

Solving Bi-objective Reliability Optimization Problem of Mixed System by Firefly Algorithm

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Abstract— The main objective of reliability optimization of a particular system, such as a mixed or complex system, is to determine the best way to obtain an increase in the final reliability of this system by allocating high reliability to the important components of the system. In this paper, the problem of optimization reliability of the mixed system is studied using the firefly algorithm based on two cost functions, which are the logarithmic function and the exponential function. Some modifications have been made to these two functions to suit the reliability system under study, where very good results have been obtained for the overall reliability of the mixed system at a reasonable cost. Also, it was found that the overall system reliability is directly proportional to the total cost.

Keywords— reliability, optimization, mixed system, Firefly Algorithm, allocation.

I. INTRODUCTION

Most industrial products consist of several units or components. According to the concept of reliability, an element is any component that does not decompose into simpler objects [1]-[3]. The item can be a light bulb, two electrical components contact point, or a piston in an engine. When connecting several elements, the so-called system is formed, and the system is simple (series, parallel and mixed) or complex, depending on how its elements are connected. For example, the electrical system consisting of a lamp, a socket, a switch, and wires is a simple system [4], [5]. A spacecraft system, which contains more than ten thousand mechanical, hydraulic, or electrical components, is a complex system. The ultimate system reliability depends on the reliability of its elements, their number and how they are connected [2], [6], [7]. A proper arrangement can increase system reliability. The reliability function of any system can take a variety of different forms, depending on the system structures and the interrelationship of its components [6], [8].

In addition to the problems of developing modern and advanced technology, reliability constituted another problem until the optimization reliability problem for mixed or complex systems became one of the modern problems, which has evolved with the evolution of solutions methods coinciding with the evolution of new technology [2], [9]. The task of optimization reliability of a particular system requires the development of perceptions of variable options and

restrictive conditions and requires high capabilities in order to describe the overall performance of engineering designs for a given product, then to experiment with the overall values and their suitability for the options, so that the best targets can be identified. Any issue related to the design of a particular product has its own planning objectives and may need to be changed according to the requirements and conditions. Reliability has become a major concern for producers in recent years due to high-tech industrial processes with the rapid development of most engineering systems [10], [11].

For most designers of a reliable system, it is always a good idea to simultaneously optimize several conflicting design goals such as reliability and cost because the problem of system design deserves a lot of attention. The goal of the reliability optimization problem is to maximize system reliability estimates and minimize cost.

II. RELIABILITY OF MIXED SYSTEM

A mixed system is any system consisting some of sub-systems whose components are connected in series, parallel, sires – parallel, or parallel - sires [12]-[15], as the system evident in Fig. 1, which is the focus of our study.

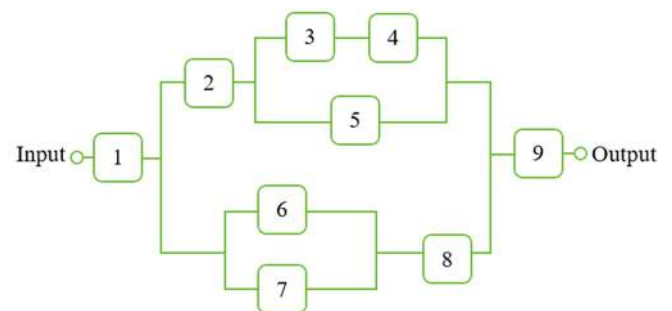


Fig. 1. Mixed System

If we consider that R_i the reliability of i – th component for $i = 1, 2, \dots, 9$, such that $0 \leq R_i \leq 1$. Then the total reliability of the system $R_S \in [0, 1]$ is given by:

$$R_S = R_1 R_2 R_5 R_9 + R_1 R_6 R_8 R_9 + R_1 R_7 R_8 R_9 + R_1 R_2 R_3 R_4 R_9 \\ + R_1 R_2 R_5 R_6 R_7 R_8 R_9 + R_1 R_2 R_3 R_4 R_5 R_6 R_8 R_9 \\ + R_1 R_2 R_3 R_4 R_5 R_7 R_8 R_9 + R_1 R_2 R_3 R_4 R_6 R_7 R_8 R_9 \\ - R_1 R_6 R_7 R_8 R_9 - R_1 R_2 R_3 R_4 R_5 R_9 - R_1 R_2 R_5 R_6 R_8 R_9$$

$$\begin{aligned}
 & -R_1R_2R_5R_7R_8R_9 - R_1R_2R_3R_4R_6R_8R_9 \\
 & -R_1R_2R_3R_4R_7R_8R_9 - R_1R_2R_3R_4R_5R_6R_7R_8R_9 \quad (1)
 \end{aligned}$$

We will find the best value for the system under study using the polynomial in equation (1) after the reliability of all components of the system is allocated by improving the reliability by firefly algorithm [2], [16], [17].

