Sensitivity Dependent on Initial Condition of Rossler System

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Abstract

In this work we search the chaotic behavior for the Rossler system through employment sensitive depends on initial condition by using the software (Matlab) we get sensitive depends on initial condition (chaos) by varying the parameter of system.

الخلاصة

في هذا العمل درسنا السلوك الفوضوي لنظام روسيلر (Rossler System) من خلال الاعتماد على الشروط الابتدائية (sensitive dependent on initial condition) ولهذا الغرض استخدمنا برنامج (Matlab) وأجرينا تغيرات في قيم المعالم (Parameters) لهذا النظام.

1-Introduction

Rossler systems is introduced in the 1970s as prototype equations with the minimum ingredients for continuous times chaos.

Since the Poincar´e-Bendixson theorem precludes the existence of other than steady periodic, attractors in au tonomous systems defined in one- or two -dimensional manifolds such as the line, the circle, the plane, the sphere, or the torus (Hartman, 1964), the minimal dimension for chaos is three. On this basis, Otto Rossler came up with a series of prototype systems of ordinary differential equations in three-dimensional phase spaces (Rossler 1976a,c, 1977a, 1979a). systems He also proposed four-dimensional for hyper chaos, that is chaos with more than one positive Lyapunov exponent (Rossler 1979a,b).

Rossler was inspired by the geometry of flows in dimension three and, in particular, by the re-injection principle, which is based on the feature of relaxation-type systems to often present a Z-shaped slow manifold in their phase space. On this manifold, the motion is slow until an edge is reached whereupon the trajectory jumps to the other branch of the manifold, allowing not only for periodic relaxation oscillations in dimension two, but also for higher types of relaxation behavior as noted by Rossler (1979a). In dimension three, the re-injection can induce chaotic behavior if the motion is spiraling out on one branch of manifold). In this way, Rossler invented a series of systems, the most famous of which is probably (Rossler 1979a).

2-In this section we study the chaotic behavior of Rossler system depend on the definition of Gulick which is referred to in section two.

2- Definition

In this section we introduce many fundamental definitions we use in this work

- **Definition 1 [Periodic attracting]**

Let x be a periodic –n point for a function f then x is attracting period-n point if x is an attracting fixed point of f^n [Gulick,1992]

- **Definition 2 [lyapunov expoent]**

Let J be abounded interval, and f:J→J continuously differentiable on J. fix x in J ,and let λx be defined by

$\lambda x=\text{Lim}\ 1/n\ |f^{(n)}|\ (x)$……(1)

$n\to\infty$

provided that the limit exist . in that case λx is the lyapunov exponent of f at x [Gulick,1992]

- **Definition 3 [sensitive dependent on initial condition]**

Let J be an interval, and $f:J\to J$ has asensitive dependent on initial condition if ther exist $\xi>0$ such that for any $x \in J$ and any neighborhood N of x ,ther exist $y\in N$ and $n>0$ such that

$| f^n(x)-f^n(y) | > \xi$ [Deveny,1989]

- **Definition 4 [Chaoic]**

A function f is Chaoic if satisfies at least one of the follwing conditions

(i) f has appositive lyapunov exponent at each point in its domain

(ii) f has a sensitive dependent on initial condition on its domain [Gulick,1992]

- **Definition 5 [Capacity and Fractal dimension]**

Let S be subset of R^n**,**wher n=1,2 or 3 the capacity dimention of S is given by $\mathrm{Dim}_c S = \lim_{\epsilon\to 0} \ln(N(\epsilon))/\ln(1/\epsilon)$ …….(2)

If the limit exist and is not integer then S is said to be have Fractal dimension [Gulick,1992]

- **Definition 6 [Bifurcation]**

Consider the differential equation :

$\dot{x} = f_\mu(x)$ ……..(3)

one is especially concerned how the phase portrait of (3) chang as μ varies ,A value μ_0 where there is a basic structural change in this phase portrait is called a bifurcation point [Gulick,1992]

- **Definition 7 [Bifurcation diagram]**

One method of displaying the points at which a parameterized family of function $\{f_\mu\}$ bifurcates and is designed to give information about the behavior of higher interates of arbitrary member of the domain of f_μ for all value of parameter μ [Gulick,1992]

3-Rossler Model

Rossler was able to obtain the simplest nonlinear vector field capable of generating chaotic behavior [Rossler,1976]see however, [Sprott,1994] This attractor is written in the following form :

$$\begin{aligned} \dot{x} &= -x-y \\ \dot{y} &= x+ay \qquad \text{.......(4)} \\ \dot{z} &= b+z(x-c) \end{aligned}$$

such that it has a single nonlinear term *xz* in $\dot{z}$.

