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Computing the reliability of a complex network using two techniques

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Computing the reliability of a complex network using two techniques

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Abstract. To find the reliability of any complex network, we need to convert it into a simple network to facilitate the process of reliability calculation. In this paper we simplify a complex network into a simple network by using two techniques, in the first technique "reduction method" the complex network was converted into a parallel network, while in the second technique "delta - star method", the complex network has been transformed into a series network. The 2nd technique is based on the 1st technique in finding the reliability of a subnetwork. At the end of the research, a comparison between these two methods was made.

1. Introduction

Reliability has come to be an extra problem in latest years, because advanced technology commercial tactics with ever increasing ranges of class contain maximum engineering structures nowadays [1,8]. So, reliability engineers are referred to as upon to evaluate the reliability of the machine during the layout phase of a product. Reliability concept describes the capacity of a network to complete the task for which it's far responsible at a specific time [2,11]. Network reliability evaluation gets first rate interest for the making plans, effectiveness, and protection of many actual worldwide networks, including computers, communications, electrical circuits, aircraft, linear accelerators, and electricity networks [3, 4]. The devices of a network are subject to random failures, as many groups and institutions become dependent upon networked computing packages. The failure of any component of a network might also additionally right away have an effect on the operation of a community, for that reason the probability of every component of a network is a totally important whilst thinking about the reliability of a network. Hence the reliability consideration is an essential aspect in networked computing [5, 6]. There are number of ways to calculate the reliability of complex network. Such are, for examples, reduction to parallel elements technique and delta-star transformation which are depend on the graph of a network. The above-mentioned methods reduce complex networks and convert them into simple networks for easy reliability calculation, as will be evidenced by our discussion of those methods [7, 8].

In this article, although we refer to our previous work [8-14] and [17], the emphasis is different from other authors'. We use reduction and delta star techniques to determine the reliability of a given network.

The aim of this paper is to reduce the complex network in order to simplify the network to the simplest method, and these strategies help one to deal with a given network as the simplest way to learn certain things that are connected to the work properly or optimize the system, such as reliability allocation, reliability value, redundancy, and so on, as potential work.

2. Minimal path

A path is a chain of lines which connects the beginning node of the network to its end node. A **minimal path** is a path from which no line can be eliminated without disconnecting the link it creates between the start node and the stop node [6,7].

3. Structure function

Consider a network with m components, each component of them could have two feasible states success and failure. The state of component k is given by the binary variable x_k

$$x_k = \begin{cases} 1 & \text{if component } k \text{ success} \\ 0 & \text{if component } k \text{ fails} \end{cases}$$

The state of this network can be described by the binary function $\emptyset(x) = \emptyset(x_1, x_2, \cdots, x_m)$, where

$$\emptyset(x) = \begin{cases} 1 & \text{if the network success} \\ 0 & \text{if the network fails} \end{cases}$$

which is called the structure function of the network [7-10].

The network that is working if and only if all of its m components are success, is called a series structure, and the structure function of it is given by:

$$\emptyset(x) = x_1 x_2 \cdots x_m \tag{1}$$

Assume that R_k is the reliability of a component k. Then the reliability of the network (R_N) with m components in series is

$$R_N = R_1 R_2 \cdots R_m \tag{2}$$

While, the network that is working whenever at least one of its components is success, is called a parallel structure. The structure function of this network is given by:

$$\emptyset(x) = 1 - (1 - x_1)(1 - x_2) \cdots (1 - x_m) \tag{3}$$

So that the reliability of the network with m components in parallel is given by [11,13]:

$$R_N = 1 - (1 - R_1)(1 - R_2) \cdots (1 - R_m) \tag{4}$$

Now we'll use two methods to convert the complex network in figure 1 into a simple network to find the structure function and reliability of it

