Journal of Kerbala University , Vol. 10 No.3 Scientific . 2012

ON WEAKLY λ-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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Abstract

. As a generalization of λ -continuous functions, we introduce and study several properties of weakly λ -continuous functions in Bitopological spaces and we obtain its several characterizations

Keywords and phrases. Bitopological spaces, λ -open sets ,weakly λ -continuous function.

ألخلاصه

ان موضوع λ -continuous في فضاء ثنائي التبولوجي الذي تم تعريفه في [1] قد تم استخدامة في البحث لتعريف انواع اضعف في الاستمرارية وهي weakly λ -continuous و almost λ-continuous مع بعض النظريات و الأمثلة

1. Introduction

The notion of λ -open sets due to al-talkany[1] , semi-preopen sets due to Andrijević [2] plays a significant role in general topology. In [3] the concept of λ -continuous functions is introduced and further Popa and Noiri[5] studied the concept of weakly λ -continuous functions. .In this paper,we introduce and study the notion of weakly λ -continuous functions in bitopological spaces further and investigate the properties of these functions.

Throughout the present paper,(X,T,T^{α})(resp.(X,τ))) denotes a bitopological (resp.topological) space.Let (X,τ) be a topological space and A be a subset of X.The closure and interior of A are denoted by Cl(A) and Int(A) respectively.

Let ,(X,T,T^{α}) be a bitopological space and let A be a subset of X.The closure and interior of A with respect to T or T^{α} are denoted by $Cl_T(A)$, $int_T(A)$or $Cl_T{}^{\alpha}(A)$ and $Int_T{}^{\alpha}(A)$,respectively.

2. basic definition

In this section we give all basic definition and some theorems and lemma we needs in this paper.

Definition 2.1 [1]. A subset A of a bitopological space ,(X,T,T^{α})) is said to be

(i)regular open if $A=Int_T((Cl_T{}^{\alpha}(A))$.

(ii).regular closed if $A=Cl_T\ ((Int_T{}^{\alpha}(A))$.

(iii).preopen if $A\subseteq Int_T((Cl_T{}^{\alpha}(A))$.

Remark 2.1:

1. λ-interior mean that the interior w.r.t. λ-open set.
2. λ-cl mean clouser w.r.t. λ-open set.

Definition 2.2.[1] A subset A of a bitopological space ,(X,T,T^{α}) is said to be λ -open if there exist T^{α}-open set U such that $A\subseteq U$, $A\subseteq Cl_T(U)$.

Lemma 2.1.[1]Let (X,τ1,τ2) be a bitopological space and A be a subset of X.Then

(i).A is λ-open if and only if A=λInt(A).

(ii).A is λ-closed if and only if A=λCl(A).

Lemma 2.2.For any subset A of a bitopological space ,(X,T,T^{α}),$x\in\lambda Cl(A)$if and only if $U\cap A\neq \emptyset$ for every λ-open set U containing x.

Definition 2.3.[4]A function f,$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha})$ is said to be λ-continuous if $f^{-1}(V)$ is λ-open in X for each K-open set V of Y.

3.Weakly λ- continuous

In this section we define weakly **λ** –continuous with some theorems

Definition 3.1.(i). A function f,$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha})$ is said to be weakly precontinuous if for each $x\in X$ and each K-open set V of Y containing f(x),there exists preopen set U containing x such that $f(U)\subseteq Cl_T^{\alpha}(V)$.

(ii). A function f:$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha})$ is said to be weakly- λ-continuous if for each $x\in X$ and each K-open set V of Y containing f(x),there exists λ-open set U containing x such that $f(U)\subseteq Cl_T^{\alpha}(V)$. A function f,$(X,T,T^{\alpha}) \rightarrow (Y,K, K^{\alpha})$ is said to be pairwise weakly precontinuous (resp.pairwise weakly λ-continuous) if f is weakly precontinuous and weakly -precontinuous (resp. if f is weakly λ-continuous)

Example 3.1 . Let X={a,b,c,d},T={ X, φ,{a}, {b,c} , {a,b,c}}, $T=T^{\alpha}$

λ – open (X) =0

Y={1,2,3},K={Y, φ,{1}},K^{α}={X, φ,{1},{1,2},{2,3}}

let f:$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha})$ defined by f(a)=1,f(b)=f(c)=2 then f is weakly λ- continuous .