By fixing *a* and *b* in the value *a*=*b*=0.2, one has a

period-doubling route to chaos where a period-2 orbit is created at *c*=2.6, and being *c*~4.2 the accumulation point of the period doubling cascade, beyond which one has deterministic chaos, excepting for the presence of a number of periodic windows. The system has an unstable fixed point near the origin whose 2D unstable manifold presumably spans the strange attractor. It appears that the strange attractor does not exhibit a remerging tree (or period-doubling reversal) [Stone,1993], at least for not too large values.

4-Description of Plots

In Fig.(1) one can see the scatter-plots for the Rossler attractor.

The left column of plots Fig. (1a,1c and 1e) are the results for the new algorithm, whereas the column on the right-hand side, Fig.(1b, 1d and 1f) shows the results for the Wolf algorithm. Both plots 1a and 1b have the x-coordinate of the Rossler attractor as abscissa. Analogously, plots 1c and 1d have the y-coordinate of the Rossler attractor as abscissa and plots 1e and 1f the z-coordinate. The ordinate of all cases is the value of the positive local Lyapunov exponent λ1 (t).

Fig. (2) shows the pair wise Renyi spectra corresponding to the plots of Figs. (1.) The dashed line is the spectrum for the Wolf algorithm and the full line for the new one. Specifically, parts Rossler a, Rossler b and Rossler c denote for the pairs of spectra that correspond to the pairs of point sets (1a,1b), (1c,1d), (1e,1f), respectively [Grond and Diebner,2005].

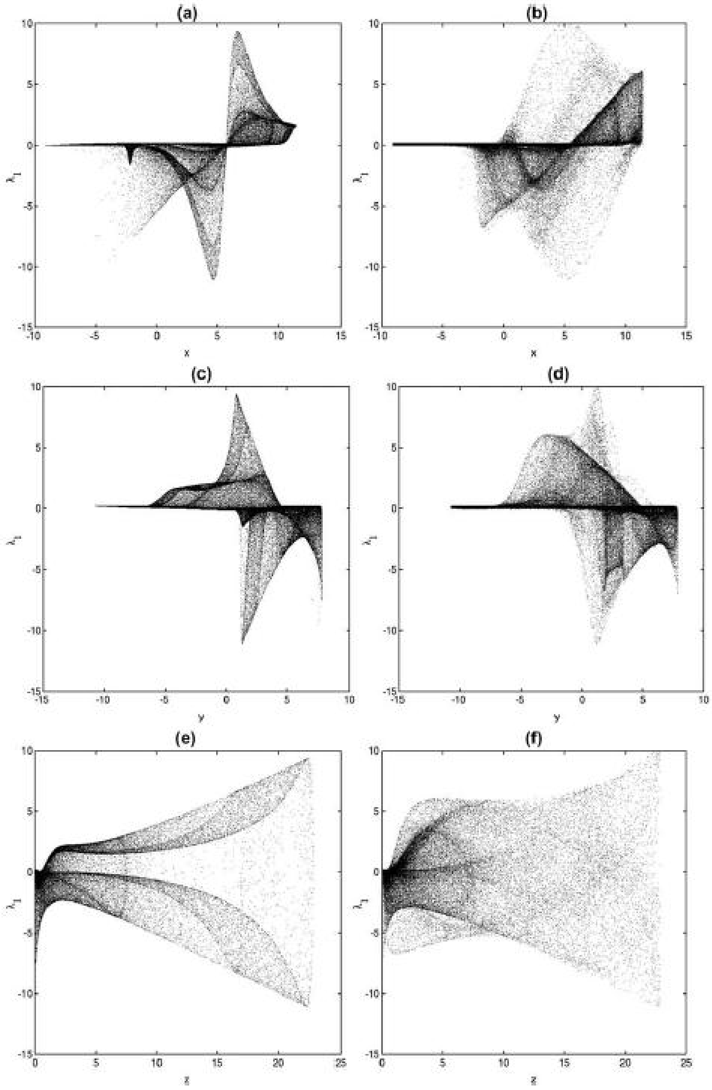


Fig. (1) Plots of the x-, y-, and z-coordinates of the Rossler attractor against the local Lyapunov exponent k1. The left column (a, c, and e) shows the results for the new algorithm, the right column (b, d, and f) for the Wolf algorithm.

