Int. Journal of Math. Analysis, Vol. 8, 2014, no. 23, 1105 - 1117
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ijma.2014.45133

Compactness with "Gem-Set"

Luay A. AL-Swidi and Maryam A. AL-Ethary

Mathematics Department
College of Education for Pure Sciences University of Babylon
Babylon, Iraq

Abstract

This paper introduced a new compactness space called Gem-compact space , point-compact space and Prefect-compact space under the idea of "Gem-Set" in topological spaces and study some of their properties and relations among them with basic compact space.

Keywords: Gem-compact space , point-compact space and Prefect-compact space

1. Introduction

The concept of compactness modulo of ideal was first defined by Newcomb 1967 [11], also investigated by Rancin 1972 [12] , an extensively studied by Hamlett and Jankovic in 1990 [10] and [2] . Furthermore , Hamlett et al. [12], have also studied the concept under the term accountably I-compact . Abd El-Monsef et al . [8] used semi-open sets to define the class of δI-compact spaces and countably δI-compact spaces . Arafa A. Nasef [1] introduced γI-compact space and countably γI-compact space by using γ-open sets . Rodyna A . Hosny [9] studied some types of compactness modulo an ideal called βI-compact spaces and countably βI-compact spaces . Dontchev [4] introduced the notions of contra-continuity , a function $f : X \to Y$ to be contra-continuous if the inverse image of every open set of Y is closed in X . Recall that a door space [5] is a topological space in which every subset is either open or closed . Attention : There are some mistakes appear in our first article [7] . The mistakes are (Theorem (3.11) and theorem (3.13)) and the true of this mistakes are (Every $I^* - map$ function is $A - map$. And Every $I^* - map$ function is $AO - map$ if the function is bijective . respective) . This current article is our second one . The aim of the present paper is to introduce and study some types of compactness modulo an ideal under the idea of "Gem-Set" , called Gem-compact , point-compact and Prefect-compact space.

2. Preliminaries

Recall that the "Gem-set" [6] . Let (X , τ) be a topological space , $A \subseteq X , x \in X$, we define A^{*x} with respect to space (X , τ) as follows : $A^{*x} = \{ y \in X : G \cap A \notin I_x , for\ every\ G \in T(y)\}\ where\ T(y) = \{ G \in T : y \in G \}$, A set A^{*x} is called '' Gem-Set " . An ideal for a one point and denoted by I_x [4] , the I_x is an ideal on a topological space (X , τ) at point x is defined by $I_x = \{U \subseteq X : x \in U^c\}$, where U is a non-empty subset of X .

2.1 Definition [3] : A space X is said to be compact if every open covering ϑ of X contains a finite subcollection that also covers X .

2.2 Definition [6]: Let (X , τ) be a topological space , $A \subseteq X$, We define ${}^{*x}pr(A) = A^{*x} \cup A$, for each $x \in X$.

2.3 Definition [6]: A subset A of a topological space (X , τ) is called prefected set if $A^{*x} \subseteq A$, for each $x \in X$.

2.4 Definition [6]: A topological space (X , τ) is called $I^* - T_2$- space if and only if for each pair of distinct points x, y of X, there exist non-empty subsets A , B of X such that $A^{*x} \cap B^{*y} = \emptyset, with\ y \notin A^{*x}\ and\ x \notin B^{*y}$.

2.5 Definition [6] : A mapping $f : (X , \tau) \longrightarrow (Y , \rho)$ is called I^*- map if and only if , for every subset A of X , $x \in X$, then $f(A^{*x}) = (f(A))^{*f(x)}$.

2.6 Definition [6]: A mapping $f : (X , \tau) \longrightarrow (Y , \rho)$ is called I^{**}- map if and only if , for every subset A of Y , $y \in Y$, then $f^{-1}(A^{*y}) = (f^{-1}(A))^{*f^{-1}(y)}$.

2.7 Definition [6]: Let (X,T) be a topological space ,for each $x \in X$, a non-empty subset A of X is called a strongly set if and only if (A^{*x} is an open set and $x \in A$) .

2.8 Definition [6] : A topological space is said to be a strongly space (briefly s-space)if and only if , for each non-empty subset A of X is a strongly set .

2.9 Definition [7] : A mapping $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be :

- f is $A-map$ at $x \in X$ iff $\forall B \subseteq Y$ $,\exists A \subseteq X$ $s.t$ $f(A^{*x}) \subseteq B^{*f(x)}$.
- f is $AO-map$ $iff \forall A \subseteq X$ $\exists B \subseteq Y$ $s.t$ $B^{*y} \subseteq f[A^{*f^{-1}(y)}]$.