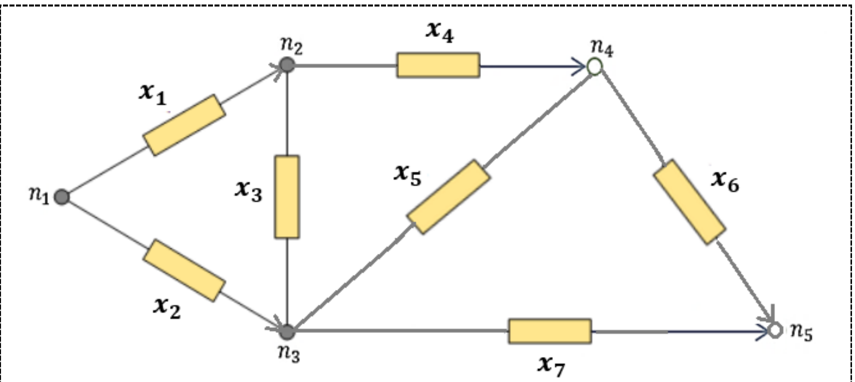


Figure 1. Complex network

4. Reduction to parallel elements Footnotes

In this technique we systematically replace each series minimal path by an equivalent single path, and finally reduce the given network to another with two parallel elements [10-12]. We can convert the complex network to a series-parallel one depending on minimal path sets.

From figure 1 the minimal paths of our complex network are $P_1 = \{x_2 x_7\}$, $P_2 = \{x_1 x_3 x_7\}$, $P_3 = \{x_1 x_4 x_6\}$, $P_4 = \{x_2 x_5 x_6\}$, $P_5 = \{x_2 x_3 x_4 x_6\}$, $P_6 = \{x_1 x_3 x_5 x_6\}$, $P_7 = \{x_1 x_4 x_5 x_7\}$ and $P_8 = \{x_2 x_3 x_4 x_5 x_7\}$. So, our network can be becoming as in figure 2 below

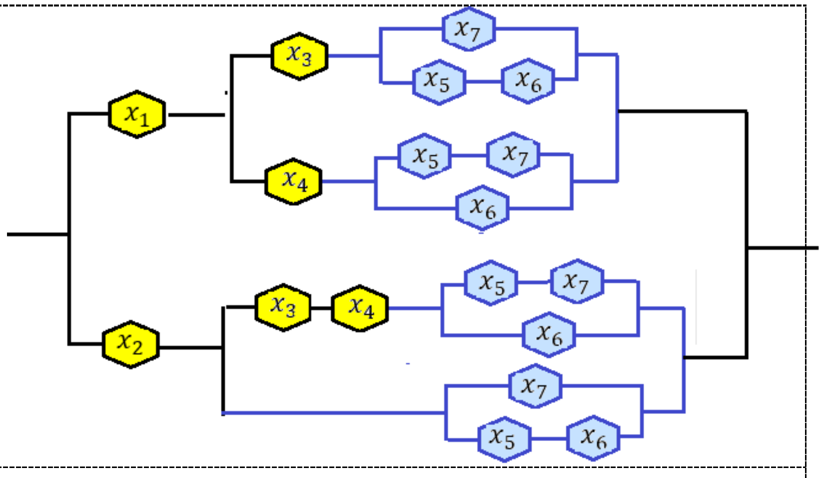
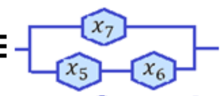
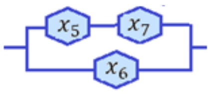


Figure 2. Series- parallel network

By assuming that $x_a \equiv$  and $x_b \equiv$ 

Applying equations (1) and (3) to find the structure functions x_a and x_b.
$x_a = 1-(1-x_7)(1-x_5x_6)$ and $x_b = 1-(1-x_6)(1-x_5x_7)$.
Thus, $R_a = 1-(1-R_7)(1-R_5R_6)$ and $R_b = 1-(1-R_6)(1-R_5R_7)$. Or

$$R_a = R_7 + R_5R_6 - R_5R_6R_7 \quad (5)$$
$$R_b = R_6 + R_5R_7 - R_5R_6R_7 \quad (6)$$

The complex network becomes simple as in figure 3.