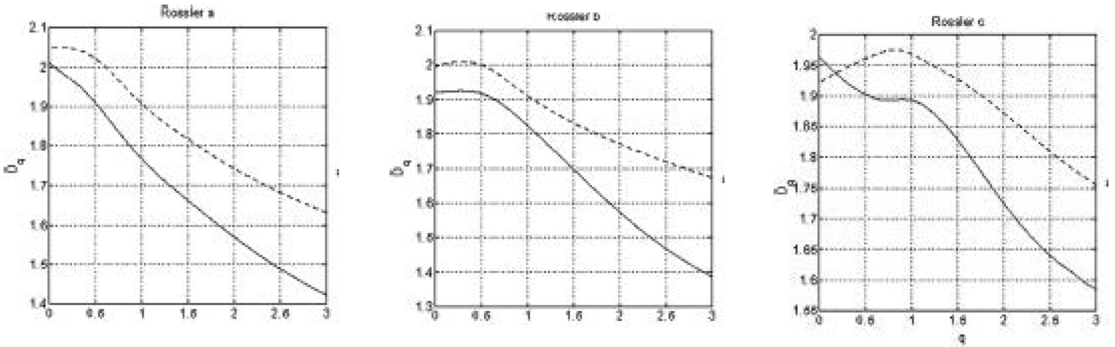


Fig. 2. Plots of the Renyi spectra computed from the point sets of Figs. 1 and 2. The left column shows the results for the Rossler attractor, The curve corresponding to the Wolf algorithm is shown as the dashed line. The full line belongs to the new variant.

Fig.(3) show scatter plots of all three local exponents λ1 (t), λ2 (t), λ3 (t) that have been computed for the Rossler attractors. Again, the two parts on the left-hand side show the results for the new algorithm and those corresponding to the Wolf algorithm on the right.

Fig(4) shows the Renyi spectra computed from the point clouds of Figs.(3.) The dashed lines denote for the Wolf algorithm, as before. Fig.(4a) corresponds to Fig.(3) and Fig.(4b) the curve belonging to the new one, can be observed for small values in three cases (Figs. 2 c, and 4b). In general, the calculation of the fractal dimension is less robust (which is between the information dimension and the capacity dimension), as discussed in [Kantz andSchreiber,2002]. Systematic errors have to be taken into account in those cases. There are some cases where the dashed line (corresponding to Wolf_s algorithm) increases as a function of q (Figs. 2 b, 2c, and 4a) which indicate systematic errors [Ground 2005]

fig.(3)

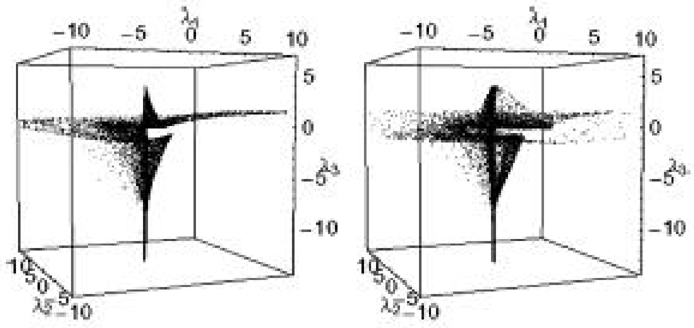
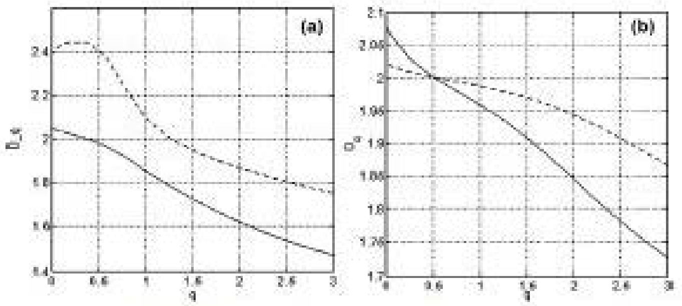


Fig. 4. Scatter plots of λ_1 vs. λ_2 vs. λ_3 for the Rossler attractor. The left part belongs to the new algorithm and the right one to the Wolf algorithm.

fig. (4)



5-

Fig. 6. Renyi spectra from the point sets in Figs. 4 (corresponding to (a)) and 5 (corresponding to (b)). The dashed line corresponds to the Wolf algorithm, the full line to the new one.

