

# A modified technique to evaluate the reliability of Atomium Bridge

Zahir Abdul Haddi Hassan  
 Department of Mathematics,  
 College of Education for pure  
 Sciences, University of Babylon  
 Babylon, Iraq  
 mathzahir@gmail.com

Ahmed A. Ouda  
 Mechanical Eng. Dept.  
 Engineering College/ University of  
 Thi-Qar, Iraq  
 Ouda1978@gmail.com

Abeer A. Abdul-Razaq  
 mathematics department  
 University of Babylon  
 Iraq  
 44abeer60@gmail.com

**Abstract-**This paper provides a new innovation in order to evaluate the reliability of Atomium Bridge, which represent a network with minimal/minimum cycle paths between entry/exit node, as well as computer programming has been used to a create an algorithm which calculates the reliability of this bridge.

**Key Words:** Reliability; minimal path; minimal cycle path; reliability network; adjacency matrix, Atomium Bridge.

## I. INTRODUCTION

Network reliability analysis receives nice attention for the design, effectiveness, and protection of the many real word systems, like computers, communications, electrical circuits, spacecraft system, nuclear reactor system or power networks [1].

The Atomium is that the most well-liked attraction in Belgian capital. The building was created by Andre Waterkeyn, Andre and Jean Polak. it had been created in 1958 for the globe Exhibition and was sculptural on the atomic structure of an iron crystal however on a scale of 1: a hundred sixty-five billion. A visit to the Atomium involves exploring the extraordinary building's tube walkways and spheres in addition as seeing the permanent exhibition that covers the history of the building. Temporary exhibitions on a spread of scientific and style themes are command often. The structure is 102 meters high and like a true atom it consists of "lines" or tubes connecting spheres. The tubes square measure every twenty-three to twenty-nine meters long. every of the nine spheres has been named in honor of a known human or Nobel prize winner and half-dozen square measure hospitable the general public. The spheres live eighteen meters in diameter. In 2006 renovations were created to the structure recreating the initial lighting and adding a tent at the bottom of the building with stores and a business office. The most sphere homes the permanent exhibition that covers the history of the Atomium.

Evaluating the network reliability is an important topic in the planning, designing, and control of the system. The network reliability theory has been applied extensively in many real-word systems such as computer and communication systems, transportation systems,...., etc. [2].

Most designs have two or more than one terminal, in the Atomium design have one terminal (source is the same sink). Most ways to solve the reliability system are

suitable for models of two or more than on terminal that have not cycle paths, like minimal path, minimal cut method, path tracing method,...., etc. [3], so in this paper we devised technique to find cycle minimal paths, and use it to find the reliability network of Atomium bridge.



Fig 1. Atomium Bridge

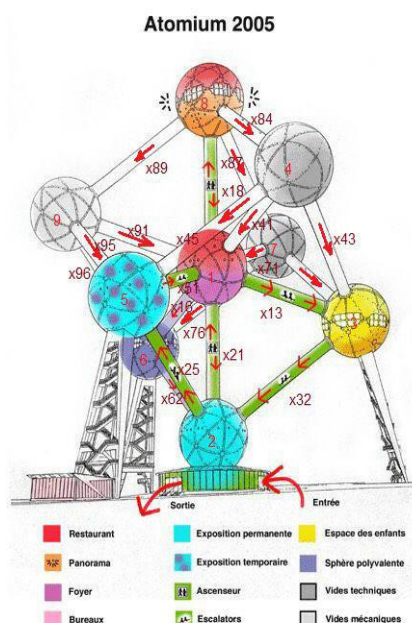


Fig.2 Graph of Atomium Bridge

## II. BASIC CONCEPTS

A. A path for network is as set of components such that if all components in the set are successful, the system will be successful.

B. Minimal path is a set of components that comprise a path, but the removal of any component will cause the resulting set to not be path.

