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## Soft Mathematical System to Solve Black Box Problem through Development the FARB based on Hyperbolic and Polynomial Functions

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**Abstract**— This work attempts to design a method for computer-based mathematical model of multi-parametric datasets that their features are fully-automatic structural and parametric optimization of models. This method has different inputs and one output as a subset of components for the base function F. In addition, several functions in modeling the classified rules are used including 12 polynomials and 6 hyperbolic functions. This work combines the advantage of data mining algorithms and Fuzzy All-Permutation Rule Base (FARB). In order to evaluate the performance of the proposed method, two classifications based on association rules case studies were used in an attempt to design a dynamic mathematical method. The first case study takes the virtual rules which consist of four rules, while in the second case study actual rules are selected. According to the experimental results, not only enhanced accuracy can be achieved by the proposed method, but also it can be used for intelligent data analysis for huge and small datasets.

**Keywords**- FARB, Hyperbolic Functions, Polynomial Functions, Black Box, Mathematical Model.

### I. INTRODUCTION

Soft Mathematical model is one of the several commonly used methods to construct mathematical equations of real-life situations. It uses a set of variables to describe a system and a set of equations are used to establish relationships between those variables and a set of inequalities. The values of the variables can be real or integer numbers, Boolean or strings values [2]. There are several reasons behind using modelling, it can be considered as a fundamental tool to better understand complex systems. And it is complement to theory, experiments and often integrate between them.

The main goal of mathematical modeling is to provide a better understand of science by using mathematical models on computers. This paper focuses on mathematical analysis and rigorous design methods for decisions of KDD system (classification based on association rules). For the following reasons:

- Mathematical model more flexible than KDD system
- Any mathematical model can be simplified to given accurate results.
- Can combine more one mathematical equation to produce true solutions but, this is impossible among the structures of KDD systems.
- Mathematical model can be composited from many mathematical models.

FARB technique is considered as one of the most widely used statistical tools for extracting the knowledge embedded in a network. It has the ability to combine the features of neural network and fuzzy logic. However, this work uses the FARB to combine the advantage of data mining algorithm and fuzzy logic. The outcome of this combination will lead to perform equivalents among different tools, which include Neural Network, FARB and KDD algorithms as shown in Figure1.

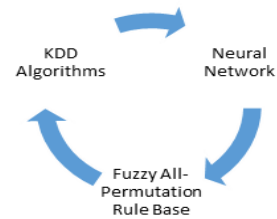


Figure 1. Equivalent among three concept (NN, FARB, KDD).

### II. DESIGN MATHEMATICAL MODEL OF CLASSIFIED RULES

Statistical analysis can be used to develop, represent and explain the relationship between variables in a model. Several types of methods were done for linear and nonlinear regression (function fitting), discriminant analysis, logistic regression, support vector machines, neural networks and decision trees [2]. These proposed methods have its advantages and they were designed for a specific application and purpose, but there is no single method that is suitable for all applications.

This paper aims at providing a method which combines the results of data mining techniques and FARB. When using the proper method to solve a problem, the analyst will be able to match, exceed the predictive ability of any other modeling program not only effectively, but also efficiently.

But, the main theme of this step is the attempt to design mathematical model of classification rules and the main question “Is it possible to convert the black- box data mining algorithm to white-box (clear box) for a given dataset?”

In traditional Fuzzy all permutation rule, the hyper polic function (Tanh) of one variable is used to transfer functions in the modeling [3]. This paper extends FARB by allowing the selection of functions which may be used in the model and various functions in modeling the classified rules were used. These functions are provided in Table I.

Linear Name of Functions	One # Variable	Functions F(Y)=P1+P2*Y1	Sinh Hyperbolic Cosh Functions	One # Variable	Functions F(Y) = $\frac{\exp(Y1) - \exp(-Y1)}{2}$
Polynomial Linear Functions	Two	F(Y)=P1+P2*Y1+P3*Y2	Hyperbolic Cosh Functions	Variable	$F(Y) = \frac{\exp(Y1) - \exp(-Y1)}{2}$
Linear	Three	F(Y)=P1+P2*Y1+P3*Y2+P4*Y3	Tanh	One	$F(Y) = \frac{\exp(Y1) - \exp(-Y1)}{\exp(Y1) + \exp(-Y1)}$
Quadratic	One	F(Y)=P1+P2*Y1+P3*Y1^2	Sinh <sup>-1</sup>	One	$F(Y) = \frac{2}{\exp(Y1) - \exp(-Y1)}$
Quadratic	Two	F(Y)=P1+P2*Y1+P3*Y1^2+P4*Y2+P5*Y2^2+P6*Y1*Y2	Cosh <sup>-1</sup>	One	$F(Y) = \frac{2}{\exp(Y1) + \exp(-Y1)}$
Cubic	One	F(Y)=P1+P2*Y1+P3*Y1^2+P4*Y1^3	Tanh <sup>-1</sup>	One	$F(Y) = \frac{\exp(Y1) + \exp(-Y1)}{\exp(Y1) - \exp(-Y1)}$
Product	Two	F(Y)=P1+P2*Y1*Y2			
Ratio	Two	F(Y)=P1+P2*(Y1/Y2)			
Logistic	One	F(Y)=P1+P2/(1+exp(P3*(Y1-P4)))			
Log	One	F(Y)=P1+P2*Log(Y1+P3)			
Exponential	One	F(Y)=P1+P2*exp(P3*(Y1+P4))			
Asymptotic	One	F(Y)=P1+P2/(Y1+P3)			