2.10 Theorem [3] : Every closed subset of a compact space is compact .

2.11 Theorem [6]: For a topological space (X,τ), then the following properties are held :

1- Every T_2- space is a $I^* - T_2$- space.

2- Every T_2- space is a $I^{**} - T_2$- space.

2.12 Theorem [7] : If $f:(X,\tau) \longrightarrow (Y,\sigma)$ is $AO-map$, open. and bijective map then f is $I^{**} - map$.

2.13 Theorem [7] : A bijective $A-map$ function f is $I^* - map$ if f is a continuous function .

2.14 Theorem [3] : Every compact subspace of Hausfdorff space is closed .

3. The new compact space and it's properties

In this section , a new compact space is introduced under the idea of "Gem-Set" , namely , Gem-compact , Point-compact , Prefect-compact and Strongly Prefect-compact space , besides , their properties as well as their relationships with compact space are studied .

3.1 Definition : A topological space (X,τ) is said to be :

1. **Gem-compact** space iff for each a non-empty subset A of X with $X = \bigcup_{x \in X} A^{*x}$ $\exists$ a finite element $x_1, x_2, \dots x_n$ in X such that $X = \bigcup_{i=1}^{n} A^{*x_i}$.

And **Gem-compact subset** space iff for each a non-empty subset A of X with $A \subseteq \bigcup_{x \in X} C^{*x}$ $\exists$ a finite element $x_1, x_2, \dots x_n$ in X such that $A \subseteq \bigcup_{i=1}^{n} C^{*x_i}$.

2. **point-compact** space iff for $x \in X$ and each collection $\varsigma = \{A_\lambda \subseteq X ; \lambda \in \Omega\}$, with $X = \bigcup_{\lambda \in \Omega} A_\lambda^{*x}$ $\exists$ a finite subcollection in X such that $X = \bigcup_{i=1}^{n} A_i^{*x}$.

And **point-compact subset** iff for $x \in X$ and each collection of subsets D of X , $\varsigma = \{D_\lambda \subseteq X ; \lambda \in \Omega\}$ such that $A \subseteq \bigcup_{\lambda \in \Omega} D_\lambda^{*x}$ $\exists$ a finite subcollection in X such that $A \subseteq \bigcup_{i=1}^{n} D_i^{*x}$.

3. **Prefect-compact** space iff for $x \in X$ and each open cover $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ with $X = \bigcup_{\lambda \in \Omega} A_\lambda$ $\exists$ a finite subcollection $A_1, A_2, \dots, A_n$ in X such that $X = \bigcup_{i=1}^{n} {}^{*x}pr(A_i)$.

And **Prefect-compact subset** space iff for $x \in X$ and each open cover $\varsigma = \{D_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ with $A \subseteq \bigcup_{\lambda \in \Omega} D_\lambda$ $\exists$ a finite subcollection $D_1, D_2, \dots, D_n$ in X such that $A \subseteq \bigcup_{i=1}^{n} {}^{*x}pr(D_i)$.

4. **Strongly Prefect-compact** space iff for $x \in X$ and each collection $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ with $X = \bigcup_{\lambda \in \Omega} A_\lambda$ $\exists$ a finite subcollection $A_1, A_2, \dots, A_n$ in X such that $X = \bigcup_{i=1}^{n} {}^{*x}pr(A_i)$.

And **Strongly Prefect-compact subset** space iff for $x \in X$ and each collection $\varsigma = \{D_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ with $A \subseteq \bigcup_{\lambda \in \Omega} D_\lambda$ $\exists$ a finite subcollection $D_1, D_2, \dots, D_n$ in X such that $A \subseteq \bigcup_{i=1}^{n} {}^{*x}pr(D_i)$.

3.2 Theorem : If (X, τ) is a compact and strongly space then (X, τ) is a Gem-compact space [point compact space] .

Proof :- Straight from definition of strongly space and compact space .

3.3 Theorem : If X is prefected space then X is compact space if and only if X is Prefect -compact space .

Proof :- Straight from definition of compact , Prefect-compact space and prefected space .

3.4 Theorem : Let X be strongly , prefected space , then , if X is Prefect -compact space then X is Gem-compact space [point-compact space] .

Proof:- Straight from definition of strongly , prefected space and by theorem (3.2) , (3.3) .

3.5 Theorem : Let X is prefected space , Strongly Prefect -compact space then (X, τ) is a point compact space .

Proof :- Straight from definition of prefected space and Strongly Prefect-compact space .

