Gamma\,(m + \omega_2 + 1)\ \ (K\ +\ \beta_2\ )}$$

$$= \frac{m + \omega_3 + 1}{K\ +\ \beta_3} \qquad (26)$$

D. Using Exponential and Gamma and Chi-squarere Distributions as Three Priors

In this method, we suggested using three different distributions as priors Distributions of θ with hyperparameter a_4 , b_4 , c_3 Where it has (pdf) respectively as it comes .The first Prior distribution of *θ to* be exponential with hyperparameter c_4

$$V_{41}(\theta) = \quad c_3\ e^{-\,\theta\, c_3}\ , \quad \theta > 0\,;\ c_3\ > 0 \qquad (27)$$

The second Prior Distributions of θ is density functions pdf for Gamma Distributions with hyperparameters a_4, b_4 , where it possesses (pdf)

$$V_{42}(\theta) = \frac{\theta^{a_4-1}\ \ {b_4}^{a_4}\ \ e^{-\,\theta\, b_4}}{\Gamma\,(a_4)}\ ,\theta > 0\,;\ a_4\ and\ \ b_4 > 0 \quad (28)$$

The three Prior Distributions of θ is density functions pdf for Chi - square Distributions with hyperparameters a_4 , where it possesses (pdf)

$$V_{43}(\theta) = \frac{\theta^{\frac{a_4}{2}-1}\ \ e^{-\frac{\theta}{2}}}{2^{\frac{a_4}{2}}\ \Gamma\left(\frac{a_4}{2}\right)}\ , \qquad \theta > 0\ and\ a_4 > 0 \qquad (29)$$

We will define three prior for θ as follows

$$V_{T3}(\theta) \propto V_{41}(\theta) * V_{42}(\theta) * V_{43}(\theta) = \theta^{\frac{3a_4}{2}-2}\ \ e^{-\,\theta(c_3+\,b_4\ +\frac{1}{2}\ )} \qquad (30)$$

And by using " (5)", and "(30)", Then joint density function of T and θ described in will be as follows

$$f^*(t_1, t_2, \dots, t_m, \theta) = L(t_1, t_2, \dots, t_m|\theta)\, V_{T3}(\theta) = W\, 2^m\, \theta^{m+\boldsymbol{\omega_4}} \prod_{i=1}^{m} t_i \;\; e^{-\theta(\,K+\boldsymbol{\beta_4}\,)} \qquad (31)$$

Where $K = \sum_{i=1}^{m} t_i^{\,2} + (n-m)\, t_0^{\,2}$ and $\omega_4 = \frac{3a_4}{2} - 2$ and $\beta_4 = c_3 + b_4 + \frac{1}{2}$

Hence from (23), we find marginal density function of T is given by

$$f^*(t_1, t_2, \dots\dots t_m) = \frac{2^m\, W \prod_{i=1}^{m} t_i \;\Gamma\,(m + \boldsymbol{\omega_4} + 1)}{(K + \beta_4)^{m+\boldsymbol{\omega_4}+\mathbf{1}}} \qquad (32)$$

By using (31) and (32), then pdf of the posterior distribution of $\boldsymbol{\theta}$ is given by

$$H_4^{\,*}(\theta\,|\underline{t}) = \frac{f^*(t_1, \dots t_m, \theta_1)}{f^*(t_1, \dots, t_m)} = \frac{(K+\beta_4)^{m+\boldsymbol{\omega_4}+\mathbf{1}}\, \theta^{\,m+\boldsymbol{\omega_4}}\;\; e^{-\theta(K+\beta_4)}}{\Gamma\,(m+\omega_4+1)} \qquad (33)$$

By using the quadratic loss function $c\big(\,\hat{\theta} - \theta\,\big)^2$

Then $\hat{\theta}_{BAY3} = \int_0^{\infty} \theta \;\; H_4^{\,*}\,(\theta\,|t_i)\; d\theta$

$$\therefore \hat{\theta}_{BAY4} = \frac{m + \omega_4 + 1}{K + \beta_4} = \frac{m + \frac{3a_4}{2} - 1}{K + c_3 + b_4 + \frac{1}{2}} \qquad (34)$$

IV. Practical Aspect (Simulation)

Formulation of a model simulation includes the following essential and important steps for estimation of the scale parameter of Rayleigh distribution that is respectively:

a. The initial values for the parameter θ

This step is important upon which later steps depend. Then we assume the initial values (θ =1.5, 2.5, 3.5) for scale parameter θ of the of Rayleigh distribution.

b. Selected sample size (n)

We chose different sizes of the sample proportionally to the effect of sample size on the accuracy and efficiency of the results obtained from the estimation methods used, so we take the sizes (25, 50,100).

c. The initial values for the time estimation of reliability function (t_0):

We take three values of the time $t_0 = 1, 2, 3$

d. Select values for the constants in the estimators:

We take the value parameter β_i and ω_i . All the results we took uniform values where hyperparameters $a_i = b_i = c_i = 4$

e. Step of data generation:

In this step, the generation of Rayleigh distribution data using the inverse method is as follows

$$F\,(t_i;\theta) = U_i = 1 - \;e^{-t_i(\theta-1)} \Longrightarrow$$

$$t_i = \sqrt{\frac{\log\,(U_i - 1)}{\theta}}$$

$F(t_i;\theta)$ =The distribution function and U_i = Uniformly distributed random variable (0, 1)

f. **Measure comparison :** We adopt the mean square error $MSE\big(\,\hat{\theta}\,\big) = \frac{\sum_{i=1}^{R}(\,\hat{\theta}_i - \theta)^2}{R}$ Where R=1000 is the number of replications. The tables below show the results of the estimation using the simulation. Program the simulation written by **(Mathlab-2015a)**.