C. Adjacency matrix , let  $G=\{V,E\}$  be a graph with  $V=\{v_1,v_2,\dots,v_n\}$ , in Atomium bridge the number of  $V$ (nodes) is nine and the number of edges is twenty. The adjacency matrix ,  $A=(a_{ij})$ , is the  $n$ -by- $n$  matrix whose rows and columns are indexed by the elements of  $V$ . The element  $a_{ij}$  is defined by

$$a_{ij} = \begin{cases} k & \text{if } v_i \text{ are joined by } k \text{ common edges} \\ 0 & \text{if else} \end{cases}$$

D. Acycle path in a graph is a non-empty trail in which the only repeated vertices are the first and last vertices.

E. Minimal(short)cycle path A cycle  $C$  is called a minimal(short)cycle if for every pair of nodes  $u$  and  $v$  it contains a minimal(shortest) path from  $u$  to  $v$  or minimal(shortest) path from  $v$  to  $u$  in graph  $G$  is contained in  $C$

## III. GENERATION MINIMAL PATH SETS

A Minimal path method is suitable way to find all paths of a graph of two-terminal, in Atomium there is one terminal, so we developed the minimal path method, and find all minimal path of Atomium.

As we will explain it,

In two-terminal network, the edges are prone to failure and the nodes are perfect. In this method will construct connection matrix to create minimal path. We will combine  $n \times n$  adjacency matrix of a simple graph with the identity matrix as the following

$$CM = \begin{matrix} & & \text{To node} \\ & & 1 & 2 & \dots & n \\ \text{From node} & \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ a_{21} & 0 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{2n} & \dots & 0 \end{bmatrix} \end{matrix}$$

$$+ \begin{matrix} & & \text{To node} \\ & & 1 & 2 & \dots & n \\ \text{From node} & \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \end{matrix}$$

After the combination results

$$CM = \begin{matrix} & & \text{To node} \\ & & 1 & 2 & \dots & n \\ \text{From node} & \begin{matrix} 1 \\ 2 \\ \vdots \\ n \end{matrix} & \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{21} & 1 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{2n} & \dots & 1 \end{bmatrix} \end{matrix} \quad (1)$$

Where  $\{1,2,\dots,n\}$ , the set of nodes/sphere and  $a_{ij}=(i,j)$  the edge between node  $i$  and  $j$ , if there is a connection between node  $i$  and node  $j$  then  $a_{ij}=x_{ij}$ ,

otherwise  $a_{ij}=0$ , therefore removes nodes that are neither the source nor the sink from CM, one by one until the only nodes left in matrix are the source node and the sink node[4], [5].

When you remove a node, the entries of the connection matrix with the remaining nodes are modified using the equation:  $a_{ij}^1 = a_{ij} + a_{il}a_{lj}$  (2)

If node  $l$  is removed, where  $i \neq j$ ,  $i \neq l$ ,  $j \neq l$ ,  $1 \leq i < n$ ,  $1 < j \leq n$  for  $i=1,2,\dots,n$ .

It is better in this way make the source node as the first node and the sink node as the last node. Through this method is glorified all minimal path of two terminal systems. **The question here is whether technique is useful for finding minimal path sets for one-terminal network?**

The answer is no there is at yet.

So we found a way to take advantage of the adjacency matrix by add the first column that contains the source node after the last column of the matrix, and add the row that contains the source node, before the first row, in order to ensure the creation of matrix to be  $(n+1) \times (n+1)$ . By this way we can find all minimal path of Atomium bridge.

In order to complete this task, we have designed a program to find the minimal path sets as well as calculate the reliability of this bridge, because this processes need more time, effort and accuracy. So the CM matrix of Atomium is