3.6 Theorem : Every Strongly Prefect -compact space is Prefect -compact space .

Proof:- To prove X is Prefect -compact space . Let $\varsigma = \{A_\lambda \subseteq Y\ ;\ \lambda \in \Omega\}$ be an open cover for X , with $X = \bigcup_{\lambda \in \Omega} A_\lambda$. By hypothesis X is Strongly Prefect -compact, so there exists a finite subcover of X such that $X = \bigcup_{i=1}^{n}({}^{*x}pr(A_i))$. Thus X is Prefect -compact space.

3.7 Theorem : If X is strongly and prefected space then X is compact space if and only if X is Strongly Prefect -compact space .

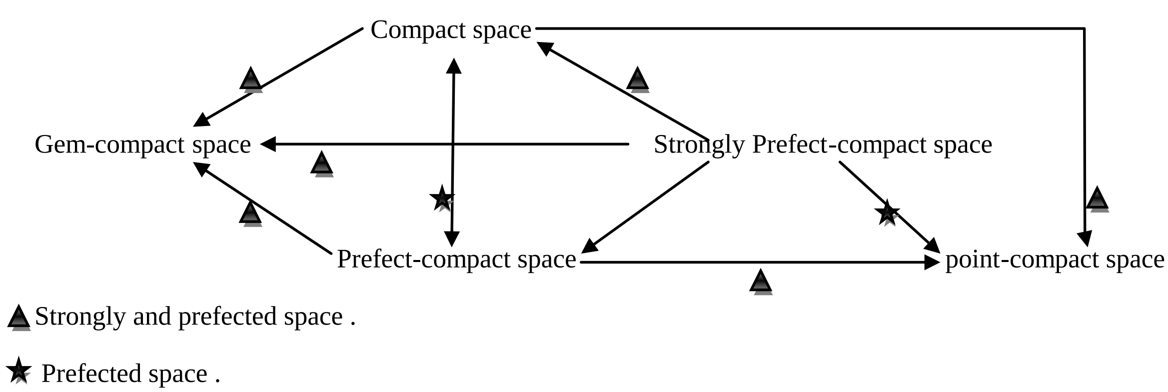
Proof :- Let X is compact space . To prove X is Strongly Prefect -compact space . Let $\varsigma = \{A_\lambda \subseteq Y\ ;\ \lambda \in \Omega\}$ with $X = \bigcup_{\lambda\in\Omega} A_\lambda$. Since X is strongly and perfected . So $\{A_\lambda \subseteq Y\ ;\ \lambda \in \Omega\}$ are open sets and ${A^{*x}}_\lambda \subseteq A_\lambda$, so ${A^{*x}}_\lambda$ are open sets therefore A_λ are open sets in X . Then we get that $\bigcup_{\lambda\in\Omega} A_\lambda$ is an open cover for X so there exists a finite sub cover of X such that $X = \bigcup_{i=1}^{n} A_i$. Therefore $X = \bigcup_{i=1}^{n}({}^{*x}pr(A_i))$, since X is prefected space . Thus X is Strongly Prefect -compact space .

Conversely : To prove that X is compact space . Let $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ are open cover of X . By hypothesis X is a Strongly Prefect -compact space . So there exist a finite subcollection $A_1, A_2, \dots, A_n$ in X such that $X = \bigcup_{i=1}^{n}({}^{*x}pr(A_i))$, since$X$ is prefected space . Therefore $X = \bigcup_{i=1}^{n} A_i$, thus X is compact space .

3.8 Theorem : Let X is strongly and prefected space then X is Gem-compact space if X is Strongly Prefect -compact space .

Proof :- Since X is strongly , prefected space and Strongly Prefect -compact space so by theorem (3.7) X is compact space . By theorem (3.2) X is Gem-compact space .

3.9 Remark : the following diagram holds :



3.1 Diagram : The relationship between compact space , Point-compact , Gem-compact , Prefect-compact and Strongly Prefect-compact space .

3.10 Theorem : Every open subset of Strongly Prefect-compact subset is Strongly Prefect -compact subset .

Proof :- Let A is an open set in X and $A \subseteq B$ such that B is Strongly Prefect -compact subset . To prove that A is Strongly Prefect -compact subset . Let $\varsigma = \{D_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ with $A \subseteq \bigcup_{\lambda\in\Omega} D_\lambda$. Then $B \subseteq \bigcup_{\lambda\in\Omega} D_\lambda \cup A^c$; B is Strongly Prefect -compact , so there exists a finite subcollection $D_1, D_2, \dots, D_n$ in X such that $B \subseteq \bigcup_{i=1}^{n}({}^{*x}pr(D))_i \cup {}^{*x}pr(A^c)$. Since A^c is closed so $A^c = {}^{*x}pr(A^c)$. So we get that $B \subseteq \bigcup_{i=1}^{n}({}^{*x}pr(D))_i \cup A^c$. Therefore $A \subseteq \bigcup_{i=1}^{n}({}^{*x}pr(D))_i$. Thus A is Strongly Prefect -compact subset .

