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Description and Study of Energy-Levels and the Deformation for Even-Even Ytterbium Isotopes

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Abstract. The program of interacting bosons was used for the purpose of finding the information about the isotopes of ytterbium, and through this information, the electrical and magnetic transitions probability was calculated, which had an impact on calculating some of the nuclear properties of these isotopes. The energy levels and deformation in these nuclei and contour shapes were also calculated and plotted. The energy ratios and branching ratios were calculated, which showed that the isotopes under study are between rotational determination and unstable gamma. The computed results are in very excellent agreement with experimental data for the isotopes under investigation.

INTRODUCTION

"The success in computing parity, spin, ϒ-, and beta decay in the ground levels, and the calculation of magnetic momentum derived from the (singly particle) shell model, already had resulted in the evolution of geometrical models, but it had failed to evaluate and calculate quadruple momentum and transition rates, particularly between closed shells transition region." This model assumes that a vast number of nucleons are travelling in a mutual collective. IBM is one of these models that studies the spectra of low energy levels, the nucleus has an even atomic number and mass number [1].

In the parity shell, these bosons are explained as interconnected pairs of protons and interconnected pairs of neutron. This explanation limits the number of bosons, which is calculated by calculating the number of particle pairs (independently for neutrons and protons), the shell is less than half full if the shell is less than half filled, and the number of hole pairs if the shell is more than half filled. If protons' and neutrons' bosons were measured indistinguishably.

If the bosons of protons and neutrons are indistinguishable, the IBM is in its most basic form, which is known as IB M-1 [2-4].

It is possible to distinguish between the wave functions of neutrons and protons. This model generates electric transitions and energy levels by adjusting a number of parameters in the model to fit the results with experimental data [5-7]

The nuclei form of $^{160\text{-}166}$Yb isotopes has been investigated analytically using IBM-1, and they have several infinite energy levels, with just the electric transition 2_1^+ to 0_1^+ only explored by T. K. Rz aca-Urban and Rz aca-Urban Nomura [8], Nomura In the IBM-1 model, the software set (IBM) version is used to approximate a set of parameters known as the Hamiltonian operator, as indicated in equation (1).

These parameters are handled as free parameters to get the best fit between theoretical estimated values and experimental data. The calculations allow for the efficient state that must occur in these parameters based on the nature of the neutron number decrease or increase for each nucleus.

The program set (IBMT) is used also to calculate the reducing electromagnetic transition prospects transitions in IBM-1.

INTERACTING BOSONS MODEL-1(IBM-1)

Since the IBM-1 assumed that the Hamiltonian only had one- and two-body interactions, it added the creation (s†, d†m) and annihilation (s, dm) operators, with m=0,±1,±2.

The IBM-1 imaginary that the "Hamiltonian comprises only one- body and two- body relations, thus introduced creation) and annihilation (s, d_m) operators, where m=0,±1,±2. The Hamiltonian with one –boson term in boson-boson interaction is the most general Hamiltonian" [9-11]:

$$H = \in nd + a_0\tilde{P}.\,\tilde{P} + a_1L^2 + a_2Q^2 + a_3T_3{}^2 + a_4T_4{}^2\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

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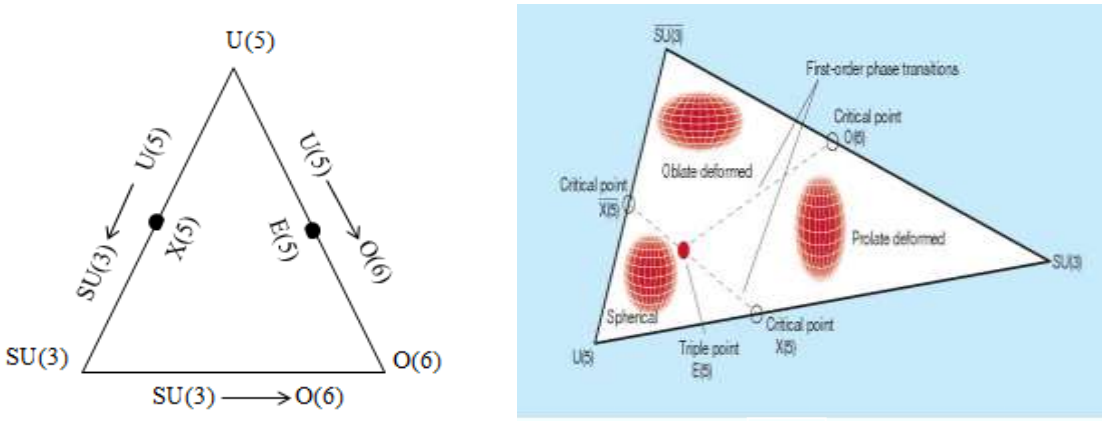
Where a_0,a_1,a_2, a_3, and a_4 are the parameters that we resolve in this model to achieve the output levels, and $\in$, L, $\tilde{P}$, T_3, Q and T_4 are the energy, angular momentum, pairing, octopole, quadruple, and hexadecapole operators, respectively.