$$CM = \begin{matrix} & & \text{Node} \\ & & 2 & 1 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \text{node} & \begin{matrix} 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix} & \begin{bmatrix} 1 & X_{21} & 0 & 0 & X_{25} & 0 & 0 & 0 & 0 \\ X_{12} & 1 & X_{13} & 0 & 0 & X_{16} & 0 & X_{18} & 0 \\ X_{13} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_{41} & X_{43} & 1 & X_{45} & 0 & 0 & 0 & 0 \\ 0 & X_{51} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ X_{62} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & X_{71} & X_{73} & 0 & 0 & X_{76} & 1 & 0 & 0 \\ 0 & X_{81} & 0 & X_{84} & 0 & 0 & X_{87} & 1 & X_{89} \\ 0 & X_{91} & 0 & 0 & X_{95} & X_{96} & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$CM = \begin{matrix} & & \text{Node} \\ & & 2 & 1 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 2 \\ \text{node} & \begin{matrix} 2 \\ 1 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 2 \end{matrix} & \begin{bmatrix} 1 & X_{21} & 0 & 0 & X_{25} & 0 & 0 & 0 & 0 & 0 & 1 \\ X_{12} & 1 & X_{13} & 0 & 0 & X_{16} & 0 & X_{18} & 0 & X_{12} & 0 \\ X_{13} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{13} \\ 0 & X_{41} & X_{43} & 1 & X_{45} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X_{51} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{62} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & X_{62} \\ 0 & X_{71} & X_{73} & 0 & 0 & X_{76} & 1 & 0 & 0 & 0 & 0 \\ 0 & X_{81} & 0 & X_{84} & 0 & 0 & X_{87} & 1 & X_{89} & 0 & 0 \\ 0 & X_{91} & 0 & 0 & X_{95} & X_{96} & 0 & 0 & 1 & 0 & 0 \\ 1 & X_{21} & 0 & 0 & X_{25} & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

So the minimal paths of the bridge are  $\{\{X_{13}X_{21}X_{32}\}, \{X_{12}X_{25}X_{51}\}, \{X_{16}X_{21}X_{62}\}, \{X_{13}X_{25}X_{32}X_{51}\}, \{X_{16}X_{25}X_{51}X_{62}\}, \{X_{18}X_{21}X_{32}X_{43}X_{84}\}, \{X_{18}X_{21}X_{32}X_{73}X_{87}\}, \{X_{18}X_{21}X_{62}X_{76}X_{87}\}, \{X_{18}X_{21}X_{62}X_{89}X_{96}\}, \{X_{18}X_{25}X_{32}X_{43}X_{51}X_{84}\}, \{X_{18}X_{25}X_{32}X_{51}X_{73}X_{87}\}, \{X_{18}X_{25}X_{51}X_{62}X_{76}X_{87}\}, \{X_{18}X_{25}X_{51}X_{62}X_{89}X_{96}\}\}$ .

### III. EVALUATION THE RELIABILITY OF ATOMIUM BRIDGE

In this technique of minimal path, the system is connected as series-parallel [6],[7] and the reliability of the system calculated by

$$R_S = \prod_{k=1}^m (1 - \prod_{i=1}^{n_k} (1 - R_{ik})) \quad (3)$$

Where k is number of series subsystems,  $R_{ik}$  is the number of parallel components in each series subsystem and  $n_k$  is the number of parallel subsystems. [8],[9],[10].

By using equation(3) we obtain a very big polynomial which represents the reliability polynomial of Atomium bridge which consisting of (270) terms, as the following

$$R_S = R_{13} R_{21} R_{32} + R_{12} R_{25} R_{51} + R_{16} R_{21} R_{62} + R_{13} R_{25} R_{32} R_{51} + R_{16} R_{25} R_{51} R_{62} R_{73} R_{76} R_{87} + \dots + R_{13} R_{16} R_{18} R_{21} R_{25} R_{32} R_{43} R_{51} R_{62} R_{73} R_{76} R_{84} R_{87} R_{89} R_{96}. \quad (6)$$

**Algorithm** minimal-path is

**input:** Connection Matrix (CM)

**output :** Sumation of all paths (Sum path)

**for** i=1 to size(CM)-1do

    step 1: change the value of remain element of CM according to equation (2)

$$a_{ij}^1 = a_{ij} + a_{il} a_{lj}$$

    If node l is removed, where  $i \neq j$ ,  $i \neq 1$ ,  $j \neq 1$ ,  $1 \leq i < n$ ,  $1 < j \leq n$  for  $i=1, 2, \dots, n$ .