III. FUZZY ALL PERMUTATION RULE BASE (FARB)

IV. SOFT MATHEMATICAL ALGORITHM (SMMA) TO

Fuzzy rule bases (FRBs) consist of a set of if-Then rules, it uses natural language to describe the relationship between inputs and outputs to be clearly expressed and easily understood by humans.. The FRBs are successfully applied to express partial or self-contradicting knowledge because of their crucial ability to handle imprecision and ambiguity. In summary, FRBs can provide an efficient means for converting verbally-stated knowledge into understandable forms. On the other side, the FRBs lack of a systematic algorithm to determine the fuzzy sets and the fuzzy operators that are appropriate for a given problem.

- Every input variable xi (t) is characterized by two verbal terms, say, term<sup>1</sup> and term<sup>i+</sup>. These terms are modeled using two membership functions (MFs): u<sup>i</sup>(.) and u<sup>i+</sup>(.).
- The rule-base contains 2<sup>m</sup> fuzzy rules spanning, in their if-part, all the possible verbal assignments of the m input variables
- Then-part of each rule is a combination of these functions. Specification the rule are:

In summary, In the FARB each input variable is characterized by two verbal terms; the terms are modeled by the use of MFs that satisfy the above equation; the rule-base contains exactly 2<sup>m</sup> rules; and the values in the Then-part of the rules are not independent, but rather they are a linear combination of the m+ 1 functions a<sub>0</sub>(t), . . . , a<sub>m</sub>(t)

$R_1 : \text{If } (x_1(t) \text{ is } term_1^-) \& (x_2(t) \text{ is } term_2^-) \& \dots \& (x_m(t) \text{ is } term_m^-)$ Then $f(t) = a_0(t) - a_1(t) - a_2(t) - \dots - a_m(t).$
$R_2 : \text{If } (x_1(t) \text{ is } term_1^+) \& (x_2(t) \text{ is } term_2^+) \& \dots \& (x_m(t) \text{ is } term_m^+)$ Then $f(t) = a_0(t) + a_1(t) - a_2(t) - \dots - a_m(t).$
⋮
$R_{2^m} : \text{If } (x_1(t) \text{ is } term_1^-) \& (x_2(t) \text{ is } term_2^+) \& \dots \& (x_m(t) \text{ is } term_m^+)$ Then $f(t) = a_0(t) + a_1(t) + a_2(t) + \dots + a_m(t).$

HANDEL BLACK BOX PROBLEM

The main steps were used to convert the Classified Association Rule base (CAR) to mathematical model are as follow:

**Input:** Set of Classified association Rules (CAR) of KDD system

**Output:** Dynamic Mathematical Models of CAR

• **Step 1:** determine type of mathematical model base one Table I.

• **Step 2 :** Convert CAR Base to Fuzzy Classified Rule Base (FCRB)

• **Step 3:** Covert each FCRB to Fuzzy all permutation rule base (FACRB) classified

Characterize Every Input Variable by Two Terms (term<sup>+</sup>, term<sup>-</sup>)

Construction 2<sup>m</sup> of Fuzzy Rules

IF-Part Contain all possible verbal assignment of m input variables.