TRANSITIONS REGIONS IN IBM-1

Pairs of symmetries are an important aspect of symmetry notions since they serve as benchmarks and, as a result, transitional region limits. The nuclei concatenation in such a phase transitional area can be computed quite easily. Generally, their location along the appropriate leg of the symmetry triangle is determined by a variation of a single parameter. This parameter is usually calculated as the ratio of the coefficients in the Hamiltonians of the two symmetries conquering the triangle's apexes at the transition leg's termini. The Casten triangle displays the three dynamical symmetries as well as the transitional zones, as seen in Fig. (1).

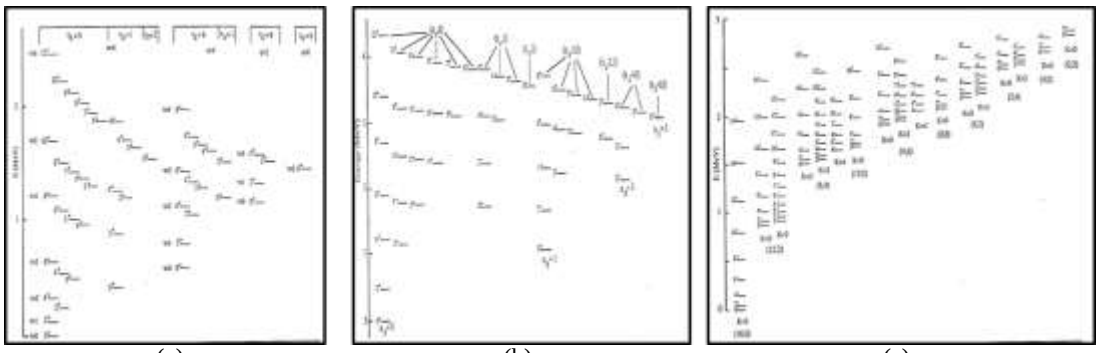


(a) (b)

FIGURE 1. The Casten triangle [12]

Each peak in Fig. (b) represents an exact symmetry that is identical to one of the three forms shown. Transition points and their crucial symmetries, as well as first-order phase transitions, are defined. According to [12], " transition between a spherical nuclear form and a prolate or oblate distorted one. "there is a nuclear triple point that represents the second-order

Fig.(2) represents a typical spectrum for the three dynamical symmetries, Fig.(2a) is O(6) typical spectrum symmetry, Fig.(2b) is SU(5) typical spectrum symmetry and Fig.(2c) is SU(3) typical spectrum symmetry.



(a) (b) (c)

FIGURE 2. A typical spectrum with SU(3), SU(5) and O(6) symmetry [13]

The limits previously discussed give a set of analytical solutions that can be easily tested, as the number of the nuclei that can be characterized by these limits is so few, because most of nuclei have common properties between these limits called transition region, which can be divided into four classes:

Class A: $\mathbf{U(5) \rightarrow SU(3)}$

This nuclei has properties between vibrational and rotational limit in the transition region, and the Hamiltonian operator is given as follows [14]:

$$\hat{H}^{I+II} = \varepsilon\hat{n}_d + a_1\hat{L}.\hat{L}a_2\hat{Q}.\hat{Q} \quad \text{……..……..…(2)}$$

The ratio ($\varepsilon\hat{n}_d/a_2$) determines the properties of nuclei in this region. Thus, when the ratio gets higher, the properties become closer to U(5) limit, but when the ratio gets lower, the properties becomes closer to SU(3) limit.

Class B: $SU(3) \rightarrow O(6)$

The nuclei has properties between rotational limit and γ_ unstable limit in the transition region and the Hamiltonian is [16]:

$$\hat{H}^{II+III} = a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} \quad \text{……..……..…(3)}$$

The ratio (a_0/a_2) determines the properties of nuclei in this region. Thus, when the ratio gets higher, the properties become closer to O(6) limit, but when the ratio gets lower, the properties becomes closer to SU(3) limit.

Class C: $U(5) \rightarrow O(6)$

The nuclei has properties between vibrational limit and γ- unstable limit in the transition region and the Hamiltonian is [16]:

$$\hat{H}^{(I+III)} = \varepsilon\hat{n}_d + a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L}+a_3\hat{T}_3.\hat{T}_3 \quad \text{……..……..…(4)}$$

Properties of this limit depend on the ratio ($\varepsilon\hat{n}_d/a_0$).