Let us start by briefly describing two typical solutions to the Rossler system [Rossler,1976] readingas (4) where *(a, b, c)* are the bifurcation parameters. The Rössler system has two fixed points given by :

$x=\pm (c \pm \surd c2 - 4ab)/2$

$y = \pm(c \pm \surd c2 - 4ab)/ 2a$ ……(5)

$z=\pm (c \pm \surd c2 - 4ab)/2a$

For $a = 0.432$, $b = 2$ and $c = 4$, the Rössler system has a chaotic attractor for solution (Fig. 5a). According to Farmer et al. [Farmer and Crutchfield andFroeling and Pachard ,1980], we designate this attractor as the *spiral* attractor. This attractor is characterized by a first-return map to the Poincaré section. For three-dimensional systems such a section is defined by the plane :

$P \equiv \{(yn, zn) \in R2|xn = x-, \dot{x}n > 0\}$ …. (6)

Thus, the map is constituted by an increasing monotonic branch and a decreasing branch separated by the critical point located at the maximum (Fig. 5b). The critical point defines the generating partition of the attractor which allows the encoding of all periodic orbits embedded within the attractor [Letellier and Dutertre and Maheu,1995] The increasing branch is close to the bisecting line and, consequently, the symbolic dynamics is almost complete. A two-symbol symbolic dynamics [Devaney ,] is complete when all periodic orbits which can be encoded with these two symbols are solutions to the Rössler system. Thus, for a=0.432, most of periodic orbits encoded with two symbols are embedded within the attractor generated by the Rössler system.

When the bifurcation parameter a is increased, new periodic orbits are created and the chaotic attractor increases in size (Fig. 6b). The corresponding first-return map is constituted by more than two branches and, for $a = 0.556$, up to eleven monotonous branches may been identified [Letellier and Dutertre and Maheu,1995]. The corresponding attractor is designated as the *funnel* attractor [Farmer and Crutchfield andFroeling and Pachard ,1980]. For a greater than 0.556, there is metastable chaos, that is the trajectory visits the neighborhood of the unstable periodic orbits solution to the Rössler attractor before being ejected to infinity [Letellier and Dutertre and Maheu,1995]. The dynamics of the Rössler system can therefore be investigated for $a < 0.556$, b and c remaining constant.

A bifurcation diagram synthesizes the evolution of the dynamics under the change of the bifurcation parameter a (Fig7). The bifurcation parameter a is varied over the interval [0.432, 0.556]. It will be shown that quite a similar bifurcation diagram is obtained when the discretization time step h of the discretization of the Rössler system is increased.