3.11 Theorem : Every closed subset from a Strongly Prefect-compact space is Strongly Prefect-compact subspace , if the topological space is prefected .

Proof :- To prove the theorem (3.13) , directly by using the definition of Strongly perfected space and definition of prefected space.

3.12 Theorem : Every Prefect -compact subset from a prefect and T_2-space is a closed set .

Proof :- Let A be Prefect -compact subset of X . We shall prove that $X - A$ is an open set . Let $x \in X - A$, since X is T_2-space . So for each point $y \in A$. There exist U and V are open subsets from X such that $x \in U$ and $y \in V$ with $U \cap V = \emptyset$. The collection $\{V_\lambda \subseteq X : \lambda \in \Omega\}$ is open covering of A but A is a prefect-compact subset . So there exists $y \in$ A and a finitely subcollection $V_1, V_2, \dots, V_n$ in X such that $A \subseteq \bigcup_{i=1}^{n}({}^{*y}pr(V_i))$ since X is prefected space so ${}^{*y}pr(V_i) = V_i$ for each i therefore $A \subseteq \bigcup_{i=1}^{n} V_i$, $R = V_1 \cup V_2 \cup \dots \cup V_n$, Contains A , and it is disjoint from the open set $K = U_1 \cap U_2 \cap \dots \cap U_n$ formed by taking the intersection of corresponding subset containing x . For if z is a point of R , then $z \in V_i$ for some i. Whence $z \notin U_i$ and so $z \notin K$. Therefore K is a neighborhood of x disjoint from A . Hence $X - A$ is open set . Thus A is a closed subset .

3.13 Theorem : Every Strongly Prefect -compact subset from a prefected and T_2-space is closed .

Proof :- By the same way of theorem (3.12) .

3.12 Theorem : If a space X is Strongly Prefect -compact space , then for any family $\{A_\lambda\ ;\ \lambda \in \Omega\}$ of open sets in X satisfying $\bigcap_{\lambda\in\Omega} A_\lambda = \emptyset$ there is a finite subfamily $A_1, A_2, \dots, A_n$ with $\bigcap_{i=1}^{n} A_i = \emptyset$.

Proof :- Let $\{A_\lambda\ ;\ \lambda \in \Omega\}$ be a family of open sets in X satisfying $\bigcap_{\lambda\in\Omega} A_\lambda = \emptyset$, then $\{(A_\lambda^C)\ ;\ \lambda \in \Omega\}$ of a closed sets in X with $X = \bigcup_{\lambda\in\Omega}(A_\lambda^C)$. X is Strongly Prefect -compact space , so there exists a finite subcover $A_1^C, A_2^C, \dots, A_n^C$ in X such that $X = \bigcup_{i=1}^{n} {}^{*x}pr(A_i^C)) = \bigcup_{i=1}^{n}(A_i^C)$, since $\{(A_\lambda^C)\ ;\ \lambda \in \Omega\}$ of a closed sets in X . Thus $\bigcap_{i=1}^{n} A_i = \emptyset$.

4. The compactness and the mapping

In this section we study our new compact space under the functions of transpositing their properties from one space to another displaying more properties of new compact spaces .

4.1 Theorem : An injective $I^{**} - map\ f$ from a topological space (X, τ) into a topological space (Y, σ)

1. If (X, τ) is a Gem-compact space , then $f[X]$ is a Gem-compact space .
2. If (X, τ) is a point-compact space , then $f[X]$ is a point-compact space .
3. If (X, τ) is a Strongly Prefect-compact space , then $f[X]$ is a Strongly Prefect-compact space .

Proof : 1. Let $B \subseteq Y\ s.t\ f[X] = \bigcup_{f(x)\in Y} B^{*f(x)}$. It follows $f^{-1}(f[X]) = f^{-1}(\bigcup_{f(x)\in Y} B^{*f(x)})$ then $X = \bigcup_{f(x)\in Y} f^{-1}(B^{*f(x)})$ [since f is $I^{**} - map$] . $X = \bigcup_{x\in X}(f^{-1}(B))^{*x}$. By hypothesis , (X, τ) is a Gem-compact space . So there exists a finite element $x_1, x_2, \dots x_n$ in X s.t $X = \bigcup_{i=1}^{n}(f^{-1}(B))^{*x_i}$. $X = \bigcup_{i=1}^{n}(f^{-1}(B^{*f(x_i)})$, since f is $I^{**} - map$. Then $f[X] = f[\bigcup_{i=1}^{n}(f^{-1}(B))^{*x_i}]$ so $f[X] = \bigcup_{i=1}^{n} f[(f^{-1}(B))^{*x_i}]$. then we get that $f[X] = \bigcup_{i=1}^{n} B^{*f(x)_i}$. Thus , $f[X]$ is a Gem-compact space .