III. FIREFLY ALGORITHM

Algorithms inspired by nature are among the most powerful algorithms used in optimization. The firefly algorithm based on swarm intelligence has proven effective in solving nonlinear optimization problems, especially multi-objective problems, where the optimization problem can have many maxima and minima [18], [19].

Firefly algorithm denoted by (FA) is a metaheuristic algorithm inspired by the blinking conduct of fireflies proposed by Xin-She Yang in 2007 [12], [13], [20]. To describe the firefly algorithm, Xin-She Yang relied on three basic rules:

- Every fireflies flash so that they are attracted to one another that may be of different gender or identical.
- The attraction increases with the brightness of the flash, so the least brightness fireflies ought to proseed towards the brighter, the gravity and the brightness are inversely proportional to the distance between the fireflies. If there is no firefly brighter than a certain firefly, the movement is random.
- The brightness of the firefly changes according to the existing scenery in nature.

Xin-She Yang concluded that the motion of the firefly i that is attracted to the firefly j can be formulated by:

$$p_i = p_i + \beta_0 e^{-\gamma r_{ij}^2} (p_j - p_i) + \alpha \epsilon_i \quad (2)$$

where p_i and p_j represented to the position of i -th firefly and j -th firefly respectively, r_{ij} is the distance between i -th firefly and j -th firefly, β_0 represents to attractiveness when $r_{ij} = 0$, γ is a fixed light absorption coefficient, $\alpha \in (0,1)$ is the randomization parameter and ϵ_i is a vector of random numbers pulled from a Gaussian distribution [11]-[14].

The flow chart of FA algorithm shown in Fig. 2.

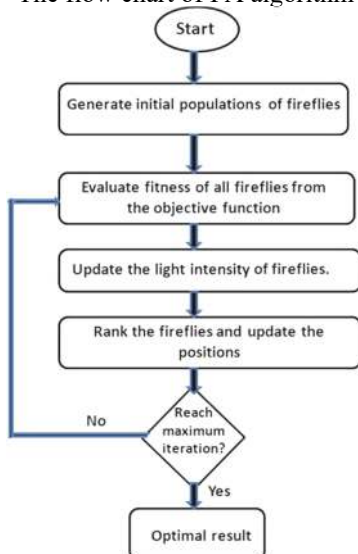


Fig. 2. Flow Chart of Firefly Algorithm

IV. BI-OBJECTIVE OPTIMIZATION PROBLEM

The bi-objective optimization problem can solve optimization problems for different objectives. The bi-objective optimization problem usually arises when the decision maker chooses two competing goals to satisfy them together.

The bi-objective optimization problem can be expressed as:

$$\begin{aligned}
 & \text{Minimize } (F_1(x), F_2(x)) \\
 & \text{Subject to } x = (x_1, x_2, \dots, x_n) \quad (3)
 \end{aligned}$$

The purpose of bi-objective optimization is to reduce the two objectives simultaneously, and if one of the two objectives is to be maximized, then we must precede it with a negative sign [21]-[24].

Here we are interested in maximizing the reliability of the system R_S and minimizing its cost C_S , so we will convert two cost functions CF_1 and CF_2 that depend on the reliability of the system components whose are given by [20], [25]:

$$CF_1: C_i(R_i) = R_i \left(\log \frac{R_i}{1 - R_i} \right) \quad (4)$$

$$CF_2: C_i(R_i) = R_i (R_i + e^{R_i/4}) \quad (5)$$

where C_i represent to the cost of i -th component.

Let C_S be the total cost of the system with n components. Then:

$$C_S = \sum_{i=1}^n C_i(R_i) \quad (6)$$

V. USE FA TO REACH THE BEST SOLUTION

It is natural that the reliability of any product device is equal to one when it is manufactured, but it begins to decrease gradually with the increase in operating time and this results from the decreasing reliability of the device's components over time [3], [26]-[29]. For this reason and to be close for reality, we required the reliability of each component to be within the range $0.45 \leq R_i \leq 0.99$.