Class D: $U(5) \rightarrow SU(3) \rightarrow O(6)$

The nuclei of this class possesses the common properties between three limits and the Hamiltonian operator is given as follows [16]:

$$\hat{H}^{(I+II+III)} = \varepsilon\hat{n}_d + a_0\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q}+a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \text{ ……..…..…(5)}$$

ENERGY RATIOS

Some ratios between the ground energy levels 8^+,6^+ and 4^+ with the first excited state 2^+ to match the three limits as exposed in Table (1) [12].
The analytical solvable dynamical symmetries SU(3) and O(6) with schematically descriptions – soft nuclei are used to describe axial symmetric rotations and spherical vibrators in IBM [13]. The unstable region γ –soft, from the ratio between $E_{8_1}^+/E_{2_1}^+$, $E_{6_1}^+/E_{2_1}^+$ and $E_{4_1}^+/E_{2_1}^+$ as in Table (1).

TABLE 1. Typical energy levels ratios for each limits [12]

Limit	$\frac{E_{4_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{2_1^+}}$	$\frac{E_{8_1^+}}{E_{2_1^+}}$
U(5)	2	3	4
SU(3)	3.33	7	1٠
O(6)	2.5	4.5	7

BRANCHING RATIO

Other important quantities show that the difference between the three limits are the branching ratio ([17]:

$$R=\frac{B(E2;4_1^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)},\ R'=\frac{B(E2;2_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)},\ R''=\frac{B(E2;0_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)} \quad \text{…...…..…….(6)}$$

$$R=\begin{cases} U(5): & 2(N_b-1)/N_b & \to 2 \\ SU(3): & 10(N_b-1)(2N_b+5)/7N_b(2N_b+3) & \to 10/7 \\ O(6): & 10(N_b-1)(N_b+5)/7N_b(N_b+4) & \to 10/7 \end{cases} \quad \text{…..……..……..(7)}$$

$$R'=\begin{cases} U(5): & 2(N_b-1)/N_b & \to 2 \\ SU(3): & 0 & \\ O(6): & 10(N_b-1)(N_b+5)/7N_b(N_b+4) & \to 10/7 \end{cases} \quad \text{……...…….(8)}$$

$$R''=\begin{cases} U(5): 2(N_b-1)/N_b \to 2 \\ SU(3): 0 \\ O(6): 0 \end{cases} \quad \text{…… …..…….(9)}$$

POTENTIAL ENERGY SURFACE (P.E.S)

"The potential energy surface is calculated using the P.E.S. for program. For 160-166Yb isotopes, the contour plots in the (-) plane resulting from E are displayed. The mapped IBM energy surfaces are form for the majority of the considered Yb nuclei. The triaxial form is linked to intermediate values (0-60). The triaxial deformation helps to understand the prolate to oblate shape transition that occurs in the considered Yb isotopes." [18].

RESULTS AND DISCUSSION

The energy levels of isotopes were estimated using the (IBM) software in Fortran and the Hamiltonian as equation (1) with the values listed in Table (2).

These parameters are gotten energy levels for ^{160}Yb, ^{162}Yb,^{164}Yb and^{166}Yb and draw it in Fig.(2, 3, 4, 5) respectively, there is an excellent corresponding with the experimentally results [19-22], and it is so near from the typical spectrum with SU(3) symmetry (Fig.(2-c)). We can see that there are many uncertain energy levels or unknown parties in this investigation.

TABLE 2. The Hamiltonian's parameters in MeV units and the number of bosons (N).

The Isotopes	Boson Number	E	a_0	a_1	a_2	a_3	a_4	χ
^{160}Yb	11	0	0.0114	0.0184	-0.0165	0	0	-0.068
^{162}Yb	12	0	0.0116	0.0164	-0.0164	0	0	-1.170
^{164}Yb	13	0	0.0126	0.0132	-0.0123	0	0	-0.280
^{166}Yb	14	0	0.0112	0.0119	-0.0114	0	0	-0.112