Then-Part is linear combination of m+1 functions a<sub>0</sub>(t),.....a<sub>m</sub>(t)

• **Step 4:** Compute Membership Function for each Terms

**IF MF (term<sup>+</sup>) + MF (term<sup>-</sup>) =1 Then Goto 10 Else**

The term model using Gaussian MF according to eq(1)

$$GMF = \exp(-(y-k)^2 / (2*sd^2)) \dots(1)$$

Set: Sig=(1/ 1+exp(-net)), a=(k1 k2)/2\*sd^2, b=(k1^2-k2^2)/4\*sd^2

Substation a, b in eq (1)

$$GMF = \text{Hyperbolic function } (ay-b) \dots(2)$$

$$GMF = 2*\text{sig}(2ay-2b) \dots(3)$$

Gotostep5

**10** : The term model using one of Hyperbolic functions such as Tanh according to eq(4)

$$TMF=(MF(y)-MF(-y)) / (MF(y)+MF(-y)) \dots(4)$$

$$\text{Set: } MF(-y)=1+MF(y) \dots(5)$$

Substation eq (5) in eq (4)

$$TMF=2*MF(y) -1 \dots(6)$$

**IF Term + = K1 and Term - < K1 Then**

Substation eq (1) in eq(6)

$$MF= 2* \exp(-((y-k)^2)/(2*sd^2))-1 \text{ Goto step5}$$

**IF Term + Small Than K1 and Term + Large Than K1 Then**

Using Logistic Functions: Set Alfa>0,

$$MF(y) \text{ ST } K=(1/1+\exp(-\text{alfa}*(y-k))) \dots (7)$$

$$MF(y) \text{ LT } K=(1/1+\exp(\text{alfa}*(y-k))) \dots (8)$$

Substation eq (7) in eq(6)

$$TMF=2*\text{sig}(\text{alfa}(y-k))-1$$

Substation eq (7) in eq(6)

$$TMF=2*\text{sig}(-\text{alfa}(y-k))-1: \text{ Goto step5}$$

**IF Term + is Positive number and Term - is negative number or posit Then**

Using Spatial MF of Positive value

$$\text{Test if } -\infty < y < \infty \text{ Then } MF_{\text{pos}}(y) = 0$$

$$\text{Test if } -\Delta \leq y-k \leq \Delta \text{ Then } MF_{\text{pos}}(y) = (1+(y/\Delta)/2)$$

$$\text{Test if } \Delta < y < \infty \text{ Then } MF_{\text{pos}}(y) = 1 \dots(9)$$

Base on eq(5) find

$$MF_{\text{Neg}}(y) = MF_{\text{pos}}(y)-1$$

Set:

$$\text{SigL}(y) = 0 \text{ if } -\infty < y < 0$$

$$\text{SigL}(y) = y \text{ if } 0 \leq y \leq 1$$

$$\text{SigL}(y) = 1 \text{ if } 1 < y < \infty$$

Substation eq (9) in eq(6)

$$TMF=2*\text{sigL}(y/(2\Delta)+0.5)-1: \text{ Goto step5}$$

**IF Term + is ST K1 and Term - is LT K1 and Sum of MF > 1 Then**

Using Spatial MF of Positive value

$$\text{Test if } -\infty < y < -\Delta \text{ Then } MF_{\text{pos}}(y) = 0$$

$$\text{Test if } -\Delta \leq y-k \leq \Delta \text{ Then } MF_{\text{pos}}(y) = (1+(y/\Delta)/2)$$

$$\text{Test if } \Delta < y-k < \infty \text{ Then } MF_{\text{pos}}(y) = 1 \dots(10)$$

Substation eq (10) in eq(6)

$$TMF=2*\text{sigL}((y-k)/(2\Delta)+0.5)-1: \text{ Goto step5}$$

• **Step 5:** Find the Center of Gravity (COG) as follow

$$COG=U(y)/D(y) \dots(11)$$

• **Step 6:** IF chose one of the hyperbolic functions in design your mathematical models we need to compute the parameters  $a_0(t), a_1(t), \dots, a_m(t)$  where the values of it find by solving the equations spontaneously as explain below. While, IF chose one of the polynomial functions in design your mathematical models we need to compute the parameters  $p_1, p_2, \dots, p_m$  where the values of it find by solving some of mathematical equation as explain below.

## V. EXPERIMENT RESULTS

To evaluate the performance of the proposed methodology, two classifications based on association rules case studies were used in an attempt to design a dynamic mathematical model. The first case study takes the virtual rules which consist of four rules, while in the second case study actual rules are selected.