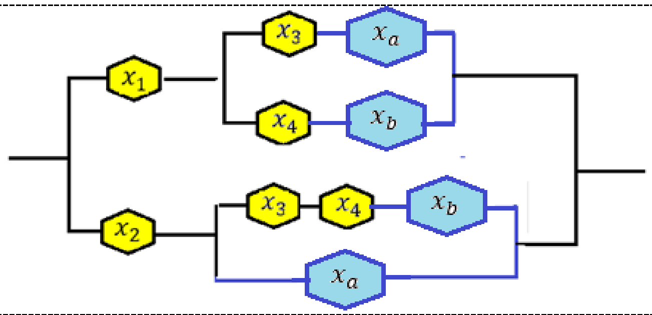


Figure 3. Simple series- parallel network

By apply equations (1) and (3) for the network in figure 3 to get
$x_{r1} = 1-(1-x_3x_a)(1-x_4x_b)$ and $x_{r2} = 1-(1-x_a)(1-x_3x_4x_b)$
So, $R_{r1} = 1-(1-R_3R_a)(1-R_4R_b)$ and $R_{r2} = 1-(1-R_a)(1-R_3R_4R_b)$.Then we get

$$R_{r1} = R_3R_a + R_4R_b - R_3R_4R_aR_b \quad (7)$$
$$R_{r2} = R_a + R_3R_4R_b - R_3R_4R_aR_b \quad (8)$$

The network becomes simpler as in figure 4.

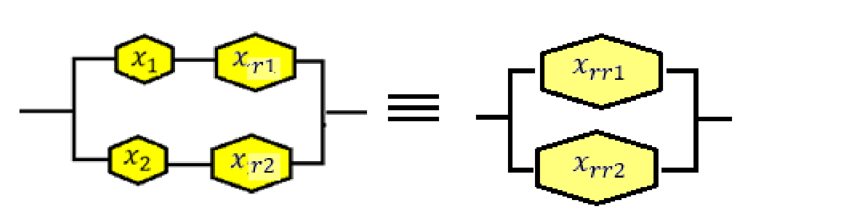


Figure 4. The simpler network

By apply equation (1) on the left diagram of figure 4 we find $x_{rr1} = x_1 x_{r1}$ and $x_{rr2} = x_2 x_{r2}$ which are implies to $R_{rr1} = R_1 R_{r1}$ and $R_{rr2} = R_2 R_{r2}$.
Then from equations (7) and (8) we get:

$$R_{rr1} = R_1 R_3 R_a + R_1 R_4 R_b - R_1 R_4 R_b R_3 R_a \tag{9}$$
$$R_{rr2} = R_2 R_a + R_2 R_3 R_4 R_b - R_2 R_a R_3 R_4 R_b \tag{10}$$

Apply equation (3) on the right diagram of figure 4 to get the structure function of our network:
$\emptyset(x) = 1 - (1 - x_{rr1})(1 - x_{rr2})$ which is means

$$\emptyset(x) = x_{rr1} + x_{rr2} - x_{rr1} x_{rr2} \tag{11}$$

So, the reliability of our complex network R_N is given by:

$$R_N = R_{rr1} + R_{rr2} - R_{rr1} R_{rr2} \tag{12}$$

5. Delta-star method

To transform a delta subnetwork to an equivalent star subnetwork we want to derive a metamorphosis method for equating the different components to each different between the diverse terminals [7,15]. Consider figure 5 below.

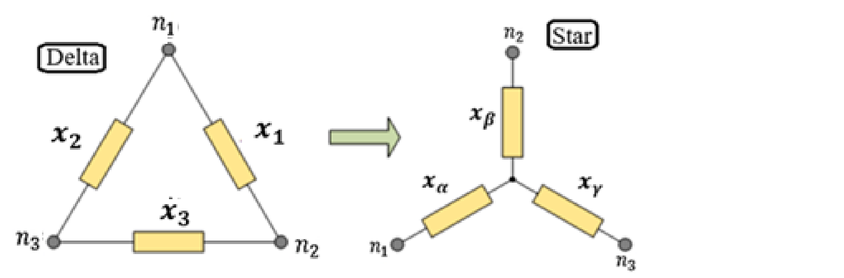


Figure 5. Delta to Star transformation

Compare the three components of delta with three components of star, then applying the equations (1) and (3) to get three relations:

$$x_\beta x_\gamma = 1 - (1 - x_1)(1 - x_2 x_3) \tag{13}$$
$$x_\beta x_\alpha = 1 - (1 - x_2)(1 - x_1 x_3) \tag{14}$$
$$x_\gamma x_\alpha = 1 - (1 - x_3)(1 - x_1 x_2) \tag{15}$$