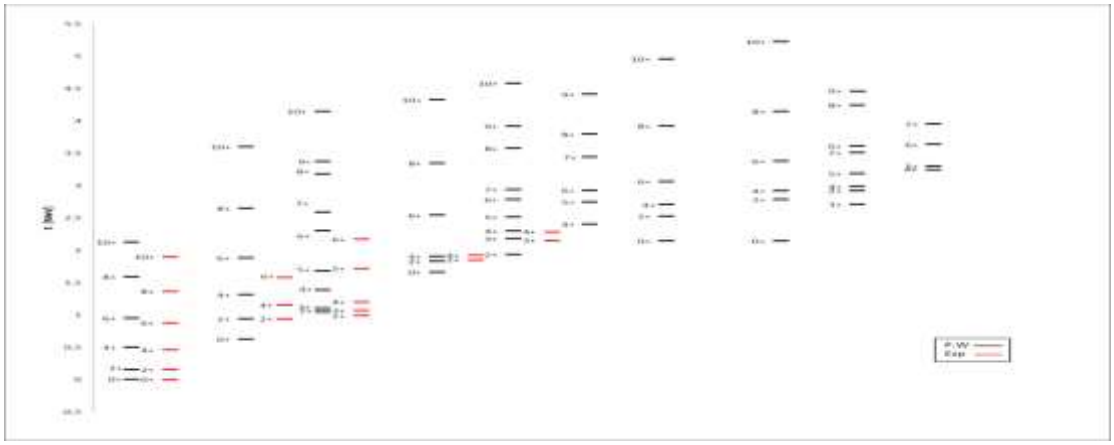


FIGURE 3. Energy Levels for ^{160}Yb**.**

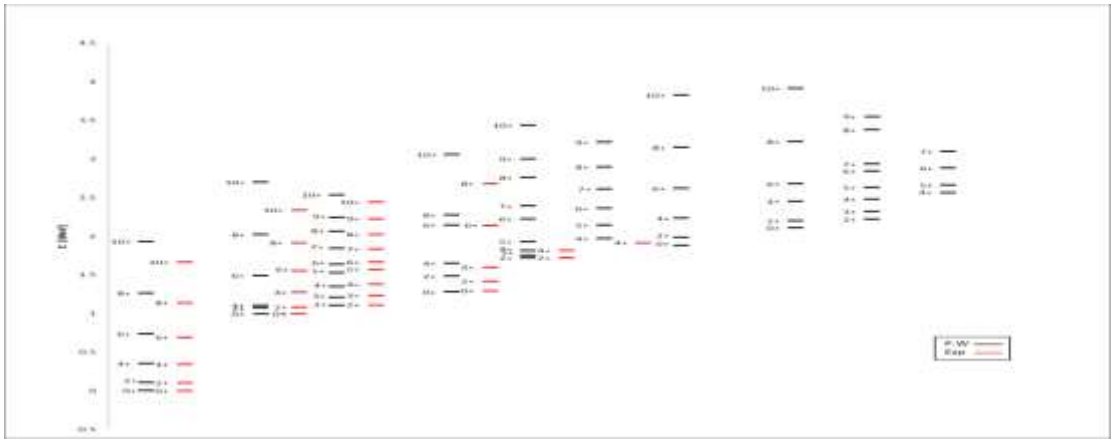


FIGURE 4. Energy Levels for ^{162}Yb**.**

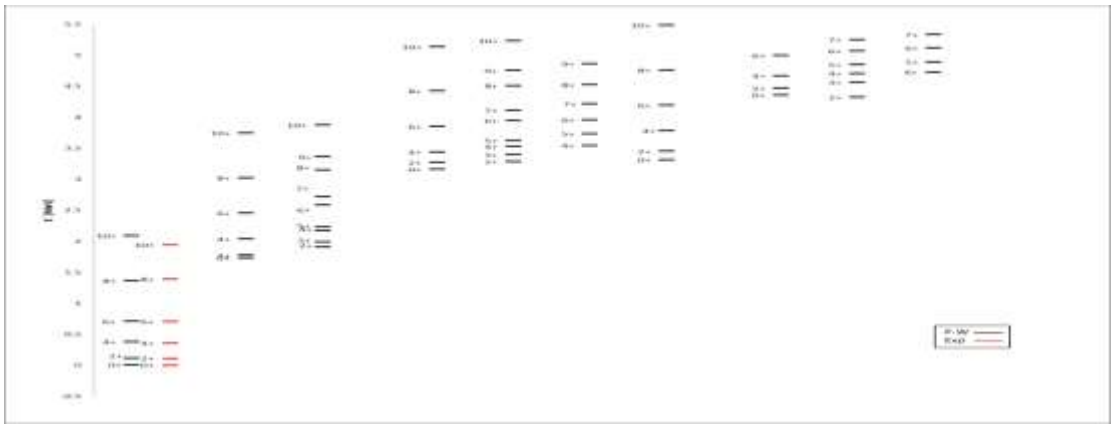


FIGURE 5. Energy Levels for ^{164}Yb.

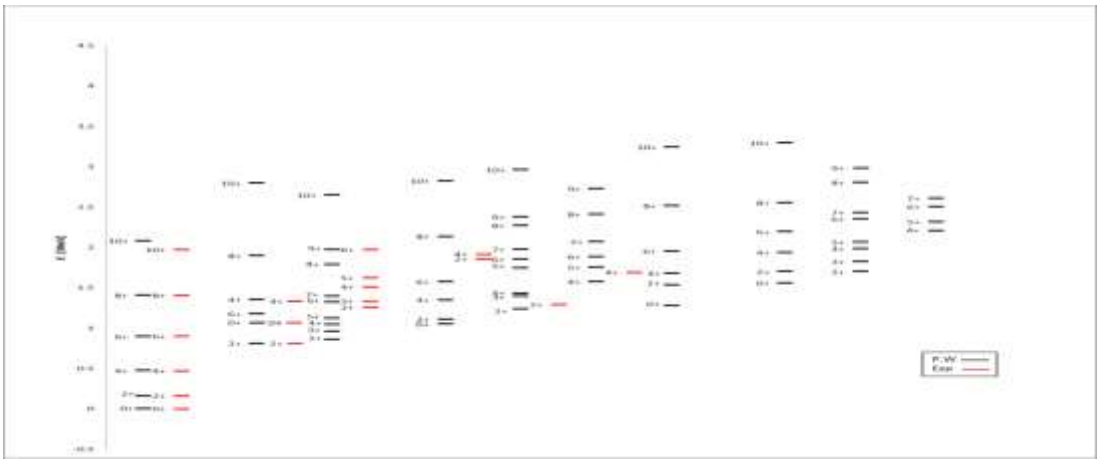


FIGURE 6. Energy Levels for ^{166}Yb.