### A. Case Study of Virtual Rules

Suppose, we take the above four FRB rules, where each rule consists of two part (smaller or larger than to 1 or 7). Therefore, it is essential to compute the MF for each sub-part, which they are called M1F and M2F in this work and to find the summation of both M3F as provided in Table 2. The center of gravity of the rules based on (eq. (11)) is explained in Table 3

TABLE II. MEMBERSHIP FUNCTION OF THE IF-PART OF RULES

RULES	M1F	M2F	M3F
R1	2.472623156634 77E-03	0.34697152516631 5	0.349444148322 949
R2	0.997527376843 365	6.36971776353819 E-03	1.003897094606 90
R3	2.472623156634 77E-03	5.97702373023027 E-246	2.472623156634 77E-03
R4	0.997527376843 365	1.07504529457153 E-227	0.997527376843 365

TABLE III. COMPUTE U(y), D(y) AND CENTER OF GRAVITY OF THE RULES

RULES	U(y)	D(y)	COG
R1	3.43171931127646 E-03	8.57929827819115 E-04	-4
R2	0	6.35396785189484 E-03	0
R3	2.95578545662458 E-248	1.47789272831229 E-248	2
R4	6.43432267609044 E-227	1.07238711268174 E-227	6

$$\text{Where, } U(y) = \sum_{i=1}^4 \text{value of Then - Part}(i) * M1F(i) * M2F(i), D(y) = \sum_{i=1}^4 M1F(i) * M2F(i)$$

TABLE IV. A: MATHEMATICAL MODELS BY HYPERBOLIC FUNCTIONS

Name of Hyperbolic Functions	Parameters of IF-Part to FARB			MODELS
	a0	a1	a2	
Sinh: One Variable	1	2	3	$F(Y)=1+2 * \text{TANH}((x1-5) * 0.5) + 3 * \text{TANH}(3 * ((x2-4) / (0.2 \wedge 2)))$
Cosh: One Variable	1	2	3	$F(Y)=1+2 \text{Cosh}(((x1-5) * 0.5))/42 + 3 \text{Cosh}(((x2-4) / (0.5 \wedge 1/3)))$
Tanh: One Variable	1	2	3	$F(Y)=1+2 * \text{TANH}((x1-5) * 0.5) + 3 * \text{TANH}(3 * ((x2-4) / (0.2 \wedge 2)))$
Sinh <sup>-1</sup> : One Variable	1	2	3	$F(Y)=1+2 \text{Sinh}^{-1}(0.412987-0.89*x1) + 3 \text{Sinh}^{-1}((x2-0.86) / (0.8 \wedge 2))$
Cosh <sup>-1</sup> : One Variable	1	2	3	$F(Y)=1+2 \text{Cosh}^{-1}((2/(x1-2.4) * 0.35) + 3 \text{Cosh}^{-1}(0.9*x2-0.54)/0.6)$
Tanh <sup>-1</sup> : One Variable	1	2	3	$F(Y)=1+2 \text{Tanh}^{-1}(0.5/(x1-5)) + 3 \text{Tanh}^{-1}(0.2 \wedge 2 / (3(x2-4)))$

TABLE IV. B: MATHEMATICAL MODELS BY POLYNOMIAL FUNCTIONS

Name of Polynomial Function	Parameters of IF-Part to FARB			Estimation Parameters of Functions						MODELS
	a0	a1	a2	P1	P2	P3	P4	P5	P6	
Linear: One Variable	1	2	3	0.666667	0.666667	-	-	-	-	F(Y)= -0.666667+0.666667*Y2
Linear: Two Variables	1	2	3	4.192982	0.824561	0.210526	-	-	-	F(Y) = -4.192982+0.824561*Y2+0.210526*Y1
Linear: Three Variables	1	2	3	-	-	-	-	-	-	can not be use because no of input variables =2
Quadratic: : One Variable	1	2	3	4.547368	1.415038	0.070677	-	-	-	F(Y) = 4.547368-1.415038*Y1+0.070677*Y1^2
Quadratic: Two Variables	1	2	3	-	-	-	-	-	-	can not be use because no of input variables =2
Cubic: One Variable	1	2	3	-	-	-	-	-	-	can not be use because no of input variables =2
Product: Two Variables	1	2	3	0.423484	0.135331	-	-	-	-	S = 0.44236+0.0377*Y2*Y1 F(Y) = 0.423484+0.135331*Y2*S
Ratio: Two Variables	1	2	3	4.523974	1.534968	-	-	-	-	S= 2.913541-0.190107*Y2/Y1 F(Y) = 4.523974-1.534968*S/Y2
Logistic: One Variable	1	2	3	2.666667	3.812506	5.042049	3.653246	--	--	F(Y) = 2.666667+3.812506/(1+exp(5.042049*Y1+3.653246))
Log: One Variable	1	2	3	0.342869	1.903954	0.16668	-	-	-	F(Y) = 0.342869+1.903954*log(Y2-0.16668)
Exponential: One Variable	1	2	3	3.0059	2.604905	0.135894	0.9	-	-	F(Y) = -3.005998+2.604905*

B. Case Study of Actual Rules

In order to validate the proposed method, a DNA dataset was used, where this dataset consists of 2000 samples and 181 features belong to three groups. The Target Variable is DEPVAR and it is divided into three classes as shown in Table 5, The predictor Variables include A63, A72, A73,, A83, A85,A88, A89, A93, A94, A74, A75, A82, A95, A97, A98, A100, and A105. After applying the classification based on the association rules on the DNA dataset, **thirty nine** rules were generated. For more details about how these rules are produced see reference [14]), the center of gravity can be computed by using the same above manner.