Remark 3.1 :The composition of two weakly λ- continuous is not necessary weakly λ-continuous

Theorem 3.2.For a function f:$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha})$,the following properties are equivalent:

(i). f is weakly λ-continuous.

(ii). $\lambda Cl(f^{-1} (Int_T^{\alpha}(Cl_T(B))))) \subseteq f^{-1} (Cl_T(B))$ for every subset B of Y.

(iii). $\lambda Cl(f^{-1} (Int_T^{\alpha}(F))) \subseteq f^{-1} (F)$ for every regular closed set F of Y.

(iv). $\lambda Cl(f^{-1} (Cl(V)) \subseteq f^{-1}(Cl_T(V))$ for every K-open set V of Y.

(v). $f^{-1} (V) \subseteq \lambda Int(f^{-1} (Cl_T^{\alpha}(V)))$ for every K-open set V of Y.

Proof. (i) →(ii). Let B be any subset of Y. Assume that $x\in X\sim f^{-1} (Cl_T (B))$.Then $f(x) \in Y\sim Cl_T (B)$ and so there exists a K-open set V of Y containing f(x) such that $V\cap B=\emptyset$, so $V\cap Int_T^{\alpha} (Cl_T (B)))= \emptyset$ and hence $Cl_T^{\alpha} (V)\cap Int_T^{\alpha} (Cl_T (B)))= \emptyset$. Therefore, there exists λ -open set U containing x such that $f(U) \subseteq Cl_T^{\alpha} (V)$.

Hence we have $U\cap f^{-1}(Int_T^{\alpha} (Cl_T (B)))= \emptyset$ and $x\in X\sim \lambda Cl(f^{-1} (((Int_T^{\alpha} (Cl_T (B))))$ by Lemma 2.3.Thus we obtain $\lambda Cl(f^{-1} (((Int_T^{\alpha} (Cl_T (B)))) \subseteq f^{-1} (Cl_T (B))$.

(ii) →(iii).Let F be any regular closed set of Y.Then $F= Cl_T (Int_T^{\alpha} (F)$ and we have $\lambda Cl(f^{-1} ((Int_T^{\alpha} (F)))= \lambda Cl(f^{-1} ((Int_T^{\alpha} (Cl_T (Int_T^{\alpha} t(F))))) \subseteq f^{-1} (Cl_T (Int_T^{\alpha} (F))) = f^{-1} (F)$.

(iii) → (iv) . For any K-open set Vof X $Cl_T (V)$ is regular closed .then $\lambda\text{-} Cl(f^{-1}(V)) \subseteq \lambda Cl(f^{-1} (Int_T^{\alpha}(Cl_T(V))\subseteq f^{-1}(Cl_T (V))$

(iv) → (v) Let V be an K-open set of Y . the $Y/ Cl_T^{\alpha}(V)$ is K-open set in Y and we have $\lambda Cl(f^{-1}(Y/ Cl_T^{\alpha}(V)) \subseteq f^{-1}(Cl_T (Y/ Cl_T^{\alpha}(V))$ and hence $X/ \lambda Int(f^{-1} (Cl_T^{\alpha}(V)) \subseteq X/ f^{-1} (Int_T (Cl_T^{\alpha} (V)) \subseteq X/f^{-1}(V)$.therefore we obtain $f^{-1}(V) \subseteq \lambda Int(f^{-1} (Cl_T^{\alpha}(V))$.

4. Weakly*- quasi continuous

Now we define the regular in the topological space(X,T,T^{α}) with some theorems

Definition 4.1.A bitopological space ,(X,T,T^{α}) is said to be regular if for each $x\in X$ and each T-open set U containing x, there exists a T-open set V such that $x\in V\subseteq Cl_T^{\alpha} (V) \subseteq U$.