For this current work and the experimental values in a very excellent matching with the theoretical values as in Table(1), we can determine that $^{160\text{-}166}$Yb isotopes are located in the SU(3) - O(6) area, because the ratio of $E_{8_1}^{+}$ $/E_{2_1}^{+}$, $E_{6_1}^{+}$ $/E_{2_1}^{+}$ and $E_{4_1}^{+}$ $/E_{2_1}^{+}$ for this present work shown in Table(3) and in Figs. (7), (8) and(9)is also in a good matching with ta typical values in Table (1) .

TABLE 3. Experimental and present work energy levels ratios for Yb isotopes.

Isotopes	**^{160}Yb**		**^{162}Yb**		**^{164}Yb**		**^{166}Yb**	
	Exp.	**IBM-1**	**Exp.**	**IBM-1**	**Exp.**	**IBM-1**	**Exp.**	**IBM-1**
$E_{4_1}^{+}/E_{2_1}^{+}$	2.965	2.98	3.109	3.096	3.193	3.25	3.247	3.263
$E_{6_1}^{+}/E_{2_1}^{+}$	5.655	6.059	6.102	6.419	6.378	6.568	6.598	6.711
$E_{8_1}^{+}/E_{2_1}^{+}$	8.865	8.905	9.780	9.798	10.353	10.04	10.898	10.884

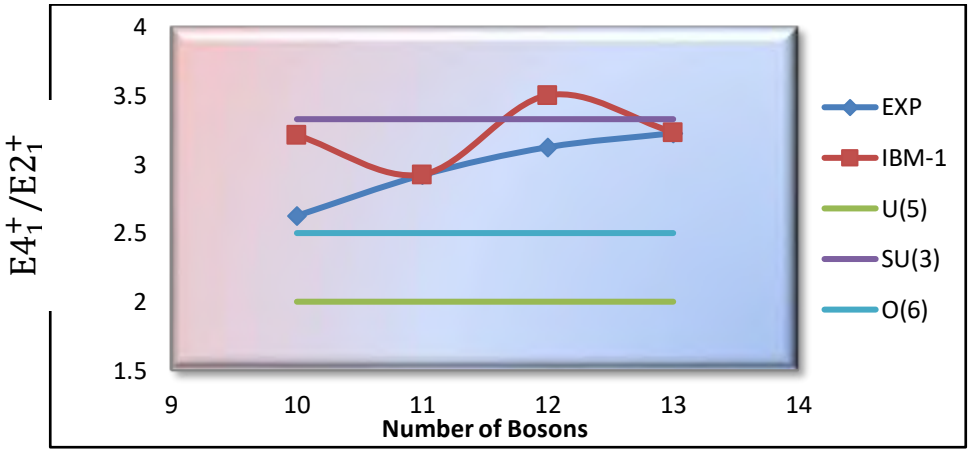


FIGURE 7. $E4_1^+/E2_1^+$ Ratios between experimental and the IBM-1 with standard ratios for $^{160\text{-}166}$Yb isotopes.

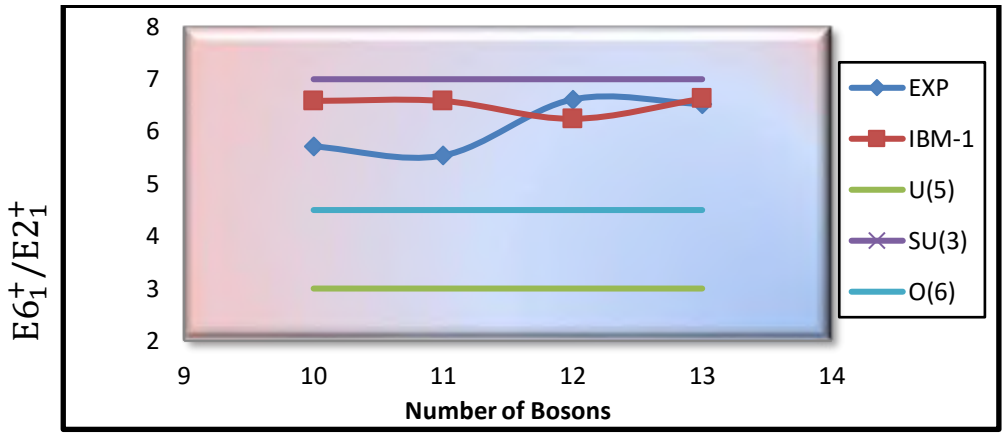


FIGURE 8. $E6_1^+/E2_1^+$ ratios between experimental and the IBM-1 with standard ratios for $^{160\text{-}166}$Yb isotopes.