Assume that $u = 1 - (1 - x_1)(1 - x_2 x_3)$, $v = 1 - (1 - x_2)(1 - x_1 x_3)$ and $w = 1 - (1 - x_3)(1 - x_1 x_2)$
Now, solving three equations above to get delta-star relationships

$$x_\beta = (uv/w)^{1/2} \quad , \quad x_\gamma = (uw/v)^{1/2} \quad \text{and} \quad x_\alpha = (vw/u)^{1/2} \tag{16}$$

Thus, our complex network in figure 1 becomes a simple network as in the figure 6

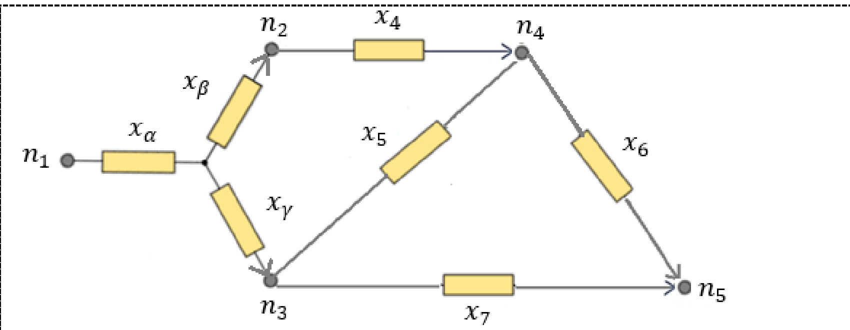


Figure 6. Simple network

Now applying the reduction method on the subnetwork resulting from deletion x_α from figure 6. The minimal paths of this subnetwork are $P_1 = \{x_\beta x_4 x_6\}, P_2 = \{x_\beta x_4 x_5 x_7\}, P_3 = \{x_\gamma x_7\}$, and $P_4 = \{x_\gamma x_5 x_6\}$, so that this subnetwork converts to a series- parallel subnetwork and can drown as in figure 7

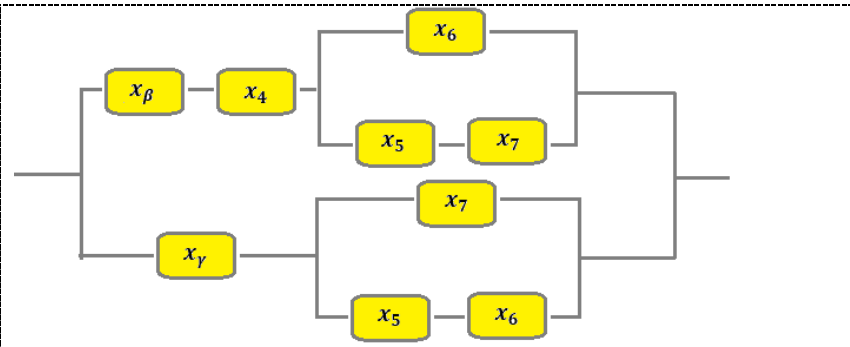
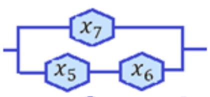
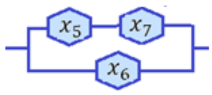


Figure 7. Simple series- parallel subnetwork

Note that in 1st method we assuming that $x_a \equiv$ 

and $x_b \equiv$ 

Then R_a and R_b as in equations (5) and (6) respectively and the subnetwork can drown as in figure 8