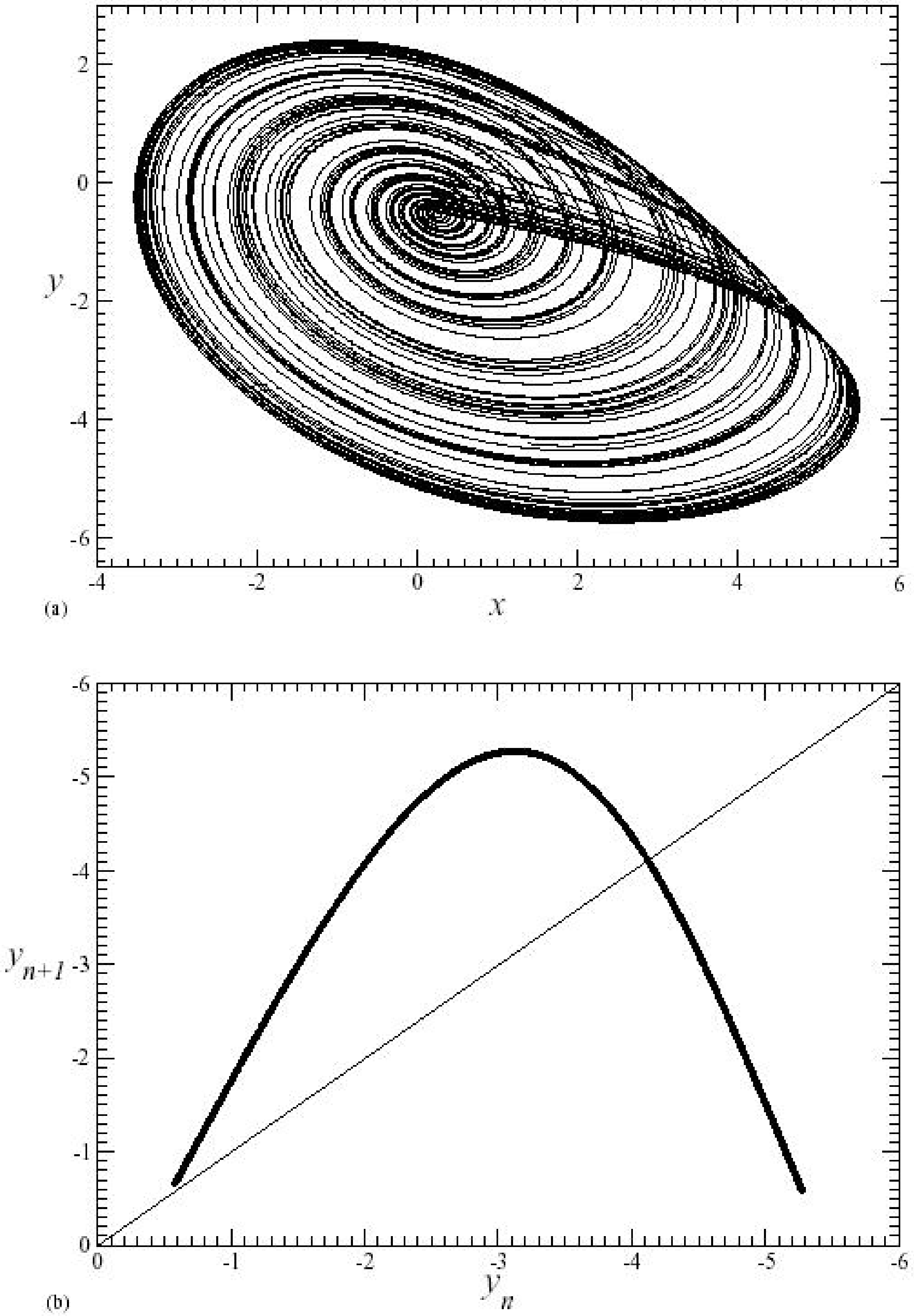
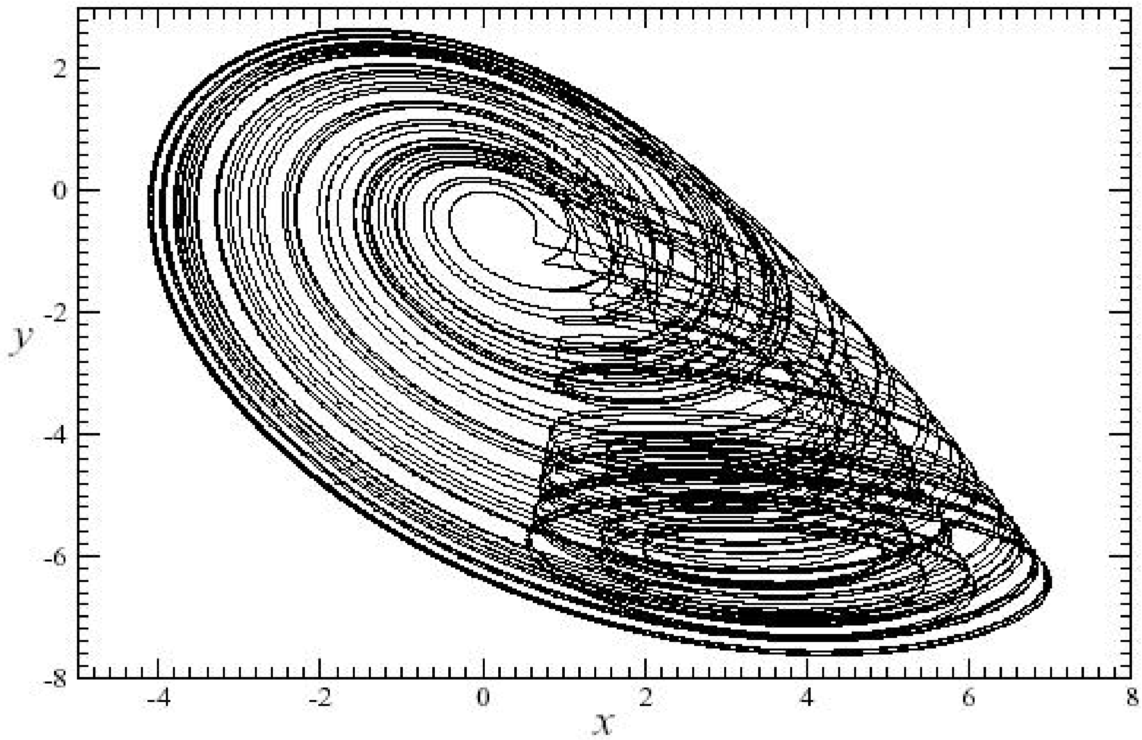