TABLE V. DISTRIBUTE CLASS IN DNA DATASET

CLASS	LEARN	%	TOTAL
1	1051	52.55	1051
2	464	23.20	464
3	485	24.25	485
<b>Total:</b>	2000	100.00	2000

TABLE VI. A: MATHEMATICAL MODELS BY HYPERBOLIC FUNCTIONS

Name of Hyperbolic Functions	MODELS
Sinh: One Variable	DEPVAR= 0.9783+1.806818*Sinh ((A93-0.25563)^2/ 0.03))+ Sinh ((A88-0.5)*2.2)+2.39854 * Sinh ( 0.519752-*A85*A83)+ Sinh( 1.204298-2.562323*A100)
Cosh: One Variable	DEPVAR= 0.9783+1.806818* Cosh( 0.051125+0.453941*A85)*Cosh( 2.515478-1.204798*A105*A100)+Cosh( 2.53012-0.919594*A93*A85)
Tanh: One Variable	DEPVAR= 0.9783+1.806818Tanh(2.412987-0.843155*A97*A93)+Tanh(2.611919-1.301574*A105)+ 0.254485*Tanh(0.681822+0.296851*A88)
Sinh <sup>-1</sup> : One Variable	DEPVAR= 0.9783+1.806818Sinh <sup>-1</sup> (2.53012-0.919594*A98*A85)+ Sinh <sup>-1</sup> ( 0.86801 -1.301574*A94)+0.86801 * Sinh <sup>-1</sup> ( - 0.919594*A93)
Cosh <sup>-1</sup> : One Variable	DEPVAR= 0.9783+1.806818*Cosh <sup>-1</sup> (2.53012-0.919594*A98*A85)+Cosh <sup>-1</sup> ( 0.3 *A105)+0.46573 * Cosh <sup>-1</sup> ( 0.48856 *A63-0.782)
Tanh <sup>-1</sup> : One Variable	DEPVAR= 0.9783+1.806818Tanh <sup>-1</sup> (1.0750452*A72-3023027 )+0.97177635 *Tanh <sup>-1</sup> (0.44443*A93+0.00229 ) *A105) - 0.5 48322949*Tanh <sup>-1</sup> (0.633881 *A85+0.63477 E-03*A63)

TABLE VI: B: TABLE IV. B: MATHEMATICAL MODELS BY POLYNOMIAL FUNCTIONS

Name of Polynomials Function	Estimation Parameters of Functions						MODELS
	P1	P2	P3	P4	P5	P6	
Linear: Three Variables	0.041587	-0.039078	-0.097369	0.999288	-	-	DEPVAR = 0.041587-0.039078*A104-0.097369*A97+0.999288*S(5) S(1) = 2.502743+0.490362*A95+0.498123*A94-0.827805*A93 S(2) = 3.077677-0.546067*A100-0.699282*A93-0.529314*A85 S(3) = -2.067273+0.707371*A93+0.820925*S(1)+0.933216*S(2) S(4) = 0.448309-0.418464*A105+0.329223*A88+0.848834*S(3) S(5) = 0.030268+0.149773*A98+0.265582*A89+0.958982*S(4)
Quadratic: One Variable	2.741228	-	+6.104781E+014	-	-	-	DEPVAR = 2.741228 -6.104781e+014*A93+6.104781e+014*A93^2
Quadratic: Two Variables	3.003096	9.979959E+012	9.979959E+012	2.178707e+014	2.178707e+014	+0.118952	DEPVAR = 3.003096+9.979959e+012*A93-9.979959e+012*A93^2-2.178707e+014*A85+2.178707e+014*A85^2+0.118952*A93*A85
Cubic: One Variable	2.24834	2.967218E+014	4.444339E+030	4.444339e+030	-	-	DEPVAR = 2.24834-2.967218e+014*A63+4.444339e+030*A63^2-4.444339e+030*A63^3
Logistic: One Variable	1.608937	1.673063	2.768504	0.26693	-	-	DEPVAR = 1.608937+1.673063/(1+exp(2.768504*(A93-0.26693)))
Log: One Variable	1.814658	-0.288681	0.040351	-	-	-	DEPVAR = 1.814658-0.288681*log(A93+0.040351)
Exponential: One Variable	-0.848695	+0.956229	0.493471	-1.905544	-	-	DEPVAR = -0.848695+0.956229*exp(0.493471*(S(1)-1.905544)) S(1) = 0.6732+1.900905*exp(-0.355391*(A105+0.1))