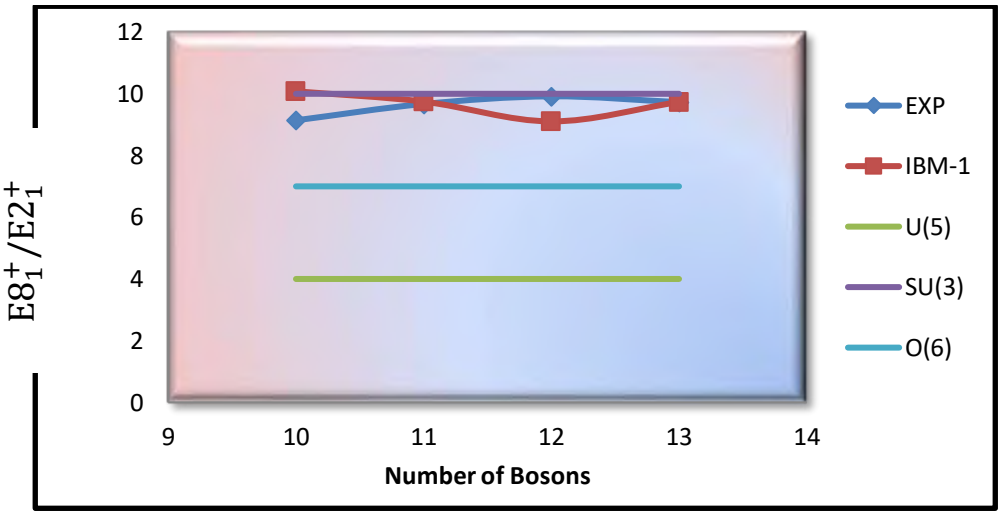


FIGURE 9. $E8_1^+/E2_1^+$ ratios between experimental and the IBM-1 with standard ratios for $^{160\text{-}166}$Yb isotopes.

The electric transition probabilities obtained from the IBMT algorithm were compared to experiment data in Table (4) for the transition from the initial level I to the final level (f), using E2DD and E2SD values. These transitions are significant because they allow us to calculate the probability of emission or ϒ-ray absorption at any energy. E2SD is equal effective charge (α_2), where (α_2) is dependent on the limit of the region of isotopes in order to obtain the appropriate electric transition, and E2DD = $(5)^{1/2}$ β_2, where (β_2) is the parameter of deformation.

TABLE 4. Represents some of permissible electric transitions calculated in IBM-1model for Yb isotope.

		$B(E2;2_1^+ \rightarrow 0_1^+)$	$B(E2;2_1^+ \rightarrow 0_1^+)$		
The Isotopes	**Boson Number**	**e^2b^2 Exp.**	**e^2b^2 IBM-1**	**E2SD**	**E2DD**
^{160}Yb	11	0.692	0.692	0.141	-0.417
^{162}Yb	12	0.857	0.857	0.100	-0.295
^{164}Yb	13	1.01	1	0.113	-0.336
^{166}Yb	14	0.876	0.876	0.106	-0.315

The branching ratio *R*, *R′* and *R″* have been found in agreement both experimentally and theoretically (Table (4)) also with in consistence with their ideal corresponding limits as in equations (7),(8) and (9) depending on these equations and the results in Table (5) it have been found that $^{160\text{-}166}$Yb isotopes are locating between O(6) and SU(3) region.

TABLE 5. The branching ratio between two electric transitions for Yb Isotopes.

The Branching Ratios	Isotopes							
	^{160}Yb		^{162}Yb		^{164}Yb		^{166}Yb	
	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1
$R=\frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	1.4	1.33	1.4	1.36	1.4	1.34	1.41	1.33
$R'=\frac{B(E2;2_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	-----	0.016	-----	0.096	-----	0.072	-----	0.028
$R''=\frac{B(E2;0_2^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	----	0.036	----	0.024	----	0.003	----	0.009

TABLE 6. The electric transitions in (e^2b^2) with positive parity for Yb isotopes.