Definition 4.2. A function f:$(X,T,T^{\alpha})\rightarrow (Y,K,K^{\alpha}))$ is said to be weakly*- quasi continuous(briefly.w*.q.c) if for every K -open set V of Y,
$f^{-1} (Cl_T^{\alpha} (V)\sim V)$ is biclosed in X.

Theorem 4.3.If a function $f:(X,T,T^{\alpha})\rightarrow(Y,K,K^{\alpha})$ is weakly- λ -continuous and w*.q.c, then f is λ -continuous.

Proof .Let $x\in X$ and V be any K-open set of Y containing f(x).Since f is weakly- λ -continuous, there exists an λ -open set U of X containing x such that $f(U)\subseteq Cl_T^{\alpha}(V)$.Hence $x\notin f^{-1}(Cl_T^{\alpha}(V)\sim V)$.Therefore,$x\in U\sim f^{-1}(Cl_T^{\alpha}(V)\sim V)=U\cap(X\sim(f^{-1}(Cl_T^{\alpha}(V)\sim V))$.Since U is λ -open and $X\sim(f^{-1}(Cl_T^{\alpha}(V)\sim V)$ is biopen , $G=U\cap(X\sim(f^{-1}(Cl_T^{\alpha}(V)\sim V))$ is λ -open. Then $x\in G$ and $f(G)\subseteq V$.For if $y\in G$, then $f(y)\notin(Cl_T^{\alpha}(V)\sim V)$ and hence $f(y)\in V$. Therefore,f is λ -continuous.

5. Almost λ –continuous

In this section we define almost λ –continuous with some theorems

Definition 5.1. A function $f:(X,T,T^{\alpha})\rightarrow(Y,K,K^{\alpha})$ is said to have a λ interiority condition if $\lambda Int(f^{-1}(Cl_T^{\alpha}(V)))\subseteq f^{-1}(V)$ for every K-open set V of Y.

Definition 5.2.A function $f:(X,T,T^{\alpha})\rightarrow(Y,K,K^{\alpha})$ is said to be almost λ -continuous if for each $x\in X$ and each K-open set V containing f(x),there exists an λ -open set U of X containing x such that $f(U)\subseteq Int_T(Cl_T^{\alpha}(V))$.

Lemma 5.1. A function $f:(X,T,T^{\alpha})\rightarrow(Y,K,K^{\alpha})$ is almost λ -continuous if and only if $f^{-1}(V)$ is λ -open for each regular open set V of Y.

Definition 5.3.A bitopological space ,(X,T,T^{α}) is said to be almost regular if for each $x\in X$ and each regular open set U containing x,there exists an regular open set V of X such that $x\in V\subseteq Cl_T^{\alpha}(V)\subseteq U$.

Theorem 5.4. Let a bitopological space (Y,K,K^{α}) be almost regular. Then a function $f:(X,T,T^{\alpha})\rightarrow(Y,K,K^{\alpha})$ is almost λ -continuous if and only if it is weakly- λ -continuous.

Proof . Necessity this is obvious

Sufficiency . Suppose that f is weakly- λ -continuous. Let V be any regular open set of Y and
$x\in f^{-1}(V)$.Then we have $f(x)\in V$. By the almost -regularity of Y, there exists an regular open set V0 of Y
such that $f(x)\in V0\subseteq Cl_T^{\alpha}(V0)\subseteq V$. Since f is weakly- λ -continuous, there exists an λ -open set U of X containing x such that $f(U)\subseteq Cl_T^{\alpha}(V0)\subseteq V$. This implies that $x\in U\subseteq f^{-1}(V)$.Therefore we have $f^{-1}(V)\subseteq\lambda Int(f^{-1}(V))$ and hence $f^{-1}(V)=\lambda Int(f^{-1}(V))$.By Lemma 2.2, $f^{-1}(V)$ is λ -open and by Lemma 5.1,f is almost λ -continuous.

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