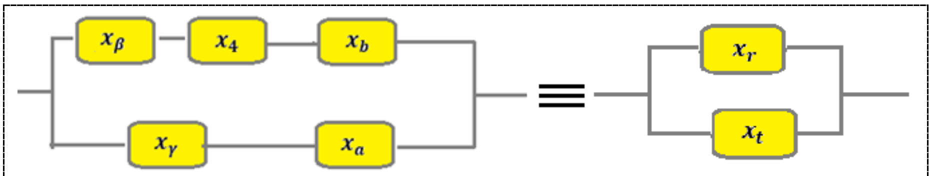


Figure 8. The simpler subnetwork

Now if we apply equation (1) to simplify the network on the left of figure (8), we will get an equivalent network as on the right of the figure above and we have $x_r = x_\beta x_4 x_b$ and $x_t = x_\gamma x_a$.
So, we obtain the structure function by apply the equation (3) on the network on the right as following $x_{rt} = 1 - (1 - x_r)(1 - x_t)$ which in turn leads to:

$$x_{rt} = x_r + x_t - x_t x_r \tag{17}$$

So, the reliability of simpler subnetwork is

$$R_{rt} = R_r + R_t - R_t R_r \tag{18}$$

where $R_r = R_\beta R_4 R_b$, $R_t = R_\gamma R_\alpha$, $\quad R_\beta = \left(\frac{(R_1 + R_2R_3 - R_1R_2R_3)(R_2 + R_1R_3 - R_1R_2R_3)}{(R_3 + R_1R_2 - R_1R_2R_3)}\right)^{1/2}$

and $R_\gamma = \left(\frac{(R_1 + R_2R_3 - R_1R_2R_3)(R_3 + R_1R_2 - R_1R_2R_3)}{(R_2 + R_1R_3 - R_1R_2R_3)}\right)^{1/2}$

And finally, our network becomes  in series so that the structure function and the reliability are given by

$$\emptyset(x) = x_\alpha x_{rt} \tag{19}$$
$$R_N = R_\alpha R_{rt} \tag{20}$$

6. Comparison between two techniques

In this section we are able to do a little computations with a purpose to make a comparison between these methods and discover which is better with the aid of substituting random values in equations (12) and (20) for each method, in addition to compensating for comparable values for all components for the identical reason, as follows

Case 1

If all components have the same reliability values such as 0.7, (i.e., $R_i = 0.7 \quad \forall\, i = 1,2,\cdots,7$)

Case 2

If we take some random values for reliability to check which techniques closed to optimal value of reliability network $(\text{i.e., let } R_1 = 0.9\,, R_2 = 0.85, R_3 = 0.7, R_4 = 0.75, R_5 = 0.8\,,$ $R_6 = 0.95, \text{and } R_7 = 0.8)$

Case 3

If R_1, R_2 and R_3 are of equal values and the other are random values$(\text{i.e., let } R_1 = R_2 = R_3 = 0.7,$ $R_4 = 0.75, R_5 = 0.8\,, R_6 = 0.9, \text{and } R_7 = 0.95).$

The results we obtained are listed in the table below

Table 1. Summary table for comparison between two methods with three cases.

Methods	R_N **Case 1**	R_N **Case 2**	R_N **Case 3**
Reduction to parallel elements	**0.85**	**0.97**	**0.89**
Delta- star	0.83	0.94	**0.89**

7. Discuss the results

From table 1, we are able to see that the very best fee of reliability of complex network in figure 1 is 0.97 in (case 2) also, the best value of reliability of complex network is 0.85 in (case 1), while the two values are equal in (case 3). So, the best value of reliability of complex network is by using “reduction to parallel elements method”.

8. Conclusions

In 1st technique “reduction to parallel elements”, we reduced all parallel minimal paths to make the given network easier, that is the complex network has been transformed into parallel form and we noticed that the reliability was calculated simply. While in the 2nd technique “delta-star”, the complex network has been transformed into series form, the network simplified techniques have been applied to calculate its reliability. We saw the ease of dealing with complex networks by using these two methods. On the other hand, in the first and second cases, the reliability value is better by using 1st technique “reduction to parallel elements”, while in the third case, the reliability value is the same in both techniques.

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