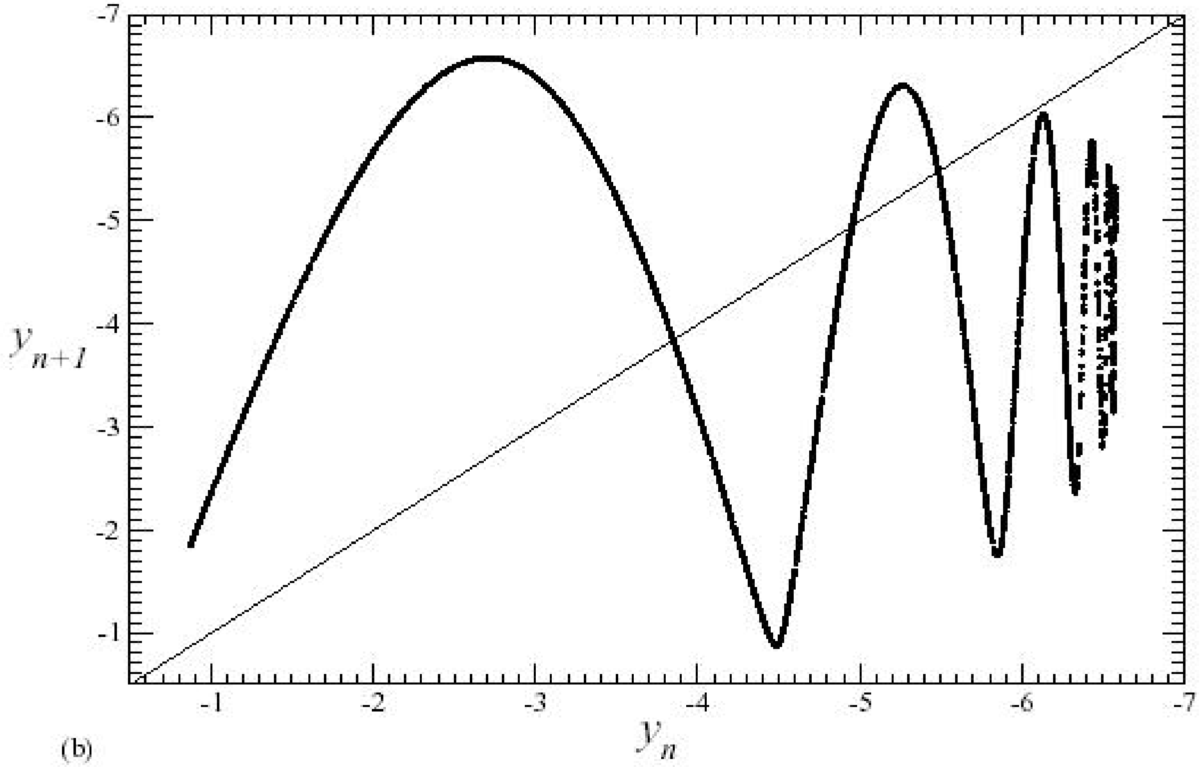