    Step 2: eliminate the second node by deleting the second row and column of CM.

$$CM(i=1:size(CM)-1, j=1:size(CM)-1) = CM(i \neq 2, j \neq 2)$$

**Return** the eliminated CM which equal to Sum path

**algorithm** to calculate Reliability is

**input:** all paths (P)

**output:** Reliability( $R_S$ )

$$R_S = 1;$$

**for each path ( $P_i$ ) do**

    step 1:  $Q_S = \text{expand}(R_S * (1 - P_i))$

    Step 2: **for each edge in ( $P_i$ )**

**do**

**Replace all square**

**Step 3: set the new value of  $R_S$**

$$R_S = 1 - Q_S$$

**Return  $R_S$**

### IV. CONCLUSIONS

This paper presented a new idea for finding / evaluating the reliability of networks that are of the type one terminal network (i.e., the source node is a sink node itself.) as represented by Atomium bridge. Path tracing method has been suggested in order to complete this task, by finding all minimal cycle paths. This technique can be applied for all networks of the type one and two terminal. As well as computer programming has been used in order to find the reliability of the given network.

### ACKNOWLEDGEMENT

I extend my thanks and appreciation to Mr Jasim fahad rahaim from Belgium to refund information about Atomium bridge.

### REFERENCES

- [1] Z.A.H. Hassan and E.K. Muter, Geometry of reliability models of electrical system used inside spacecraft," 2017 Second AI -Sadiq International Conference on Multidisciplinary in IT and Communication Science and Applications(AIC-MITCSA),Baghdad, Iraq, 2017, pp.301-306.
- [2] P.C.Kainen,Cycle construction and geodesic cycles with application to the hyper cube,2014.
- [3] Z. A. H.Hassan, and V.Balan, Fuzzy T-map estimates of complex circuit reliability, International Conference on Current Research in Computer Science and Information Technology (ICIT-2017), IEEE, Special issue, pp.136-139,
- [4] M. Javanbarg, C. Scawthorn and J. Kiyono , Minimal path sets seismic reliability evaluation of lifeline networks with link and node failures,TCLEE 2009, Lifeline Earthquake Engineering in aMultihazard Environment,vol.21.2(2009),pp.193-205
- [5] J.L. Marichal, Structure functions and minimal path sets, IEEE Transactions on Reliability, Vol. 65.2(2016),pp.763-768
- [6] Z. A. H.H.Hassan, and M. A. K.Shiker, Using of Generalized Bayes' Theorem to Evaluate the Reliability of Aircraft Systems, Journal of Engineering and Applied Sciences, 13: 10797-10801, (2018).
- [7] R.S. Douglas, Network Reliability and Algebraic Structures, Oxford Science Publications,(1991)
- [8] B. S. Dhillon, Design reliability fundamentals and applications, CRC Press LLC,Canada,1999
- [9] M. Beccuti, A. Bobbio, A new symbolic approach for network reliability analysis, IEEE 978-1-4673-1625-5/\$ 31.00,2012
- [10] K. Aggarwal, Reliability engineering Netherlands, Springer Science and Business Media,( 2012)
- [11] N.K. Goyal , On Some Aspects of Reliability Analysis and Design of Communication Networks, Indian Institute of Technology Kharagpur, India,( 2006).

[12] M.L. Mutar, B.M. Aboobaidar, A.S.Hameed., Rev Vehicle Routing Problem and Future Research Trend. International Journal of Applied Engineering Research ISSN 0973-4562 Volume 12, Number 22. pp. 12279-12283. 2017.

[13] A.S.Hameed , B.M.Aboobaidar, M.L.Mutar, N.H.choon, W.H.Bilal, A comparative study between the branch and cut algorithm and ant colony algorithm to solve the electric meter reader problem in rural areas. Opcion. Volume 34, Issue 86. pp1525-1539. 2018.

[14] R.Diestel, Graph theory, springer, Berlin, 2005.