TABLE VII. ANALYSIS OF POLYNOMIAL MODELS RESULTED BY PROPOSED SYSTEM FOR DNA DATABASE

Polynomial Functions	Maximum error	RMSE	MSE	MAE	MAPE
Linear: one variable	1,03704	0,7050831	0,4971422	0,5133745	35,202332
Linear: two variables	1,22542	0,6717494	0,4512473	0,5186096	35,345826
Linear: three variables	1,27693	0,6125859	0,3752615	0,4889608	32,638685
Quadratic: one variable	385,00000	64,170996	4117,9167	11,25	393,51852
Quadratic: two variables	2,5961E+33	4,33E+32	1,87E+65	7,21E+31	3,61E+33
Cubic: one variable	1,00000	7,64E-01	5,83E-01	5,83E-01	37,962963
Product: two variables	1,20650	0,6976609	0,4867307	0,507345	34,331393
Logistic: one variable	1,03704	0,7050831	0,4971422	0,5133744	35,202332
Log: one variable	1,03704	0,7050832	0,4971423	0,513374	35,202337
Exponential: one variable	1,09897	0,7062308	0,4987619	0,5438002	35,49973
Asymptotic: one variable	1,03704	0,7050831	0,4971422	0,5133745	35,202332

The results of the best model of that dataset by using the polynomial functions is linear of three variables is provided in Table 7, while the worst model is quadratic of two variables Figure. 2 shows the analysis of polynomial models resulted from the proposed method based on predicate error models of the DNA dataset.

After finding the best model using the polynomial functions and verified their correctness, the model can be then analysed based on the hyperbolic functions as explain in Table 8 which compute the membership function of that rules.

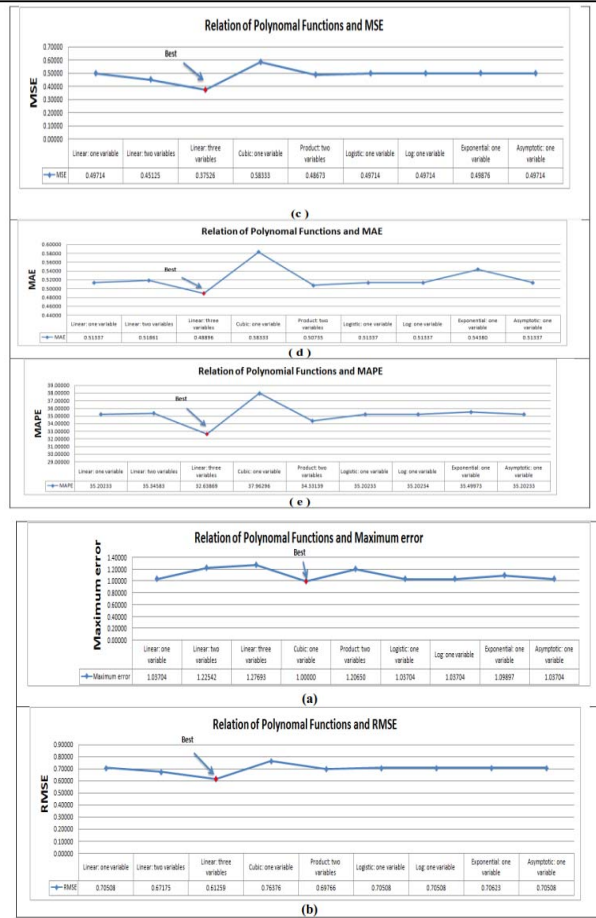


Figure 2. Analysis of Polynomial Models resulted from the proposed System based on Predicate Error Models of DNA Database. (a) Relation of Polynomial Functions and Maximum error, (b) Relation of Polynomial

Functions and RMSE, (c) Relation of Polynomial Functions and MSE, (d) Relation of Polynomial Functions and MAE and (e) Relation of Polynomial Functions