For optimize our problem in equation (1) we formulate it as follows:

$$\text{Minimize } (C_S(R_i), -R_S(R_i))$$

$$\text{Subject to: } 0.45 \leq R_i \leq 0.99$$

$$R_S \geq R_G;$$

$$0 < C_i \leq 4$$

$$C_S \leq C_G$$

We chose the system reliability goal $R_G = 0.9$ and total cost target $C_G = 25$.

To find out the best reliability of a mixed system, we used FA by 100 from iterations with the aforementioned two cost functions [30]-[32]. The results of R_i were rounded to three decimal places. All results of FA algorithm with CF_1 and CF_2 were recorded in Table I.

TABLE I. VALUES OF AND BY FA

i	CF_1		CF_2	
	R_i	C_i	R_i	C_i

1	0.982	3.927	0.976	2.198
2	0.953	2.868	0.949	2.104
3	0.865	1.607	0.870	1.838
4	0.812	1.188	0.743	1.447
5	0.930	2.406	0.930	2.038
6	0.948	2.752	0.938	2.066
7	0.898	1.953	0.905	1.954
8	0.915	2.174	0.920	2.004
9	0.968	3.300	0.961	2.146
$R_S C_S$	0.945	22.175	0.932	17.795

VI. DISCUSS THE RESULTS

Through the values of table I, we can record the following observations:

- (1) Best system reliability $R_S > 0.93 > R_G$ by using FA with two cost functions which is a very acceptable value.
- (2) Best total cost $C_S < 22.2 < C_G$ by using FA with two cost functions.
- (3) $R_S = 0.945$ when use FA with CF_1 , while $R_S = 0.932$ when use FA with CF_2 , which is meaning that the system reliability R_S is the best by CF_1 .
- (4) $C_S = 22.175$ when use FA with CF_1 , while $C_S = 17.795$ when use FA with CF_2 , which is meaning that the total cost C_S is the best by CF_2 .
- (5) $R_1 > R_9 > R_2 > R_6 > R_5 > R_8 > R_7 > R_3 > R_4$ when use FA with two cost functions. From this observation, we can put a classification of the importance of the components of the system, where the first and ninth components were at the first level, while the second and sixth components were at the second level, and the third level was for the fifth, seventh and eighth components, while the third and fourth components were at the last level and we see this clearly in Figure (2).

From observations (1) to (4), we conclude that the cost increases with the reliability of the system.

From observation (5), a classification can be made for the importance of the components of the system into four following levels:

- 1. First level: the first and ninth components are located in the first level of importance.
- 2. Second level: the second and sixth components are in the second level
- 3. Third level: is for the fifth, seventh and eighth components.
- 4. Fourth level: the third and fourth components fall in the last level of importance.

The bar chart in Fig.3 shown clearly the levels of importance for all components of the system.

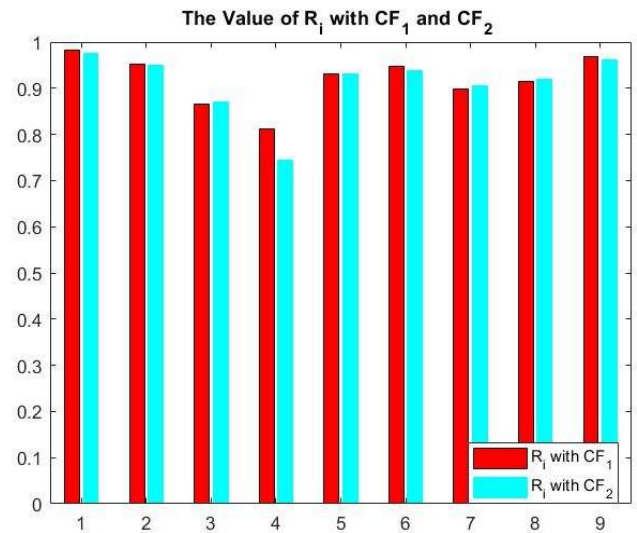


Fig. 3. The Reliability of all Components

II. CONCLUSIONS

Despite the difference in the algorithm results, according to the two costs used, it can be depending on it for solve the problem of optimization reliability for mixed system. The results of the algorithm showed that the best value for system reliability and the highest cost was when using a logarithmic function, while the exponential function gave less reliability and a lower cost. From the above, we conclude that system reliability is directly proportional to cost.

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