Fig. (5). Spiral attractor generated by the Rössler system (5) with the bifurcation

parameters *(a, b, c)* = (0.432, 2, 4).





(b)

Fig.(6). Funnel attractor generated by the Rössler system (5) with the bifurcation parameters *(a, b, c)* = *(*0.556, 2, 4).

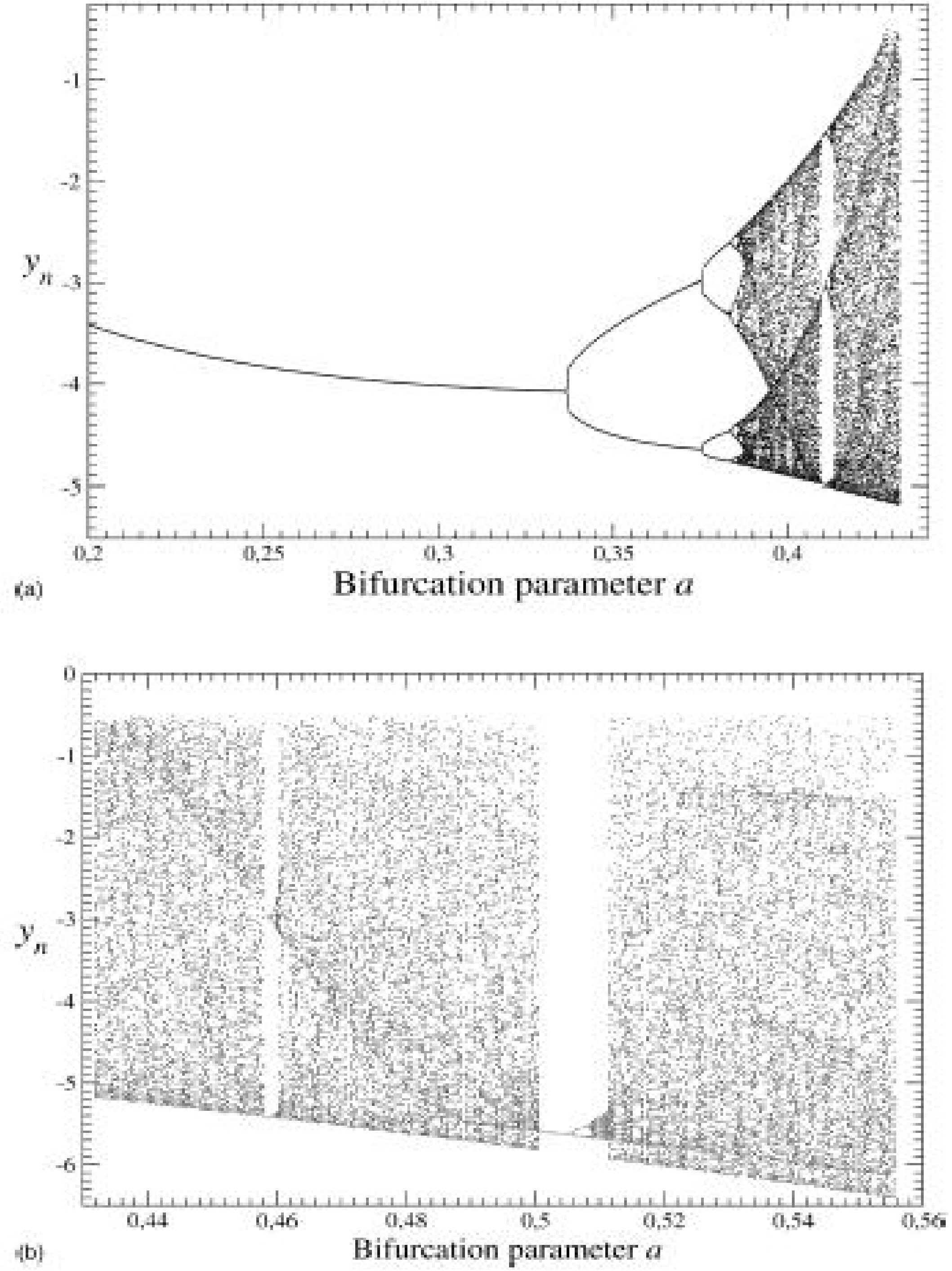


Fig. (7). Bifurcation diagram vs. the bifurcation parameter *a* of the Rössler system (5). Part (a) corresponds to values smaller than 0.432 here used as a reference and (b) for values larger than this reference.

6-Sensitive dependent on initial condition of Rossler System :

In this section we study the chaotic behavior of Rossler system is depend on definition of Gulick which is refered to in section two.

New , we study the sensitivity to initial condition of system (4) by verying the control parameters (a,b,c) by using (matlab) to analysis of view for sensitivity dependent on initial condition .this jop show as appendix New concider the system (4) we get sensitivity to initial condition on the initial point (xi,yi,zi) as follows (i=1,2)

(i) (0,0,0.01) and (0.03,0,0.01) with parameter a=10,b=2,c=8 as show in fig (8).