2. By the same way of proof theorem (4.1) (1) .

3. By the same way of proof theorem (4.1) (1) .

4.2 Theorem : An injective continues and $I^{**} - map\ f$ from a topological space (X, τ) into a topological space (Y, σ) then : If (X, τ) is Prefect-compact space then $f[X]$ is a Prefect-compact space .

Proof : By the same way of theorem (4.1) .

4.3 Corollary : A bijective $I^{**} - map\ f$ from a topological space (X, τ) into a topological space (Y, σ) , then :

1. If (X, τ) is a Gem-compact space , then (Y, σ) is a Gem-compact space .
2. If (X, τ) is a point-compact space , then (Y, σ) is a point-compact space .
3. If (X, τ) is a Strongly Prefect -compact space , then (Y, σ) is a Strongly Prefect -compact space .

Proof : 1. Since f is onto , so $Y = f[X]$ so by theorem (4.1) (1) , (Y, σ) is a Gem-compact space .

2. By using theorem (4.1) (2) and the surjective of mapping .

3. By using theorem (4.1) (3) and the surjective of mapping .

4.4 Corollary : A bijective continues , $I^{**}-map\ f$ from a topological space (X,τ) into a topological space (Y,σ) , then : If (X,τ) is a Prefect -compact space then (Y,σ) is a Prefect -compact space .

Proof : Since f is onto so we get that $Y=f[X]$. By theorem (4.2) . Y is a Prefect -compact space .

4.5 Corollary : A bijective $AO-map$ and open mapping f from a topological space (X,τ) into a topological space (Y,σ) , then :

1. If (X,τ) is Gem-compact space then (Y,σ) is a Gem-compact space .
2. If (X,τ) is point-compact space then (Y,σ) is a point-compact space .
3. If (X,τ) is a Strongly Prefect -compact space then (Y,σ) is a Strongly Prefect -compact space .

Proof : 1. By theorem (2.12) f is $I^{**}-map$. Therefore we get that (Y,σ) is Gem-compact space by Corollary (4.3) (1) .

2. By theorem (2.12) and theorem (4.3) (2).

3. By theorem (2.12) and theorem (4.3) (3).

4.6 Corollary : A homeomorphism $AO-map\ f$ from a topological space (X,τ) into a topological space (Y,σ) , then : If (X,τ) is a Prefect -compact space then (Y,σ) is a Prefect -compact space.

Proof : By theorem (2.12) so f is $I^{**}-map$. Therefore we get that (Y,σ) is Prefect -compact space by Corollary (4.4).

4.7 Theorem : A bijective $I^{*}-map\ f$, from a topological space (X,τ) into a topological space (Y,σ) , then :

1. If (Y,σ) is a Gem-compact space , then (X,τ) is Gem-compact space .
2. If (Y,σ) is a point-compact space , then (X,τ) is point-compact space .
3. If (Y,σ) is a Strongly Prefect -compact space , then (X,τ) is Strongly Prefect -compact space .

Proof : 1. Let $A\subseteq X$ and $X=\bigcup_{x\in X}A^{*x}$ then $Y=f(X)=f(\bigcup_{x\in X}A^{*x})$, so we get $Y=\bigcup_{x\in X}f(A^{*x})$ it follows $Y=\bigcup_{f(x)\in Y}(f(A))^{*f(x)}$, since f is $I^{*}-map$. But (Y,σ) is a Gem-compact space , so there exists a finite element $f(x_1),f(x_2),\dots,f(x_n)$ in Y such that $Y=\bigcup_{i=1}^{n}(f(A))^{*f(x_i)}$. $Y=\bigcup_{i=1}^{n}f(A^{*x_i})$ so $X=f^{-1}(Y)=f^{-1}(\bigcup_{i=1}^{n}f(A^{*x_i}))$;

therefore , $X = \bigcup_{i=1}^{n} f^{-1}(f(A^{*x_i}))$, then we get $X = \bigcup_{i=1}^{n} A^{*x_i}$. Thus (X,τ) is a Gem-compact space .