**Step3: Compute the Center of Gravity**

TABLE IV: COMPUTE U(Y), D(Y) AND CENTER OF GRAVITY OF THE RULES

R1	8,5073532374867E-07	2,83578444124955E-07	3
R2	7,29444120074296E-08	3,64722060037148E-08	2
R3	2,50705766658860E-08	2,50705766658860E-08	1
R4	1,27031181419033E-07	4,23437271396777E-08	3
R5	7,76994840931832E-05	3,88497420465916E-05	2
R6	2,66138117805355E-26	8,87127059351183E-27	3
.....			
.....			
R37	3,27148237803129E-09	1,09049412601043E-09	3
R38	2,88574722808446E-03	9,61915742694821E-04	3
R39	1,26839170263147E-02	4,22797234210489E-03	3

**Step4: Generate the hyperbolic mathematical models**

TABLE X: MATHEMATICAL MODELS BY HYPERBOLIC FUNCTIONS

Hyp. Functions	Mathematical Models
Sinh: One Variable	Class= a0+a1*Sinh(0.017885*(-((A88-0.67977)^2)/0.710772))+ a2*Sinh(A85*206526)+ a3*Sinh(A93+0.897432)+a4* 0.5* Sinh(A105-0.745341) +a5*Sinh(0.34591*A100-14)+a6* Sinh(A88*0.5)/(0.63748*A97)+a7* Sinh(A72)+ a8* 0.2*Sinh(A98+0.4392)+a9*0.3*Sinh(A82-0.9386)+a10*Sinh(A89*2)+a11*(-0.3572*A63)/Sinh(0.2*A83)+a12*Sinh(A95+0.4518)/0.2842)+a15* Sinh(A94+0.5)
Cosh: One Variable	Class= a0+a1*(0.4+1.806818*Cosh(2.412987-0.843155*A97))+a2* Cosh(A93)+a3*Cosh(2.611919-1.301574*A105)+a4*0.254485* Cosh(0.681822+0.296851* A88)+a5* Cosh(3.28238*A85)+a6*0.3*Cosh(A83*0.5)+a7*Cosh(0.9*A63)+ a8*0.7854 Cosh(A89*0.2)+a9* Cosh(A72+0.4)+a10* Cosh(A94/0.8)+a11* Cosh(A82*0.3)+a12*0.9* Cosh((A95+3.98)*2))+a15* Cosh(0.6*A100)
Tanh: One Variable	Class= a0+a1*Tanh(3.013885*((A88+0.63933)/0.310330))+ a2*Tanh(A85*006506)+ a3*Tanh(A93+0.893430)+ a4*0.5* Tanh(A105+ 0.345341)+ a5* Tanh(0.34591*A100+14)+ a6*Tanh(A88*0.5)/(0.63348*A93)+ a7* Tanh(A30)+ a8*0.8* Tanh(A98+0.4390)+a9*(0.3/Tanh(A80+0.9386)))+a10*Tanh(A89*4)+a11*(+0.3530*A63)/Tanh(0.9*A83)+a12*Tanh((A95+0.4518)/0.0840)+ a15* Tanh((A94+0.5)/(0.4+A74))
Sinh <sup>-1</sup> : One Variable	Class= a0+a1*Sinh <sup>-1</sup> (-2.6502*A85)+a2*0.5* Sinh <sup>-1</sup> (A93-2.6502)+ a3*Sinh <sup>-1</sup> (A105/0.2)*A95+ a4*Sinh <sup>-1</sup> ((A100*0.87316)+0.281)+a5* Sinh <sup>-1</sup> (0.363*A88-0.257)*0.801+ a6*Sinh <sup>-1</sup> (0.546+0.928/A97)+ a7*Sinh <sup>-1</sup> (A72)+a8*0.8* Sinh <sup>-1</sup> (0.847*A98)+a9* Sinh <sup>-1</sup> (A82/0.2)+a10*0.4* Sinh <sup>-1</sup> (A89+0.32)+a11*Sinh <sup>-1</sup> ((A63*0.2)-0.8)+a12*Sinh <sup>-1</sup> (A95-0.3)+a15* Sinh <sup>-1</sup> ((A74*0.3)+0.5239)
Cosh <sup>-1</sup> : One Variable	Class= a0+a1*0.8310* Cosh <sup>-1</sup> (0.82645*A88)+ a2*Cosh <sup>-1</sup> (-0.3*A98)+a3* 0.46573*Cosh <sup>-1</sup> (0.4*A89-0.7)+a4* Cosh <sup>-1</sup> (0.7*A100+0.5)+ a5*Cosh <sup>-1</sup> (-0.2*A105))+a6* Cosh <sup>-1</sup> (0.352*A82)+ a7*Cosh <sup>-1</sup> ((A74/0.7)+ 0.27938)+a8* Cosh <sup>-1</sup> (A83*0.43)+a9* Cosh <sup>-1</sup> ((A95*0.3)-0.392)+a10* Cosh <sup>-1</sup> ((A94*0.00281)-20.77819)+ a11* Cosh <sup>-1</sup> (A93/0.3829)+a12* Cosh <sup>-1</sup> ((A72*0.3)-0.5)+A15* Cosh <sup>-1</sup> (A85*0.2)
Tanh <sup>-1</sup> : One Variable	Class= a0+a1* Tanh <sup>-1</sup> (-((A85-0.333483)/0.359)-0.4862)+a2* Tanh <sup>-1</sup> (-((A93-0.483)/0.3596)-0.4862)+ a3*0.5326* Tanh <sup>-1</sup> (-((A105-0.267692)+ a4*Tanh <sup>-1</sup> (-((A100-0.333483)/2)+0.42029))+ a5*Tanh <sup>-1</sup> (A88*0.3)+a6* Tanh <sup>-1</sup> (A97-0.5)+ a7* Tanh <sup>-1</sup> (A72)+ a8* Tanh <sup>-1</sup> (0.32*A98)+a9* Tanh <sup>-1</sup> (A82+0.28*0.5)+a10* Tanh <sup>-1</sup> (A89)+ a11* Tanh <sup>-1</sup> (0.2*(A63-0.3256))+a12*0.8* Tanh <sup>-1</sup> (A95)+a15* Tanh <sup>-1</sup> (4.83-A94)