The Isotopes	^{166}Yb		^{166}Yb		^{170}Yb		^{172}Yb	
$J_i \rightarrow J_f^+$	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1
$2_1\rightarrow 0_1$	0.712	0.6934	1.173	0.8564	0.684	1.0068	0.831	0.8764
$2_1\rightarrow 0_2$	-----	0.1278	-----	0.0214	-----	0.01891	-----	0.0435
$2_1\rightarrow 0_3$	-----	0.0006	-----	0.0003	-----	0.0158	-----	0.0013
$2_2\rightarrow 0_1$	-----	0.2314	-----	0.0494	-----	0.0645	-----	0.0914
$2_2\rightarrow 0_2$	-----	0.0159	-----	0.0050	-----	0.0033	-----	0.0002
$2_2\rightarrow 0_3$	-----	0.00003	-----	0.0193	-----	0.0040	-----	0.0002
$2_3\rightarrow 0_1$	-----	0.0002	-----	0.0014	-----	0.0030	-----	0.0019
$2_3\rightarrow 0_2$	-----	0.1315	-----	0.7084	-----	0.3441	-----	0.1794
$2_3\rightarrow 0_3$	-----	0.1073	-----	0.0053	-----	0.0915	-----	0.1131
$2_4\rightarrow 0_1$	-----	0.0022	-----	0.00009	-----	0.0018	-----	0.0041
$2_4\rightarrow 0_2$	-----	0.1908	-----	0.0009	-----	0.0981	-----	0.1637

Finally, the deformation in nucleus is shown in Figs. (10-13), The parameters used in the IBMP-code for $^{160\text{-}166}$Yb isotopes shown in Table (7).

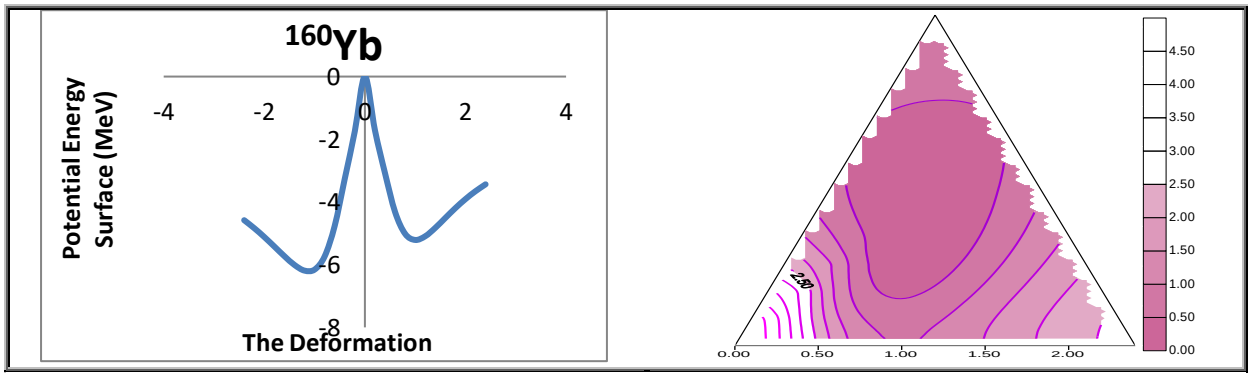


FIGURE 10. Potential Energy Surface with the Deformation for Yb

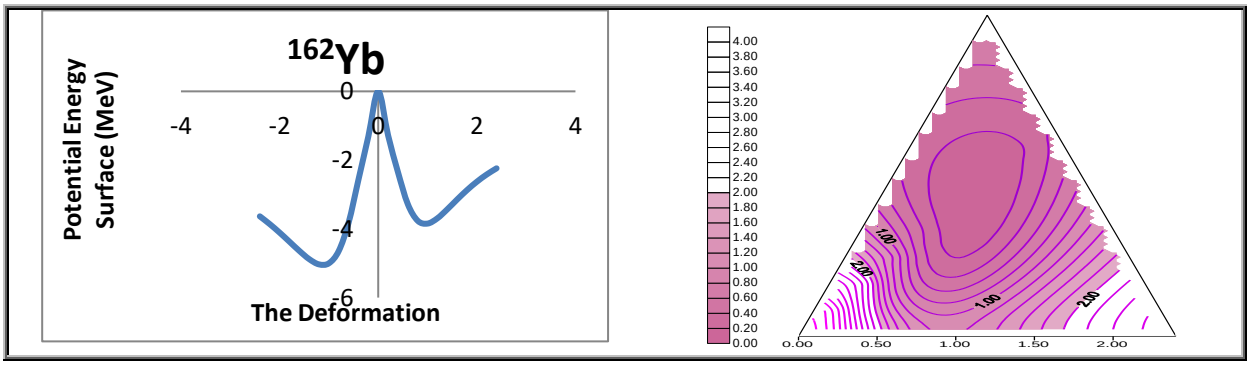


FIGURE 11. Potential energy surface with the deformation for ^{162}Yb.