2. By the same way of proof theorem (4.7) (1) .

3. By the same way of proof theorem (4.7) (1) .

4.8 Theorem : A bijective open , $I^* - map\ f$, from a topological space (X,τ) into a topological space (Y,σ) , then : If (Y,σ) is Prefect -compact space then (X,τ) is Prefect -compact space .
Proof : Let $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ is an open cover for X , with $X = \bigcup_{\lambda\in\Omega} A_\lambda$ it follows $Y = f(X) = f(\bigcup_{\lambda\in\Omega} A_\lambda)$ is an open cover for Y , since f is open mapping , then $Y = \bigcup_{\lambda\in\Omega} f(A_\lambda)$. By hypothesis Y is a Prefect-compact space . So there exist a finite subcollection $f(A_1), f(A_2), \dots, f(A_n)$ in Y such that $Y = \bigcup_{i=1}^{n}({}^{*y}pr(f(A_i)))$, since f is $I^{**} - map$. Then $Y = \bigcup_{i=1}^{n}(f({}^{*x}pr(A_i)))$ .So $X = \bigcup_{i=1}^{n} f^{-1}(f({}^{*x}pr(A_i)))$. Therefore $X = \bigcup_{i=1}^{n}({}^{*x}pr(A_i))$. Thus (X,τ) is Prefect -compact space .

4.9 Corollary : A bijective $A - map$ and continuous mapping f from a topological space (X,τ) into a topological space (Y,σ) , then :

1. If (Y,σ) is a Gem-compact space , then (X,τ) is Gem-compact space .
2. If (Y,σ) is a point-compact space , then (X,τ) is point-compact space .
3. If (Y,σ) is a Strongly Prefect -compact space , then (X,τ) is Strongly Prefect -compact space .

Proof : 1. By theorem (2.13) f is $I^* - map$ .So by above theorem (4.7) (1) we get that (X,τ) Gem-compact space.

2. By theorem (2.13) and theorem (4.7) (2) .

3. By theorem (2.13) and theorem (4.7) (3) .

4.10 Corollary : A homeomorphism $A - map\ f$ from a topological space (X,τ) into a topological space (Y,σ) , then : If (Y,σ) is Prefect -compact space then (X,τ) is Prefect -compact space .

Proof : By theorem (2.13) and theorem (4.8) .

4.11 Theorem : A bijective $A - map\ f$ from a topological space (X,τ) into a topological space (Y,σ), if $(Y,\sigma)$is a point -compact space , then (X,τ) is a point -compact space .

Proof :- To prove that (X,τ) point -compact space . Let $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ such that $X = \bigcup_{\lambda\in\Omega} A_\lambda^{*x}$. Then $Y = f(X) = f(\bigcup_{\lambda\in\Omega} A_\lambda^{*x})$. It follows

$Y = \bigcup_{\lambda\in\Omega} f(A_\lambda^{*x}) \subseteq \bigcup_{\lambda\in\Omega} B_\lambda^{*f(x)}$, because f is $A - map$. Since $\bigcup_{\lambda\in\Omega} B_\lambda^{*f(x)} \subseteq Y$ so we get that $Y = \bigcup_{\lambda\in\Omega} f(A_\lambda^{*x}) = \bigcup_{\lambda\in\Omega} B_\lambda^{*f(x)}$. By hypotheses (Y,σ) is point -compact space . So there exists a finite subcover such that $Y = \bigcup_{i=1}^{n} f(A_i^{*x}) = \bigcup_{i=1}^{n} B_i^{*f(x)}$. So $X = f^{-1}(Y) = \bigcup_{i=1}^{n} f^{-1}[f(A_i^{*x})]$. Therefore $X = \bigcup_{i=1}^{n} A_i^{*x}$. Thus (X,τ) is a point -compact space .

4.12 Corollary : A bijective f from a topological space (X,τ) into a compact and strongly space (Y,σ), then (X,τ) is point -compact space if f is $A - map$.

Proof :- By theorem (3.2) and theorem (4.11) .

4.13 Theorem : A bijective open f from a strongly space (X,τ) into topological space (Y,σ) , if (Y,σ) is a compact space , then (X,τ) is a point -compact space .