TABLE XI ANALYSIS OF HYPERBOLIC MODELS RESULTED FOR DNA DATABASE

Hyperbolic Functions	Maximum error	RMSE	MSE	MAE	MAPE
Sinh: One Variable	1.5838329	1.0298364	0.6939226	0.5865136	44.38253
Cosh: One Variable	1.4637213	0.9287354	0.6009831	0.7263500	37.97845
Tanh: One Variable	<b>1.0384229</b>	<b>0.705042</b>	<b>0.4970842</b>	<b>0.5139815</b>	<b>35.206526</b>
Sinh <sup>-1</sup> : One Variable	1.3821754	0.8090813	0.5294630	0.6399816	39.99182
Cosh <sup>-1</sup> : One Variable	1.5372091	0.9520185	0.8372625	0.6725341	41.66715
Tanh <sup>-1</sup> : One Variable	1.6536910	1.3001711	0.9204526	0.7625348	43.29281

TABLE VIII. Membership Function of the IF-Part of Rules

RULES	M1F	M2F	M3F
R1	2,51663153643448E-03	1,12681749401712E-04	2,62931328583619E-03
R2	7,38629132354838E-04	4,93782392354835E-05	7,88007371590322E-04
R3	1,46606335315023E-02	1,71006093372535E-06	1,46623435924360E-02
R4	3,31573645219465E-05	1,27705346158169E-03	1,31021082610364E-03
R5	6,15816912727624E-03	6,30865136108642E-03	1,24668204883627E-02
R6	1,27276246368792E-13	6,97009131445210E-14	1,96977159513313E-13
.....			
.....			
R37	2,35483875940171E-05	4,63086536883523E-05	6,98570412823694E-05
R38	3,05042383901418E-01	3,15338390158165E-03	3,08195767803000E-01
R39	8,83921768177557E-01	4,78319744384392E-03	8,88704965621401E-01

The Tanh of one variable is the best model of that dataset using hyperbolic functions as shown in Table 11, while the worst model is Inver of Tanh of one variable.

**VI. CONCLUSIONS**

The main assumption of this paper is to design dynamic mathematical models from the results of classification based on association rules. These models are based on the type of model that user's needs.

The proposed method aims at combining the advantage of data mining algorithms and FARB. To evaluate the performance of the proposed methodology, two classifications based on association rules case studies were used in an attempt to design a dynamic mathematical model. The first case study takes the virtual rules which consist of four rules, while in the second case study actual rules are selected.

According to the experimental results, not only enhanced accuracy can be achieved by the proposed method, but also it can be used for intelligent data analysis for huge and small datasets.

According to experimental results, this combination in designing of dynamic mathematical models not only leads to increase the accuracy in results, but also it can be used for intelligent data analysis for huge and small datasets.

In the designing phase of the mathematical models, the proposed method can be considered as Meta Knowledge (i.e., knowledge about knowledge) system. This means extraction of new knowledge (i.e., mathematical models) from the original knowledge (i.e., classified association rules).

Huge datasets have a fixed behavior, the best model is generated by linear of three variables function and the worst model is generated by cubic of one variable or quadratic function related to polynomial models. In addition, the best model of their generated by Tanh of one variable function and the worst model of their generated by more than one of other functions related to hyperbolic models

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