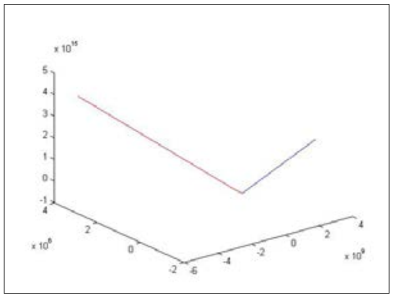


Fig (8)

(ii) (0,0,0.01) and (0.01,0,0.01) with parameter a=10,b=2,c=8 as show in fig (9) .

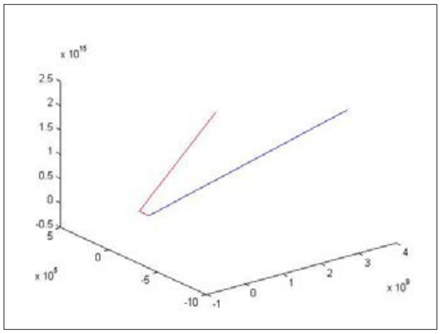


Fig (9)

(iii) (0,0,1,0.01) and (0.1,0.1,0.01) with parameter a=0.2,b=0.2,c=6.7 as show in fig (10).

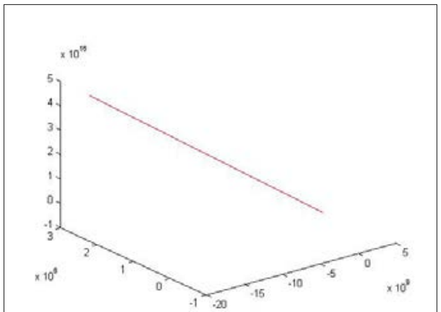


Fig (10)

(iv) (2.05,4,6) and (2, 4,6) with parameter a=0.55,b=2,c=6 as show in fig (11)

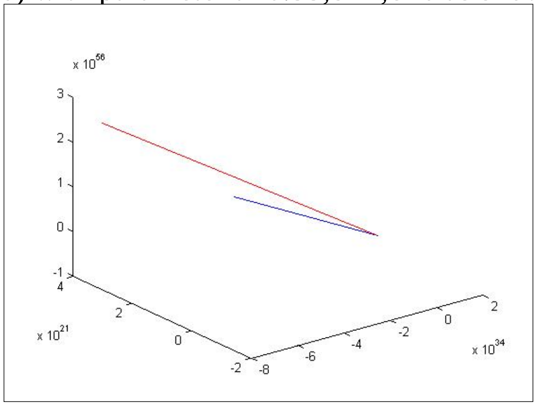


Fig (11)

(v) (0.1,0,0.3) and (0.1,0.2,0.3) with parameter a=1,b=2,c=4 as show in fig.(12).

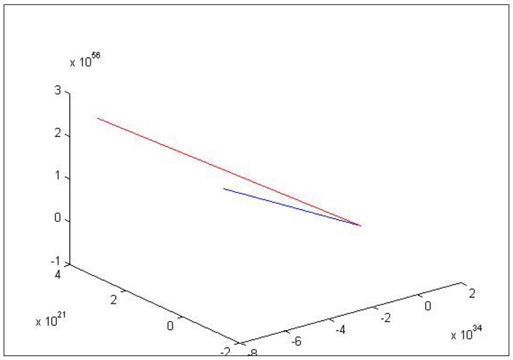
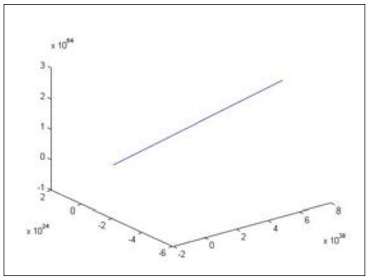


Fig (12)

But,when we change the parameter, we did not get any sensitive to the above initial condition as show in fig(13,14).



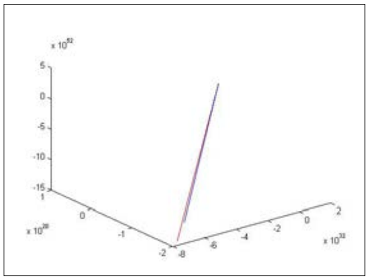


Fig (13) **Fig (14)**

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