Proof :- To prove that (X,τ) point-compact space . Let $\varsigma = \{A_\lambda \subseteq X\ ;\ \lambda \in \Omega\}$ such that $X = \bigcup_{\lambda\in\Omega} A_\lambda^{*x}$. Since (X,τ) is strongly space so we get that $\{A_\lambda^{*x} \subseteq X\ ;\ \lambda \in \Omega\}$ are open sets in X . Then $Y = \bigcup_{\lambda\in\Omega} f(A_\lambda^{*x})$ is open cover for Y . By hypothesis Y is a compact space so there exists a finite subcover such that $Y = \bigcup_{i=1}^{n} f(A_i^{*x})$. So $X = f^{-1}(Y) = \bigcup_{i=1}^{n} f^{-1}[f(A_i^{*x})\ ]$. Therefore $X = \bigcup_{i=1}^{n} A_i^{*x}$. Thus (X,τ) is a point-compact space .

4.14 Theorem : A bijective continuous f from a topological space (X,τ) into a strongly space (Y,σ) , if (X,τ) is a compact space then $(Y,\sigma)$is a point -compact space .

Proof :- To prove that (Y,σ) is a point-compact space . Let $\varsigma = \{B_\lambda \subseteq Y\ ;\ \lambda \in \Omega\}$ such that $Y = \bigcup_{\lambda\in\Omega} B_\lambda^{*y}$. Since (Y,σ) is strongly space so we get that $\{B_\lambda^{*y} \subseteq X\ ;\ \lambda \in \Omega\}$ are open sets in Y .Then $X = f^{-1}(Y) = \bigcup_{\lambda\in\Omega} f^{-1}(B_\lambda^{*y})$ is an open cover for X . By hypothesis X is a compact space so there exists a finite subcover such that $X = \bigcup_{i=1}^{n} f^{-1}(B_i^{*y})$ .So $Y = f(X) = \bigcup_{i=1}^{n} f(f^{-1}(B_i^{*y})$. Therefore $Y = \bigcup_{i=1}^{n} B_i^{*y}$ .Thus (Y,σ) is a point-compact space .

4.15 Theorem : A bijective continuous f from a topological space (X,τ) into a prefected space (Y,σ), then : If X is compact space then Y is Prefect -compact space .

Proof :- By the same way of theorem (4.14) .

4.16 Theorem : An injective and I^{**}-map f from prefected and Hausdorff (X,τ) space onto Strongly Prefect –compact (Y,σ) space , then f is contra-continuous .

Proof :- By theorem(3.10) , theorem (4.3 (3) and theorem (3.13) .

4.17 Theorem : A bijective open mapping f from prefected space (X,τ) into a topological space (Y,σ), if $B \subseteq Y$ is compact subset then $f^{-1}(B) \subseteq X$ is Prefect -compact space .

Proof :- Let $B \subseteq Y$ is compact subset. To prove that $f^{-1}(B) \subseteq X$ is Prefect-compact space. Let $\varsigma = \{D_\lambda \subseteq X \,;\, \lambda \in \Omega\}$ be an open cover for $f^{-1}(B)$ with $f^{-1}(B) \subseteq \bigcup_{\lambda\in\Omega} D_\lambda$. So$B = f(f^{-1}(B)) = \bigcup_{\lambda\in\Omega} f(D_\lambda)$ is an open cover for B, since B is compact subset. So there exists a finite subcover such that: $B \subseteq \bigcup_{i=1}^{n} f(D_i)$. Then $f^{-1}(B) \subseteq \bigcup_{i=1}^{n} D_i$, since X is prefected space. Therefore $f^{-1}(B) \subseteq \bigcup_{i=1}^{n}({}^{*x}pr(D_i))$. Thus $f^{-1}(B)$ is Prefect–compact subset.

4.18 Theorem : A bijective open mapping f from prefected space (X,τ) into compact space (Y,σ), if (X,τ) is Hausdorff space then f is homeomorphism function .

Proof :- By theorem(2.10) , theorem (4.17) and theorem (3.12) .

4.19 Theorem : A bijective $I^{**} - map\ f$ from a topological space (X,τ) into prefected space (Y,σ), then:if A is Strongly Prefact-compact subset in X then $f(A)$ is compact subset in Y .

Proof :- Let $A \subseteq X$ is Strongly Prefact-compact subset in X . To prove that $f(A) \subseteq Y$ is compact subset in Y . Let $\varsigma = \{D_\lambda \subseteq X \,;\, \lambda \in \Omega\}$ be an open cover for $f(A)$, with $f(A) \subseteq \bigcup_{\lambda\in\Omega} D_\lambda$. Then $A \subseteq \bigcup_{\lambda\in\Omega} f^{-1}(D_\lambda)$. By hypothesis A is Strongly Prefect -compact space so there exists a finite subcover such that : $A \subseteq \bigcup_{i=1}^{n} {}^{*x}pr(f^{-1}(D_i))$, $x \in X$. Since f is $I^{**} - map$ So we get that $A \subseteq \bigcup_{i=1}^{n} f^{-1}({}^{*f(x)}pr\ (D_i))$ Then $f(A) \subseteq \bigcup_{i=1}^{n} f[f^{-1}({}^{*f(x)}pr\ (D_i))]$. So $f(A) \subseteq \bigcup_{i=1}^{n} {}^{*f(x)}pr\ (D_i)$, because Y is prefected space . Therefore $f(A) = \bigcup_{i=1}^{n} D_i$. Thus $f(A)$ is compact subspace.