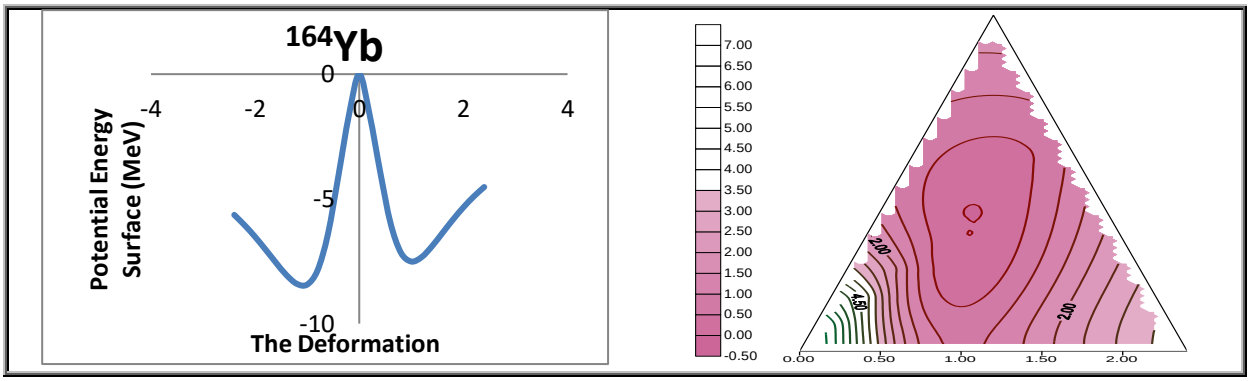


FIGURE 12. Potential energy surface with the deformation for ^{164}Yb.

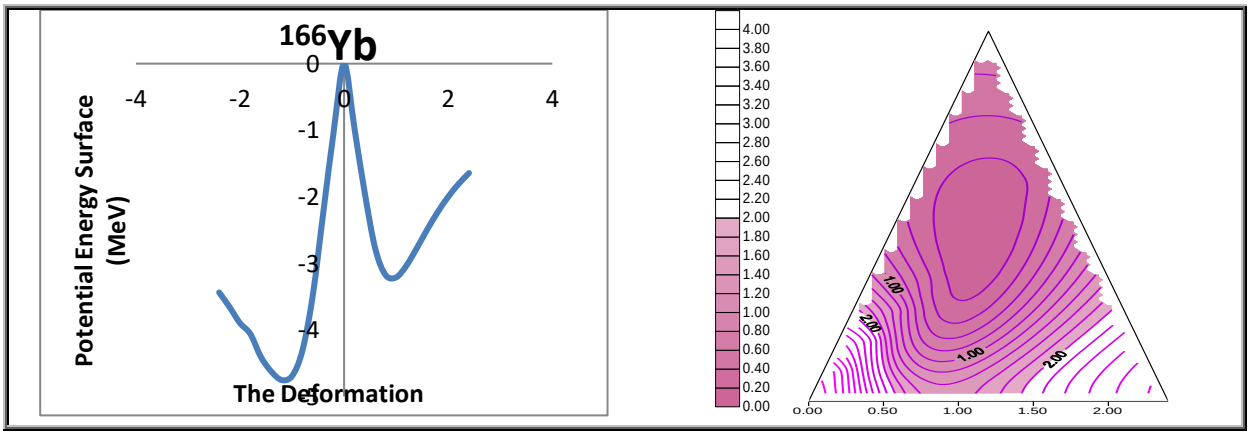


FIGURE 13. Potential energy surface with the deformation for ^{166}Yb.

TABLE 7. The parameters used in the IBMP-code for $^{160\text{-}166}$Yb isotopes.

The Isotopes	N	ES (MeV)	ED (MeV)	A1 (MeV)	A2 (MeV)	A3 (MeV)	A4 (MeV)
^{160}Yb	11	-0.223	0.086	0.004	0.021	-0.178	0.000
^{162}Yb	12	-0.150	0.071	0.003	0.021	-0.120	0.000
^{164}Yb	13	-0.241	0.014	0.011	0.014	-0.193	0.000
^{166}Yb	14	-0.100	0.038	0.002	0.019	-0.080	0.000

CONCLUSIONS

1. Ytterbium isotopes ($^{160-166}$Yb) have dynamical symmetry between SU(3) and O(6), because the ratio values are nearly identical to those in Tablet (1), and when compared to the Casten Triangle, whose Hamiltonian is eq.(5) and the branching ratios for it refer to the SU(3) region due to the close match with typical values.
2. The angular momentum and parity of many high energy levels were confirmed with the experimental data. Unknown levels were been known with positive parity.
3. The branching ratios of the electric transitions are a good exam to know the limit of the isotope and emphasis it.
4. The values of calculated B(E2) are in excellent agreement with the experimental data. Although there are many variations between them, such as the effect of deformation on the nuclei of these isotopes, there are many similarities.
5. All the isotopes of Yb in this study are between O(6) and SU(3) limit except ^{166}Yb it is nearby SU(3) limit, due to the increase in the number of its bosons than the rest of the isotopes, which leads to an increase in the interaction between them and thus an increase in distortion in the nucleus.

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