V. Explanation Results (Conclusions)

The results of Table (I) of (III) show the following:

- The results showed that the Bayes method by using **Gamma distribution as Only Prior (BAY1)** is best, especially in all size samples, because it has less (MSE).
- Estimator that is used **Exponential and Chi - square Distributions as Double Priors (BAY 3)** is second the best Especially in small samples N = 25 and at θ = 3.5 because it contains a second lower mean square error (MSE).
- We noted decreasing values of **(MES)** with an increasing sample size of all cases this corresponds to the statistical theory.

Table I: Mean squared error for $\boldsymbol{\theta}$ where N=10

Θ	to	Bay1	Bay2	Bay3	Bay4	BEST
1.5	1	0.0165549	0.034997	0.017006	0.0224857	Bay1
	2	0.0139557	0.036218	0.014102	0.086300	Bay1
	3	0.0101693	0.038636	0.0100464	0.027976	Bay3
2.5	1	0.0300040	0.0399612	0.031237	0.093120	Bay1
	2	0.01775658	0.0278309	0.0176044	0.0410449	Bay3
	3	0.0189202	0.0393275	0.0188300	0.050879	Bay3

3.5	1	0.0378265	0.177462	0.0386313	0.375175	Bay1
	2	0.025346	0.0379011	0.0250840	0.043894	Bay3
	3	0.023436	0.0316459	0.0231278	0.035417	Bay3

TABLE II: MEAN SQUARED ERROR FOR $\boldsymbol{\theta}$ WHERE N=25

θ	to	Bay1	Bay2	Bay3	Bay4	BEST
1.5	1	0.0065326	0.031118	0.006444	0.0078445	Bay3
	2	0.0066717	0.034907	0.006583	0.0080689	Bay3
	3	0.0063986	0.0313638	0.0063117	0.0076303	Bay3
2.5	1	0.01418949	0.0198577	0.0140683	0.0169965	Bay3
	2	0.0121490	0.0184798	0.0120297	0.0136799	Bay3
	3	0.0145859	0.0168570	0.0144670	0.017694	Bay3
3.5	1	0.0234345	0.0263925	0.0233862	0.0274745	Bay3
	2	0.0245431	0.0281966	0.0244063	0.0294898	Bay3
	3	0.0217613	0.0238394	0.0216045	0.0246371	Bay3

TABLE III: MEAN SQUARED ERROR FOR $\boldsymbol{\theta}$ WHERE N=50

θ	to	Bay1	Bay2	Bay3	Bay4	BEST
1.5	1	0.006479	0.030726	0.0063724	0.0070254	Bay3
	2	0.0061166	0.0313102	0.0060710	0.0066435	Bay3
	3	0.0062626	0.0310255	0.0062163	0.0068265	Bay3
2.5	1	0.01411189	0.0195880	0.0140106	0.0155103	Bay3
	2	0.0118734	0.0127633	0.01181003	0.0125634	Bay3
	3	0.0116859	0.01214055	0.0116213	0.0123304	Bay3
3.5	1	0.0234143	0.0248424	0.0232345	0.0253236	Bay3
	2	0.0221183	0.0233267	0.0221182	0.0237245	Bay3
	3	0.0206068	0.0213905	0.0205229	0.0216926	Bay3

TABLE IV: MEAN SQUARED ERROR FOR θ WHERE N=100

θ	to	Bay1	Bay2	Bay3	Bay4	BEST
1.5	1	0.0055625	0.029179	0.0055400	0.005752	Bay3
	2	0.0060607	0.0310944	0.0060370	0.0063147	Bay3
	3	0.0061764	0.0308526	0.0061525	09.006444	Bay3
2.5	1	0.1335690	0.0183423	0.0133225	0.0138874	Bay3
	2	0.0124730	0.0125735	0.0114391	0.0128871	Bay3
	3	0.0114931	0.0117062	0.0114609	0.0117948	Bay3
3.5	1	0.02175111	0.022258	0.0217074	0.022436	Bay3
	2	0.2067237	0.0210701	0.0206288	0.0212187	Bay3
	3	0.0199424	0.0202733	0.0198996	0.0204037	Bay3

REFERENCES

[1] A. J. Gross, Survival distributions: Reliability applications in the biomedical sciences, Wiles series in probability and mathematical statistics,1975.

[2] H.A. Howlader and A. Hossain, "On bayesian estimation and prediction from rayleigh based on type II consored data " Communications in Statistics- Theory and Methods, vol. 24, pp. 2251–2259, April 1995.

[3] M. Saleem and M. Aslam "On the Prior selection for the mixture of Rayleigh Distribution using predictive Intervals." Pakistan Journal of Statistics, vol. 24, pp. 21–35, Aug 2008.

[4] M.J. Daniels "Bayesain modelling of several covariance matrices and some results on the propriety of the posterior for linear regression with correlated and : or heterogeneous errors" Jornal of Multivariate Analysis, vol. 97, pp. 1185–1207, July 2008.

[5] A. Haq and M. Aslam "On the Double prior Selection for the Parameter of Poisson Distribution, Interstat, Quaid-i-Azam, University, 45320, Pakista