4.20 Theorem : A bijective $I^{**} - map\ f$ from perfected , Strongly Prefact-compact space (X,τ) into perfected space (Y,σ), if (Y,σ) is Hausdorff space then f is closed function .

Proof :- Let A be a closed subset in X . So by theorem (3.11) A is Strongly Prefect-compact subset , and by above theorem (4.19) Therefore $f(A)$ is a compact subset . Also by theorem (2.14) it follows $f(A)$ is a closed subset in Y . Thus f is closed function.

4.21 Theorem : A bijective continuous mapping f from prefected space (X,τ) into strongly space (Y,σ) , then :

1- If (X,τ) is Prefect –compact then (Y,σ) is Point-compact space.

2- If (X,τ) is Prefect –compact then (Y,σ) is Gem-compact space.

Proof :-

1- To prove Y is Point-compact space . Let $\varsigma = \{B_\lambda \subseteq Y \ ; \ \lambda \in \Omega\}$ with $Y = \bigcup_{\lambda\in\Omega} B_\lambda^{*f(x)}$, since Y is strongly so we get that $\{B_\lambda^{*f(x)} \ ; \ \lambda \in \Omega\}$ are open in Y. Then $X = \bigcup_{\lambda\in\Omega}[f^{-1}\left(B_\lambda^{*f(x)}\right)]$ is an open cover for X since f is continuous function. By hypothesis X is Prefect -compact space , so there exists a finite subcover $f^{-1}\left(B_1^{*f(x)}\right), f^{-1}\left(B_2^{*f(x)}\right), \dots, f^{-1}\left(B_n^{*f(x)}\right)$ such that $X = \bigcup_{i=1}^{n}[{}^{*x}pr\left(f^{-1}\left(B_i^{*f(x)}\right)\right)]$, X is prefected space. Therefore $X = \bigcup_{i=1}^{n}\left(f^{-1}\left(B_i^{*f(x)}\right)\right)$. So $Y = \bigcup_{i=1}^{n} B_i^{*f(x)}$. Thus Y is Point-compact space.

2- By the same way of theorem (4.25)(1) .

References

[1] A. A. Nasef , "Some Classes of Compactness in Terms of Ideals ", Soochow Journal of Mathematics , 27(2001) ,343-352 .

[2] D. V. Rancin, "Compactness Modulo an Ideal", Soviet Math. Dokl., 13(1972), 193.

[3] J. R.Munkres , “Topology” , Second edition , Prentice Hall , Incorporated , QA611.M82 , (2000) ,102-112.

[4] J. Dontchev, C ontra-continuous functions and strongly s-closed spaces, Internet. J. Math. and Math. Sci., 19(1996) , 303-310.

[5] J.L. Kelley, General topology, Princeton, NJ, D. Van Nostrand, 1955.

[6] L. A.AL-Swidi and A. B.AL-Nafee , “New Separation Axioms Using the idea of “Gem-Set” in topological Space “ Mathematical Theory and Moodeling ,3(2013) , 60-66.

[7] L. A.AL-Swidi and M. A. Al-Ethary , " New Function with “Gem-Set” In Topological Space ", International Journal of Engineering Research and Technology,3(2014),2324-2327.

[8] M. E. Abd El-Monsef , E. F. Lashien and A. A. Nasef, " δI -compactness via ideals", Tamkang Jour. of Math., 24(1993), 431-443.

[9] R. A . Hosny , "Some Types of Compactness via Ideal " , International Journal of Scientific & Engineering Research , 4(2013) ,1293-1296.

[10] R. L. Newcomb, "Topologies Which are Compact Modulo an Ideal", Ph.D. Dissertation, University of California at Santa Barbara, 1967.

[11] T. R. Hamlett and D. Jankovic, "Compactness with Respect to an Ideal", Boll. U.M./., 7(1990),849.

[12] T. R. Hamlett , D. Jankovic, and D. Rose, "Countable Compactness with Respect to an Ideal", *Math. Chron.*, 20(1991), 109.

Received: May 1, 2014